

BY
SIR JOHN F. W. HERSCHEL

PART ONE



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S C I E N C E

SIR JOHN HERSCHEL

SIR JOHN FREDERICK WILLIAM HERSCHEL, BART., the illustrious astronomer, was the only son of Sir F. William Herschel. He was born at Slough, Buckinghamshire, England, in the year 1792. His scholastic education began at Eton, whence, at the age of seventeen, he was sent to St. John's College, Cambridge, where he attained to great eminence in mathematics. After leaving the University, he entered upon the study of astronomy, and in 1820, assisted by his father, completed a mirror of 18 inches diameter and 20 feet focal length for a reflecting telescope. This, as subsequently improved, became the instrument which enabled him to effect the astronomical observations that formed the basis of his fame. In 1821-23 he undertook the re-examination of his father's double stars. For this work he received in 1826 the gold medal of the Astronomical Society, and the Lalande Medal of the French Institute. From 1824 to 1827 he was Secretary of the Royal Society, and in 1827 was elected to the Chair of the Astronomical Society, which office he filled on two subsequent occasions. In 1831 the honor of knighthood was conferred on him by William IV., and he subsequently

was made a baronet. His exploration of the southern heavens constituted an epoch in astronomy, and secured for him the distinction of memberships in almost every important society in both hemispheres. His "Outlines of Astronomy," here reproduced, first appeared in 1849, and, notwithstanding the disadvantage arising from the practice of stereotyping text-books which relate to progressive sciences, there is no more instructive volume extant on the subject of which it treats. Sir John Herschel died in 1871; his remains are interred in Westminster Abbey, close to the grave of Sir Isaac Newton.

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TO
THE REV. W. RUTTER DAWES, F.R.A.S.,
&c. &c. &c.-

MY DEAR MR. DAWES—In availing myself of your permission to dedicate to you an Edition of these Outlines, enriched by accounts of several of your own recent discoveries, I should ill acquit myself to my own feelings if I did not add to the expression of that grateful sense of your many and important services to our common Science, which every astronomer must acknowledge, that of affectionate esteem and regard, the natural result of a prolonged and most friendly intercourse.

Believe me,

Very truly yours,

J. F. W. HERSCHEL.

COLLINGWOOD, *Feb. 15, 1858.*

PREFACE TO THE FIRST EDITION

THE work here offered to the Public is based upon and may be considered as an extension, and, it is hoped, an improvement of a treatise on the same subject, forming Part 43 of the Cabinet Cyclopædia, published in the year 1833. Its object and general character are sufficiently stated in the introductory chapter of that volume, here reprinted with little alteration; but an opportunity having been afforded me by the Proprietors, preparatory to its reappearance in a form of more pretension, I have gladly availed myself of it, not only to correct some errors which, to my regret, subsisted in the former volume, but to remodel it altogether (though in complete accordance with its original design as *a work of explanation*); to introduce much new matter in the earlier portions of it; to rewrite, upon a far more matured and comprehensive plan, the part relating to the lunar and planetary perturbations, and to bring the subjects of sidereal and nebular astronomy to the level of the present state of our knowledge in those departments.

The chief novelty in the volume, as it now stands, will be found in the manner in which the subject of Perturbations is treated. It is not—it cannot be made *elementary*, in the sense in which that word is understood in these days of light reading. The chapters devoted to it must, therefore, be considered as addressed to a class of readers in possession of somewhat more mathematical knowledge than

those who will find the rest of the work readily and easily accessible; to readers desirous of preparing themselves, by the possession of a sort of *carte du pays*, for a campaign in the most difficult, but at the same time the most attractive and the most remunerative of all the applications of modern geometry. More especially they may be considered as addressed to students in that university, where the "Principia" of Newton is not, nor ever will be, put aside as an obsolete book, behind the age; and where the grand though rude outlines of the lunar theory, as delivered in the eleventh section of that immortal work, are studied less for the sake of the theory itself, than for the spirit of far-reaching thought, superior to and disencumbered of technical aids, which distinguishes that beyond any other production of the human intellect.

In delivering a rational as distinguished from a technical exposition of this subject, however, the course pursued by Newton in the section of the "Principia" alluded to, has by no means been servilely followed. As regards the perturbations of the nodes and inclinations, indeed, nothing equally luminous can ever be substituted for his explanation. But as respects the other disturbances, the point of view chosen by Newton has been abandoned for another, which it is somewhat difficult to perceive why he did not, himself, select. By a different resolution of the disturbing forces from that adopted by him, and by the aid of a few obvious conclusions from the laws of elliptic motion which would have found their place, naturally and consecutively, as corollaries of the seventeenth proposition of his first book (a proposition which seems almost to have been prepared with a special view to this application), the momentary change of place of the upper focus of the disturbed ellipse

is brought distinctly under inspection; and a clearness of conception introduced into the perturbations of the eccentricities, perihelia, and epochs which the author does not think it presumption to believe can be obtained by no other method, and which certainly is not obtained by that from which it is a departure. It would be out of keeping with the rest of the work to have introduced into this part of it any algebraic investigations; else it would have been easy to show that the mode of procedure here followed leads direct, and by steps (for the subject) of the most elementary character, to the general formulæ for these perturbations, delivered by Laplace in the *Mécanique Céleste*.¹

The reader will find one class of the lunar and planetary inequalities handled in a very different manner from that in which their explanation is usually presented. It comprehends those which are characterized as incident on the epoch, the principal among them being the annual and secular equations of the moon, and that very delicate and obscure part of the perturbational theory (so little satisfactory in the manner in which it emerges from the analytical treatment of the subject), the constant or permanent effect of the disturbing force in altering the disturbed orbit. I will venture to hope that what is here stated will tend to remove some rather generally diffused misapprehensions as to the true bearings of Newton's explanation of the annual equation.²

If proof were wanted of the inexhaustible fertility of astronomical science in points of novelty and interest, it would suffice to adduce the addition to the list of members

¹ Livre ii. chap. viii. art. 67.

² Principia, lib. i. prop. 66, cor. 6.

of our system of no less than eight new planets and satellites during the preparation of these sheets for the press. Among them is one whose discovery must ever be regarded as one of the noblest triumphs of theory. In the account here given of this discovery, I trust to have expressed myself with complete impartiality; and in the exposition of the perturbative action on Uranus, by which the existence and situation of the disturbing planet became revealed to us, I have endeavored, in pursuance of the general plan of this work, rather to exhibit a rational view of the dynamical action, than to convey the slightest idea of the conduct of those masterpieces of analytical skill which the researches of Messrs. Leverrier and Adams exhibit.

To the latter of these eminent geometers, as well as to my excellent and esteemed friend the Astronomer Royal, I have to return my best thanks for communications which would have effectually relieved some doubts I at one period entertained had I not succeeded in the interim in getting clear of them as to the *compatibility* of my views on the subject of the annual equation already alluded to with the tenor of Newton's account of it. To my valued friend, Professor De Morgan, I am indebted for some most ingenious suggestions on the subject of the mistakes committed in the early working of the Julian reformation of the calendar, of which I should have availed myself, had it not appeared preferable, on mature consideration, to present the subject in its simplest form, avoiding altogether entering into minutiae of chronological discussion.

J. F. W. HERSCHEL.

COLLINGWOOD, April 12, 1849.

PREFACE TO THE FIFTH EDITION

THE rapid progress of science renders it necessary frequently to revise and bring up elementary works to the existing state of knowledge, under penalty of their becoming obsolete. In former editions of this work, this has been done, so far as it could be done without incurring the necessity of an almost total typographical reconstruction. But Astronomy, within the last few years, has been enriched by so many and such considerable additions, that it has been considered preferable (another edition being called for), not indeed to recast the general plan of the work, but to incorporate these in it in due order and sequence, thereby materially enlarging the volume, and giving it in many respects the air of a new work. The articles thus introduced are distinguished from those of the former editions between which they have been inserted by the addition to the last current number of an *italic* letter—thus, between Arts. 394 and 395 will be found inserted 394 *a*, 394 *b*, and 394 *c*. The inclosure of any passage in brackets [] indicates its having been introduced in the Fourth or some subsequent Edition. The index references in this as in former editions, being to the articles and not to the pages, are thus preserved. Together with these recent accessions to our knowledge, I have taken the opportunity of introducing several things which might justly have been noted as deficient in the former editions—as, for instance, the account of the methods by which the mass of the Earth has been determined,

and that of the successful treatment, and it is presumed final subjugation, of those rebellious ancient Solar Eclipses which have so much harassed astronomers. A brief account of M. Foucault's remarkable pendulum experiments, and of that beautiful instrument, the gyroscope, is introduced: as are also notices of Professor Thomson's speculations on the origin of the Sun's heat, and his estimate of its average expenditure, as well as of some curious views of M. Jean Reynaud, on the secular variation of our climates, supplementary to those put forward in former editions of this work. I could have wished that its nature and limits would have permitted some account of Mr. Cooper's magnificent contributions to sidereal astronomy, in his catalogue of upward of 60,000 previously unregistered ecliptic stars; of Mr. Bishop's ecliptic charts and those of M. Chacornac; of Mr. Carrington's elaborate circumpolar catalogue; and of Mr. Jones's immense work on the zodiacal light, forming the third volume of the account of the United States' Japan Expedition, which reached me too late to allow of drawing up a fitting analysis of his results. These gentlemen will severally please to accept, however, this respectful tribute of my admiration for their important labors. Some new speculations are also hazarded; as, for instance, on the subject of the Moon's habitability, the cause of the acceleration of Encke's comet, etc., and a few numerical errors are corrected which have hitherto escaped notice and public comment as blemishes—as, for example, in some of the numbers in Art. 422 in the explanation of the phenomena of a lunar eclipse, in the evaluation of the total mass of the atmosphere, Art. 242, and in the distance of the Moon, Art. 403 (for which, however, I am not answerable).

In the numerical statement of special astronomical elements, it is unavoidable that slightly different values of the same quantity should from time to time come to be substituted for those before received, as its determination acquires additional exactness. To have altered the figures in such cases wherever they occur, throughout the letter-press, would have entailed a great probability of error and confusion; and, as the Synoptic Tables of astronomical elements at the end of the work have been carefully revised in conformity with the best current authorities, the reader is requested, whenever he may observe any discrepancy of this nature, to prefer the tabulated values.

Several of the woodcuts, which were originally drawn correctly, have been inverted right hand for left by the engraver. So far as explanation goes, this is not of the slightest moment. To a reader in the Southern Hemisphere, they are right as they stand; and one in the Northern has only to imagine himself so situated.

JOHN F. W. HERSCHEL.

COLLINGWOOD, *February 17, 1858.*

NOTE (added in 1865). The views of M. Reynaud, above referred to, though no doubt original on his part, are, however, completely anticipated by certain speculations of Sir Charles Lyell, in his "Principles of Geology" (p. 110, Ed. of 1830), expanded and further reasoned out to conclusions identical with those of M. Reynaud, in a paper by the author of the present work, "On the Astronomical Causes which may influence Geological Phenomena," read before the Geological Society on December 15, 1830, and published in the Transactions of that Society. This, however, had completely escaped his recollection when perusing the works of M. Reynaud referred to in the note to Art. 369 c, which occurring while the additions to the Fifth Edition were in preparation, with all the force of novelty, led to their insertion in Art. 369 b, and to the above notice of them in the preface.

PREFACE TO THE TENTH EDITION

SINCE the publication of the last edition of this work, the progress of astronomical discovery has been continuous and rapid; to keep up with which, so far as consistent with its general object, it has been necessary to extend some of the Notes added in former editions and to annex others. Among these recent additions to astronomical knowledge may be mentioned more especially those which relate to the physical constitution of the central body of our own system, on which so vast and unexpected an amount of information has accumulated from various quarters as to place us at the opening out of a new vista of solar discovery. Within the limits of our own system, too, the meteorites have at length acquired a firm footing as a distinct group of members, while their now established and most unexpected connection with periodic comets in some instances, presents the latter class of bodies under a new aspect, increasing, if anything can increase, the mysterious interest which already hangs about them. The group of asteroids, too, has received an accession of no less than fifteen to their list as it stood at the end of 1866. The elements of these are given in the Appendix (from the Berlin "Ephemeris" for 1871), so as to complete the list up to the present time. The correction indicated in a former edition, as required in the distance of the sun (and by necessary implication in the dimensions of all the planetary orbits and the magnitudes of the sun and planets themselves), has moreover received

an unexpected confirmation from Mr. Stone's critical re-examination of the calculations of that distance from the recorded observations of the celebrated transit of Venus in 1769, the result of which is here brought under the notice of our readers.

Beyond the limits of our own system, the application of spectrum analysis has disclosed the amazing fact of the gaseous constitution of many of the *nebulæ* (a constitution long suspected on general cosmological grounds): by which is meant their analogy to matter not in that state of luminosity presented by the photosphere of our sun, but in that in which are maintained the luminous prominences and appendages which fringe its disk in total eclipses, a state to which the epithet of *incandescent-gaseous* as distinct from *flame* or a state of *combustion* seems most appropriate. Thus a real line of demarcation between *nebulæ* proper and sidereal clusters is decisively drawn. These delicate applications of optical science to chemistry have also afforded strong grounds for concluding with considerable confidence the presence of several of our terrestrial elementary bodies in the sun and fixed stars: while in one instance prismatic observation has afforded, with a certain approach to probability, evidence of the recess of the most conspicuous among them from our system, and even supplied a measure of its rapidity.

J. F. W. HERSCHEL.

COLLINGWOOD, *July 17, 1869.*

OUTLINES OF ASTRONOMY

INTRODUCTION

(1.) EVERY student who enters upon a scientific pursuit, especially if at a somewhat advanced period of life, will find not only that he has much to learn, but much also to unlearn. Familiar objects and events are far from presenting themselves to our senses in that aspect and with those connections under which science requires them to be viewed, and which constitute their rational explanation. There is, therefore, every reason to expect that those objects and relations which, taken together, constitute the subject he is about to enter upon will have been previously apprehended by him, at least imperfectly, because much has hitherto escaped his notice which is essential to its right understanding: and not only so, but too often also erroneously, owing to mistaken analogies, and the general prevalence of vulgar errors. As a first preparation, therefore, for the course he is about to commence, he must loosen his hold on all crude and hastily adopted notions, and must strengthen himself, by something of an effort and a resolve, for the unprejudiced admission of any conclusion which shall appear to be supported by careful observation and logical argument, even should it prove of a nature adverse to notions he may have previously formed for himself, or taken up, without examination, on the credit of others.

Such an effort is, in fact, a commencement of that intellectual discipline which forms one of the most important ends of all science. It is the first movement of approach toward that state of mental purity which alone can fit us for a full and steady perception of moral beauty as well as physical adaptation. It is the "euphrasy and rue" with which we must "purge our sight" before we can receive and contemplate as they are the lineaments of truth and nature.

(2.) There is no science which, more than astronomy, stands in need of such a preparation, or draws more largely on that intellectual liberality which is ready to adopt whatever is demonstrated, or concede whatever is rendered highly probable, however new and uncommon the points of view may be in which objects the most familiar may thereby become placed. Almost all its conclusions stand in open and striking contradiction with those of superficial and vulgar observation, and with what appears to every one, until he has understood and weighed the proofs to the contrary, the most positive evidence of his senses. Thus, the earth on which he stands, and which has served for ages as the unshaken foundation of the firmest structures, either of art or nature, is divested by the astronomer of its attribute of fixity, and conceived by him as turning swiftly on its centre, and at the same time moving onward through space with great rapidity. The sun and the moon, which appear to untaught eyes round bodies of no very considerable size, become enlarged in his imagination into vast globes—the one approaching in magnitude to the earth itself, the other immensely surpassing it. The planets, which appear only as stars somewhat brighter than the rest, are to him spacious, elaborate, and habitable worlds; several of them

much greater and far more curiously furnished than the earth he inhabits, as there are also others less so; and the stars themselves, properly so called, which to ordinary apprehension present only lucid sparks or brilliant atoms, are to him suns of various and transcendent glory—effulgent centres of life and light to myriads of unseen worlds. So that when, after dilating his thoughts to comprehend the grandeur of those ideas his calculations have called up, and exhausting his imagination and the powers of his language to devise similes and metaphors illustrative of the immensity of the scale on which his universe is constructed, he shrinks back to his native sphere, he finds it, in comparison, a mere point; so lost—even in the minute system to which it belongs—as to be invisible and unsuspected from some of its principal and remoter members.

(3.) There is hardly anything which sets in a stronger light the inherent power of truth over the mind of man, when opposed by no motives of interest or passion, than the perfect readiness with which all these conclusions are assented to as soon as their evidence is clearly apprehended, and the tenacious hold they acquire over our belief when once admitted. In the conduct, therefore, of this volume, I shall take it for granted that the reader is more desirous to learn the system which it is its object to teach, as it now stands, than to raise or revive objections against it; and that, in short, he comes to the task with a willing mind; an assumption which will not only save the trouble of piling argument on argument to convince the sceptical, but will greatly facilitate his actual progress; inasmuch as he will find it at once easier and more satisfactory to pursue from the outset a straight and definite path, than to be con-

stantly stepping aside, involving himself in perplexities and circuits, which, after all, can only terminate in finding himself compelled to adopt the same road.

(4.) The method, therefore, we propose to follow in this work is neither strictly the analytic nor the synthetic, but rather such a combination of both, with a leaning to the latter, as may best suit with a *didactic* composition. Its object is not to convince or refute opponents, nor to inquire, under the semblance of an assumed ignorance, for principles of which we are all the time in full possession—but simply to *teach* what is *known*. The moderate limit of a single volume, to which it will be confined, and the necessity of being on every point, within that limit, rather diffuse and copious in explanation, as well as the eminently matured and ascertained character of the science itself, render this course both practicable and eligible. Practicable, because there is now no danger of any revolution in astronomy, like those which are daily changing the features of the less advanced sciences, supervening, to destroy all our hypotheses, and throw our statements into confusion. Eligible, because the space to be bestowed, either in combating refuted systems, or in leading the reader forward by slow and measured steps from the known to the unknown, may be more advantageously devoted to such explanatory illustrations as will impress on him a familiar and, as it were, a practical sense of the sequence of phenomena, and the manner in which they are produced. We shall not, then, reject the analytic course where it leads more easily and directly to our objects, or in any way fetter ourselves by a rigid adherence to method. Writing only to be understood, and to communicate as much information in as little space as possible, consistently with its *distinct* and *effectual*

communication, no sacrifice can be afforded to system, to form, or to affectation.

(5.) We shall take for granted, from the outset, the Copernican system of the world; relying on the easy, obvious, and natural explanation it affords of all the phenomena as they come to be described, to impress the student with a sense of its truth, without either the formality of demonstration or the superfluous tedium of eulogy, calling to mind that important remark of Bacon—"Theoriarum vires, arcta et quasi se mutuo sustinente partium adaptatione, quâ quasi in orbem cohærent, firmanantur";¹ not failing, however, to point out to the reader, as occasion offers, the contrast which its superior simplicity offers to the complication of other hypotheses.

(6.) The preliminary knowledge which it is desirable that the student should possess, in order for the more advantageous perusal of the following pages, consists in the familiar practice of decimal and sexagesimal arithmetic, some moderate acquaintance with geometry and trigonometry, both plane and spherical; the elementary principles of mechanics; and enough of optics to understand the construction and use of the telescope, and some other of the simpler instruments. Of course, the more of such knowledge he brings to the perusal, the easier will be his progress, and the more complete the information gained; but we shall endeavor in every case, as far as it can be done without a sacrifice of clearness, and of that useful brevity which consists in the absence of prolixity and episode,

¹ "The confirmation of theories relies on the compact adaptation of their parts, by which, like those of an arch or dome, they mutually sustain each other, and form a coherent whole." This is what Dr. Whewell expressively terms the *consilience* of inductions.

to render what we have to say as independent of other books as possible.

(7.) After all, I must distinctly caution such of my readers as may commence and terminate their astronomical studies with the present work (though of such—at least in the latter predicament—I trust the number will be few), that its utmost pretension is to place them on the threshold of this particular wing of the temple of Science, or rather on an eminence exterior to it, whence they may obtain something like a general notion of its structure; or, at most, to give those who may wish to enter a ground-plan of its accesses, and put them in possession of the password. Admission to its sanctuary, and to the privileges and feelings of a votary, is only to be gained by one means—*sound and sufficient knowledge of mathematics, the great instrument of all exact inquiry, without which no man can ever make such advances in this or any other of the higher departments of science as can entitle him to form an independent opinion on any subject of discussion within their range.* It is not without an effort that those who possess this knowledge can communicate on such subjects with those who do not, and adapt their language and their illustrations to the necessities of such an intercourse. Propositions which to the one are almost identical, are theorems of import and difficulty to the other; nor is their evidence presented in the same way to the mind of each. In teaching such propositions, under such circumstances, the appeal has to be made, not to the pure and abstract reason, but to the sense of analogy—to practice and experience: principles and modes of action have to be established not by direct argument from acknowledged axioms, but by continually recurring to the sources from which the axioms themselves have been

drawn: viz., examples; that is to say, by bringing forward and dwelling on simple and familiar instances in which the same principles and the same or similar modes of action take place: thus erecting, as it were, in each particular case, a separate induction, and constructing at each step a little body of science to meet its exigencies. The difference is that of pioneering a road through an untraversed country and advancing at ease along a broad and beaten highway; that is to say, if we are determined to make ourselves distinctly understood, and will appeal to reason at all. As for the method of *assertion*, or a direct demand on the *faith* of the student (though in some complex cases indispensable, where illustrative explanation would defeat its own end by becoming tedious and burdensome to both parties), it is one which I shall neither willingly adopt nor would recommend to others.

(8.) On the other hand, although it is something new to abandon the road of mathematical demonstration in the treatment of subjects susceptible of it, and to teach any considerable branch of science entirely or chiefly by the way of illustration and familiar parallels, it is yet not impossible that those who are already well acquainted with our subject, and whose knowledge has been acquired by that confessedly higher practice which is incompatible with the avowed objects of the present work, may yet find their account in its perusal—for this reason, that it is always of advantage to present any given body of knowledge to the mind in as great a variety of different lights as possible. It is a property of illustrations of this kind to strike no two minds in the same manner, or with the same force; because no two minds are stored with the same images, or have acquired their notions of them by similar habits. Accord-

ingly, it may very well happen, that a proposition, even to one best acquainted with it, may be placed not merely in a new and uncommon, but in a more impressive and satisfactory light by such a course—some obscurity may be dissipated, some inward misgivings cleared up, or even some links supplied which may lead to the perception of connections and deductions altogether unknown before. And the probability of this is increased when, as in the present instance, the illustrations chosen have not been studiously selected from books, but are such as have presented themselves freely to the author's mind as being most in harmony with his own views; by which, of course, he means to lay no claim to originality in all or any of them beyond what they may really possess.

(9.) Besides, there are cases in the application of mechanical principles with which the mathematical student is but too familiar, where, when the data are before him, and the numerical and geometrical relations of his problems all clear to his conception—when his forces are estimated and his lines measured—nay, when even he has followed up the application of his technical processes, and fairly arrived at his conclusion—there is still something wanting in his mind—not in the evidence, for he has examined each link, and finds the chain complete—not in the principles, for those he well knows are too firmly established to be shaken—but precisely in the *mode of action*. He has followed out a train of reasoning by logical and technical rules, but the signs he has employed are not pictures of nature, or have lost their original meaning as such to his mind: he has not seen, as it were, the process of nature passing under his eye in an instant of time, and presented as a consecutive whole to his imagination. A familiar parallel, or an illustration drawn

from some artificial or natural process, of which he has that direct and individual impression which gives it a reality and associates it with a name, will, in almost every such case, supply in a moment this deficient feature, will convert all his symbols into real pictures, and infuse an animated meaning into what was before a lifeless succession of words and signs. I cannot, indeed, always promise myself to attain this degree of vividness of illustration, nor are the points to be elucidated themselves always capable of being so *paraphrased* (if I may use the expression) by any single instance adducible in the ordinary course of experience; but the object will at least be kept in view; and, as I am very conscious of having, in making such attempts, gained for myself much clearer views of several of the more concealed effects of planetary perturbation than I had acquired by their mathematical investigation in detail, it may reasonably be hoped that the endeavor will not always be unattended with a similar success in others.

(10.) From what has been said, it will be evident that our aim is not to offer to the public a technical treatise, in which the student of practical or theoretical astronomy shall find consigned the minute description of methods of observation, or the formulæ he requires prepared to his hand, or their demonstrations drawn out in detail. In all these the present work will be found meagre, and quite inadequate to his wants. Its aim is entirely different; being to present to him in each case the mere ultimate *rationale* of facts, arguments, and processes; and, in all cases of mathematical application, avoiding whatever would tend to encumber its pages with algebraic or geometrical symbols, to place under his inspection that central thread of common-sense on which the pearls of analytical research are invariably strung; but

which, by the attention the latter claim for themselves, is often concealed from the eye of the gazer, and not always disposed in the straightest and most convenient form to follow by those who string them. This is no fault of those who have conducted the inquiries to which we allude. The contention of mind for which they call is enormous; and it may, perhaps, be owing to their experience of *how little* can be accomplished in carrying such processes on to their conclusion, by mere ordinary *clearness of head*; and how necessary it often is to pay more attention to the purely mathematical conditions which insure success—the hooks-and-eyes of their equations and series—than to those which enchain causes with their effects, and both with the human reason—that we must attribute something of that indistinctness of view which is often complained of as a grievance by the earnest student, and still more commonly ascribed ironically to a native cloudiness of an atmosphere too sublime for vulgar comprehension. We think we shall render good service to both classes of readers, by dissipating, so far as lies in our power, that accidental obscurity, and by showing ordinary untutored comprehension clearly what it *can*, and what it *cannot*, hope to attain.

(10 a.) To conclude: “Rome was not built in a day.” No grand practical result of human industry, genius, or meditation, has sprung forth entire and complete from the master hand or mind of an individual designer working straight to its object, and foreseeing and providing for all details. As in the building of a great city, so in every such product, its historian has to record rude beginnings, circuitous and inadequate plans; frequent demolition, renewal and rectification; the perpetual removal of much cumbrous and unsightly material and scaffolding, and constant opening out

of wider and grander conceptions; till at length a unity and a nobility is attained, little dreamed of in the imagination of the first projector.

(10 b.) The same is equally true of every great body of knowledge, and would be found signally exemplified in the history of astronomy, did the object of this work allow us to devote a portion of it to its relation. What concerns us more is, that the same remark is no less applicable to the process by which knowledge is built up in the mind of each individual, and by which alone it can attain any extensive development or any grand proportions. No man can rise from ignorance to anything deserving to be called a complete grasp of any considerable branch of science without receiving and discarding in succession many crude and incomplete notions, which so far from injuring the truth in its ultimate reception, act as positive aids to its attainment by acquainting him with the symptoms of an insecure footing in his progress. To reach from the plain the loftiest summits of an Alpine country, many inferior eminences have to be scaled and relinquished; but the labor is not lost. The region is unfolded in its closer recesses, and the grand panorama which opens from aloft is all the better understood and the more enjoyed for the very misconceptions in detail which it rectifies and explains.

(10 c.) Astronomy is very peculiarly in this predicament. Its study to each individual student is a continual process of rectification and correction—of abandoning one point of view for another higher and better—for temporary and occasional reception of even positive and admitted errors for the convenience they afford toward giving clear notions of important truths whose essence they do not affect, by sparing him that contention of mind which fatigues and dis-

tresses. We know, for example, that the earth's diurnal motion is real, and that of the heavens only apparent; yet there are many problems in astronomy which are not only easier conceived, but more simply resolved by adopting the idea of a diurnal rotation of the heavens, it being understood once for all that appearances are alike in both suppositions.

CHAPTER I

General Notions—Apparent and real Motions—Shape and Size of the Earth—The Horizon and its Dip—The Atmosphere—Refraction—Twilight—Appearances resulting from Diurnal Motion—From Change of Station in General—Parallactic Motions—Terrestrial Parallax—That of the Stars Insensible—First Step toward forming an Idea of the Distance of the Stars—Copernican View of the Earth's Motion—Relative Motion—Motions partly Real, partly Apparent—Geocentric Astronomy, or Ideal Reference of Phenomena to the Earth's Centre as a Common Conventional Station

(11.) THE magnitudes, distances, arrangement, and motions of the great bodies which make up the visible universe, their constitution and physical condition, so far as they can be known to us, with their mutual influences and actions on each other, so far as they can be traced by the effects produced, and established by legitimate reasoning, form the assemblage of objects to which the attention of the astronomer is directed. The term astronomy¹ itself, which denotes the *law* or rule of the *astra* (by which the ancients understood not only the stars properly so called, but the sun, the moon, and all the visible constituents of the heavens), sufficiently indicates this; and, although the

¹ Ἀστρον, a star; νόμος, a law; or νημειν, to tend, as a shepherd his flock; so that ἀστρονομος means "shepherd of the stars." The two etymologies are, however, coincident.

term astrology, which denotes the *reason, theory, or interpretation* of the stars,² has become degraded in its application, and confined to superstitious and delusive attempts to divine future events by their dependence on pretended planetary influences, the same meaning originally attached itself to that epithet.

(12.) But, besides the stars and other celestial bodies, the earth itself, regarded as an individual body, is one principal object of the astronomer's consideration, and, indeed, the chief of all. It derives its importance, in a practical as well as theoretical sense, not only from its proximity, and its relation to us as animated beings, who draw from it the supply of all our wants, but as the station from which we see all the rest, and as the only one among them to which we can, in the first instance, refer for any determinate marks and measures by which to recognize their changes of situation, or with which to compare their distances.

(13.) To the reader who now for the first time takes up a book on astronomy, it will no doubt seem strange to class the earth with the heavenly bodies, and to assume any community of nature among things apparently so different. For what, in fact, can be more apparently different than the vast and seemingly immeasurable extent of the earth, and the stars, which appear but as points, and seem to have no size at all? The earth is dark and opaque, while the celestial bodies are brilliant. We perceive in it no motion, while in them we observe a continual change of place, as we view them at different hours of the day or night, or at different seasons of the year. The ancients, accordingly, one or two

² Λογος, *reason, or a word*, the vehicle of reason; the interpreter of thought.

of the more enlightened of them only excepted, admitted no such community of nature; and, by thus placing the heavenly bodies and their movements without the pale of analogy and experience, effectually intercepted the progress of all reasoning from what passes here below, to what is going on in the regions where they exist and move. Under such conventions, astronomy, as a science of cause and effect, could not exist, but must be limited to a mere registry of appearances, unconnected with any attempt to account for them on reasonable principles, however successful to a certain extent might be the attempt to follow out their order of sequence, and to establish empirical laws expressive of this order. To get rid of this prejudice, therefore, is the first step toward acquiring a knowledge of what is really the case; and the student has made his first effort toward the acquisition of sound knowledge, when he has learned to familiarize himself with the idea that the earth, after all, *may* be nothing but a great star. How correct such an idea may be, and with what limitations and modifications it is to be admitted, we shall see presently.

(14.) It is evident, that, to form any just notions of the arrangement, in space, of a number of objects which we cannot approach and examine, but of which all the information we can gain is by sitting still and watching their evolutions, it must be very important for us to know, in the first instance, whether what we call sitting still is *really* such: whether the station from which we view them, with ourselves, and all objects which immediately surround us, be not itself in motion, unperceived by us; and if so, of what nature that motion is. The apparent places of a number of objects, and their apparent arrangement with respect to each other, will of course be materially dependent on the

situation of the spectator among them; and if this situation be liable to change, unknown to the spectator himself, an appearance of change in the respective situations of the objects will arise, without the reality. If, then, such be actually the case, it will follow that *all* the movements we think we perceive among the stars will not be real movements, but that some part, at least, of whatever changes of relative place we perceive among them must be merely apparent, the results of the shifting of our own point of view; and that, if we would ever arrive at a knowledge of their real motions, it can only be by first investigating our own, and making due allowance for its effects. Thus, the question whether the earth is in motion or at rest, and if in motion, what that motion is, is no idle inquiry, but one on which depends our only chance of arriving at true conclusions respecting the constitution of the universe.

(15.) Nor let it be thought strange that we should speak of a motion existing in the earth, unperceived by its inhabitants: we must remember that it is of the earth *as a whole* with all that it holds within its substance, or sustains on its surface, that we are speaking; of a motion common to the solid mass beneath, to the ocean which flows around it, the air that rests upon it, and the clouds which float above it in the air. Such a motion, which should displace no terrestrial object from its relative situation among others, interfere with no natural processes, and produce no sensations of shocks or jerks, might, it is very evident, subsist undetected by us. There is no peculiar sensation which advertises us that we are *in motion*. We perceive *jerks*, or *shocks*, it is true, because these are sudden *changes* of motion, produced, as the laws of mechanics teach us, by sudden and powerful forces acting during short times; and

these forces, applied to our bodies, are what we *feel*. When, for example, we are carried along in a carriage with the blinds down, or with our eyes closed (to keep us from seeing external objects), we perceive a tremor arising from inequalities in the road, over which the carriage is successively lifted and let fall, but we have no sense of *progress*. As the road is smoother, our sense of motion is diminished, though our rate of travelling is accelerated. Railway travelling, especially by night or in a tunnel, has familiarized every one with this remark. Those who have made *aéronautic* voyages testify that with closed eyes, and under the influence of a steady breeze communicating no oscillatory or revolving motion to the car, the *sensation* is that of perfect rest, however rapid the transfer from place to place.

(16.) But it is on shipboard, where a great system is maintained in motion, and where we are surrounded with a multitude of objects which participate with ourselves and each other in the common progress of the whole mass, that we feel most satisfactorily the identity of sensation between a state of motion and one of rest. In the cabin of a large and heavy vessel, going smoothly before the wind in still water, or drawn along a canal, not the smallest indication acquaints us with the way it is making. We read, sit, walk, and perform every customary action as if we were on land. If we throw a ball into the air, it falls back into our hand; or if we drop it, it alights at our feet. Insects buzz around us as in the free air; and smoke ascends in the same manner as it would do in an apartment on shore. If, indeed, we come on deck, the case is, in some respects, different; the air, not being carried along with us, drifts away smoke and other light bodies—such as feathers abandoned to it—appar-

ently, in the opposite direction to that of the ship's progress; but, in reality, *they* remain at rest, and we leave them behind in the air. Still, the illusion, so far as massive objects and our own movements are concerned, remains complete; and when we look at the shore, we then perceive the effect of our own motion transferred, in a contrary direction, to external objects—*external*, that is, *to the system of which we form a part*.

“Provehimur portu, terræque urbesque recedunt.”

(17.) In order, however, to conceive the earth as in motion, we must form to ourselves a conception of its shape and size. Now, an object cannot have shape and size unless it is *limited* on all sides by some definite outline, so as to admit of our imagining it, at least, disconnected from other bodies, and existing insulated in space. The first rude notion we form of the earth is that of a flat surface, of indefinite extent in all directions from the spot where we stand, *above* which are *the air* and *sky*; below, to an indefinite profundity, solid matter. This is a prejudice to be got rid of, like that of the earth's immobility—but it is one much easier to rid ourselves of, inasmuch as it originates only on our own mental inactivity, in not questioning ourselves *where* we will place a limit to a thing we have been accustomed from infancy to regard as immensely large; and does not, like that, originate in the testimony of our senses unduly interpreted. On the contrary, the direct testimony of our senses lies the other way. When we see the sun set in the evening in the west, and rise again in the east, as we cannot doubt that it is the *same* sun we see after a temporary absence, we must do violence to all our notions of solid matter, to suppose it to have made its

way *through* the substance of the earth. It must, therefore, have gone *under* it, and that not by a mere subterraneous *channel*; for if we notice the points where it sets and rises for many successive days, or for a whole year, we shall find them constantly shifting, round a very large extent of the horizon; and, besides, the moon and stars also set and rise again in *all* points of the visible horizon. The conclusion is plain: the earth cannot extend indefinitely in depth downward, nor indefinitely in surface laterally; it must have not only bounds in a horizontal direction, but also an *under side* round which the sun, moon, and stars can pass; and that side must, at least, be so far like what we see, that it must have a sky and sunshine, and a day when it is night to us, and *vice versâ*; where, in short,

—“redit a nobis Aurora, diemque reducit.
Nosque ubi primus equis oriens afflavit anhelis,
Illic sera rubens accendit lumina Vesper.”—*Georg.*

(18.) As soon as we have familiarized ourselves with the conception of an earth without *foundations* or fixed supports—existing insulated in space from contact of every thing external, it becomes easy to imagine it in motion—or, rather, difficult to imagine it otherwise; for, since there is nothing to *retain* it in one place, should any causes of motion exist, or any *forces* act upon it, it must obey their impulse. Let us next see what obvious circumstances there are to help us to a knowledge of the *shape* of the earth.

(19.) Let us first examine what we can actually *see* of its shape. Now, it is not on land (unless, indeed, on uncommonly level and extensive plains), that we can see anything of the *general* figure of the earth. The hills, trees, and other objects which roughen its surface, and break and elevate the

line of the horizon, though obviously bearing a most minute proportion to the *whole* earth, are yet too considerable with respect to ourselves and to that small portion of it which we can see at a single view, to allow of our forming any judgment of the form of the whole, from that of a part so disfigured. But with the surface of the sea or any vastly extended level plain, the case is otherwise. If we sail out of sight of land, whether we stand on the deck of the ship or climb the mast, we see the surface of the sea—not losing itself in distance and mist, but terminated by a sharp, clear, well-defined line or *offing*, as it is called, which runs all round us in a circle, having our station for its centre. That this line is really a circle, we conclude, first, from the perfect apparent similarity of all its parts; and, secondly, from the fact of all its parts appearing at the same distance from us, and that, evidently, a moderate one; and, thirdly, from this, that its apparent *diameter*, measured with an instrument called the *dip sector*, is the same (except under some singular atmospheric circumstances, which produce a temporary distortion of the outline), in whatever direction the measure is taken—properties which belong only to the circle among geometrical figures. If we ascend a high eminence on a plain (for instance, one of the Egyptian pyramids), the same holds good.

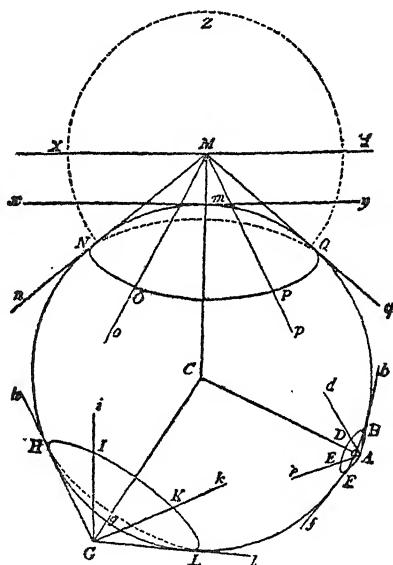
(20.) Masts of ships, however, and the edifices erected by man, are trifling eminences compared to what nature itself affords; *Ætna*, *Teneriffe*, *Mowna Roa*, are eminences from which no contemptible *aliquot* part of the whole earth's surface can be seen; but from these again—in those few and rare occasions when the transparency of the air will permit the real boundary of the horizon, the true sea-line, to be seen—the very same appearances are witnessed, but with

this remarkable addition, viz., that the angular *diameter* of the visible area, as measured by the dip sector, is materially *less* than at a lower level; or, in other words, that the *apparent size* of the earth has sensibly diminished as we have receded from its surface, while yet the *absolute quantity* of it seen at once has been increased.

(21.) The same appearances are observed universally, in every part of the earth's surface visited by man. Now, the figure of a body which, however seen, appears always *circular*, can be no other than a sphere or globe.

(22.) A diagram will elucidate this. Suppose the earth to be represented by the sphere L H N Q, whose centre is C, and let A, G, M be stations at different elevations above various points of its surface, represented by *a*, *g*, *m* respectively. From each of them (as from M) let a line be drawn, as M N *n*, a tangent to the surface at N, then will this line represent the visual ray along which the spectator at M will see the visible horizon; and as this tangent sweeps round M, and comes successively into the positions M O *o*, M P *p*, M Q *q*, the point of contact N will mark out on the surface the circle N O P Q. The area of the spherical surface comprehended within this circle is the portion of the earth's surface visible to a spectator at M, and the angle N M Q, included between the two extreme visual rays, is the measure of its apparent angular diameter. Leaving, at present, out of consideration the effect of refraction in the air below M, of which more hereafter, and which always tends, in some degree, to *increase* that angle, or render it more *obtuse*, this is the angle measured by the dip sector. Now, it is evident, 1st, that as the point M is more elevated above *m*, the point immediately below it on the sphere, the visible area, *i.e.* the spherical segment or slice N O P Q, increases;

2dly, that the distance of the visible *horizon*^s or boundary of our view from the eye, viz., the line M N, increases; and, 3dly, that the angle N M Q becomes *less obtuse*, or, in other words, the apparent angular diameter of the earth diminishes, being nowhere so great as 180°, or two right angles, but falling short of it by some sensible quantity, and that more and more the higher we ascend. The figure



exhibits three states or stages of elevation, with the horizon, etc., corresponding to each, a glance at which will explain our meaning; or, limiting ourselves to the larger and more distinct, M N O P Q, let the reader imagine n N M, M Q q to be the two legs of a ruler jointed at M, and kept extended by the globe N m Q between them. It is clear, that as the joint M is urged home toward the surface, the legs will

³ 'Opisz to terminate.

open, and the ruler will become more nearly *straight*, but will not attain *perfect* straightness till M is brought fairly up to contact with the surface at *m*, in which case its whole length will become a *tangent* to the sphere at *m*, as is the line *x y*.

(23.) This explains what is meant by the *dip of the horizon*. M *m*, which is perpendicular to the general surface of the sphere at *m*, is also the direction in which a *plumb-line*⁴ would hang; for it is an observed fact, that in all situations, in every part of the earth, the direction of a plumb-line is exactly perpendicular to the surface of still water; and, moreover, that it is also exactly perpendicular to a line or surface truly adjusted by a *spirit-level*.⁴ Suppose, then, that at our station M we were to adjust a line (a wooden ruler, for instance) by a spirit-level, with perfect exactness; then, if we suppose the direction of this line indefinitely prolonged both ways, as X M Y, the line so drawn will be at right angles to M *m*, and therefore parallel to *x m y*, the tangent to the sphere at *m*. A spectator placed at M will therefore see not only all the vault of the sky *above* this line, as X Z Y, but also that portion or zone of it which lies between X N and Y Q; in other words, his sky will be more than a hemisphere by the zone Y Q X N. It is the angular breadth of this redundant zone—the angle Y M Q, by which the *visible* horizon appears depressed below the direction of a spirit-level—that is called the *dip of the horizon*. It is a correction of constant use in nautical astronomy.

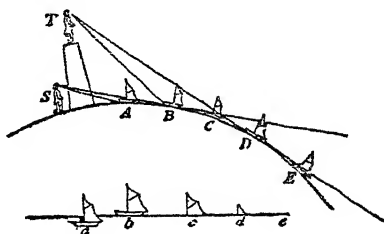
(24.) From the foregoing explanations it appears, 1st, That the general figure of the earth (so far as it can be

⁴ See these instruments described in Chap. III.

gathered from this kind of observation) is that of a sphere or globe. In this we also include that of the sea, which, wherever it extends, covers and fills in those inequalities and local irregularities which exist on land, but which can of course only be regarded as trifling deviations from the general outline of the whole mass, as we consider an orange not the less round for the roughness on its rind. 2dly, That the appearance of a *visible* horizon, or sea-offing, is a consequence of the curvature of the surface, and does not arise from the inability of the eye to follow objects to a greater distance, or from atmospheric indistinctness. It will be worth while to pursue the general notion thus acquired into some of its consequences, by which its consistency with observations of a different kind, and on a larger scale, will be put to the test, and a clear conception be formed of the manner in which the parts of the earth are related to each other, and held together as a whole.

(25.) In the first place, then, every one who has passed a little while at the seaside is aware that objects may be seen perfectly well beyond the *offing* or visible horizon—but not the *whole* of them.

We only see their upper parts. Their bases where they rest on, or rise out of the water, are hid from view by the spherical surface of the sea, which pro-



trudes between them and ourselves. Suppose a ship, for instance, to sail directly away from our station. At first, when the distance of the ship is small, a spectator, S, situated at some certain height above the sea, sees the whole of the ship, even to the *water line* where it rests on

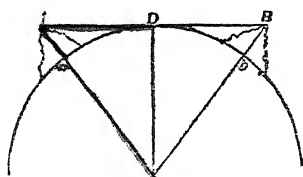
the sea, as at A. As it recedes it diminishes, it is true, in apparent size, but still the *whole* is seen down to the water line, till it reaches the *visible* horizon at B. But as soon as it has passed this distance, not only does the visible portion still continue to diminish in apparent *size*, but the hull begins to disappear bodily, as if sunk below the surface. When it has reached a certain distance, as at C, its hull has entirely vanished, but the masts and sails remain, presenting the appearance *c*. But if, in this state of things, the spectator quickly ascends to a higher station, T, whose visible horizon is at D, the hull comes again in sight; and, when he descends again, he loses it. The ship still receding, the lower sails seem to sink below the water, as at *d*, and at length the whole disappears: while yet the distinctness with which the last portion of the sail *d* is seen is such as to satisfy us that were it not for the interposed segment of the sea, A B C D E, the distance T E is not so great as to have prevented an equally perfect view of the whole.

(26.) The history of aëronautic adventure affords a curious illustration of the same principle. The late Mr. Sadler, the celebrated aëronaut, ascended on one occasion in a balloon from Dublin, and was wafted across the Irish Channel, when, on his approach to the Welsh coast, the balloon descended nearly to the surface of the sea. By this time the sun was set, and the shades of evening began to close in. He threw out nearly all his ballast, and suddenly sprang upward to a great height, and by so doing brought his horizon to *dip* below the sun, producing the whole phenomenon of a western sunrise. M. Charles in his memorable ascent from Paris in 1783 witnessed the same phenomenon.

(27.) If we could measure the heights and exact distance of two stations which could barely be discerned from each

other over the edge of the horizon, we could ascertain the actual size of the earth itself: and, in fact, were it not for the effect of refraction, by which we are enabled to see in some small degree *round* the interposed segment (as will be hereafter explained), this would be a tolerably good method of ascertaining it. Suppose A and B to be two eminences, whose perpendicular heights A a and B b (which, for simplicity, we will suppose to be exactly equal) are known, as well as their exact horizontal in-

terval $a D b$, by measurement; then it is clear that D, the visible horizon of both, will lie just half-way between them, and if we suppose $a D b$ to be the sphere of the earth, and C its centre in the figure C D $b B$, we know D b , the length of the arch of the circle between D and b —viz., half the measured interval, and $b B$, the excess of its secant above its radius—which is the height of B—data which, by the solution of an easy geometrical problem, enable us to find the length of the radius D C. If, as is really the case, we suppose both the heights and distance of the stations inconsiderable in comparison with the size of the earth, the solution alluded to is contained in the following proposition:



The earth's diameter bears the same proportion to the distance of the visible horizon from the eye as that distance does to the height of the eye above the sea level.

When the stations are unequal in height, the problem is a little more complicated.

(28.) Although, as we have observed, the effect of refraction prevents this from being an exact method of ascertaining the dimensions of the earth, yet it will suffice to

afford such an approximation to it as shall be of use in the present stage of the reader's knowledge, and help him to many just conceptions, on which account we shall exemplify its application in numbers. Now, it appears by observation, that two points, each ten feet above the surface, cease to be visible from each other over still water, and in average atmospheric circumstances, at a distance of about 8 miles. But 10 feet is the 528th part of a mile, so that half that distance, or 4 miles, is to the height of each as 4×528 or 2112 : 1, and therefore in the same proportion to 4 miles is the length of the earth's diameter. It must, therefore, be equal to $4 \times 2112 = 8448$, or, in round numbers, about 8000 miles, which is not very far from the truth.

(29.) Such is the first rough result of an attempt to ascertain the earth's magnitude; and it will not be amiss, if we take advantage of it to compare it with objects we have been accustomed to consider as of vast size, so as to interpose a few steps between it and our ordinary ideas of dimension. We have before likened the inequalities on the earth's surface, arising from mountains, valleys, buildings, etc., to the roughnesses on the rind of an orange, compared with its general mass. The comparison is quite free from exaggeration. The highest mountain known hardly exceeds five miles in perpendicular elevation: this is only one 1600th part of the earth's diameter; consequently, on a globe of sixteen inches in diameter, such a mountain would be represented by a protuberance of no more than one hundredth part of an inch, which is about the thickness of ordinary drawing-paper. Now, as there is no entire continent, or even any very extensive tract of land, known, whose general elevation above the sea is anything like half this quantity, it follows, that if we would

construct a correct model of our earth, with its seas, continents, and mountains, on a globe sixteen inches in diameter, the whole of the land, with the exception of a few prominent points and ridges, must be comprised on it within the thickness of thin writing-paper; and the highest hills would be represented by the smallest visible grains of sand.

(30.) The deepest mine existing does not penetrate half a mile below the surface: a scratch, or pin-hole, duly representing it, on the surface of such a globe as our model, would be imperceptible without a magnifier.

(31.) The greatest depth of sea, probably, does not very much exceed the greatest elevation of the continents; and would, of course, be represented by an excavation, in about the same proportion, into the substance of the globe: so that the ocean comes to be conceived as a mere film of liquid, such as, on our model, would be left by a brush dipped in color, and drawn over those parts intended to represent the sea; only, in so conceiving it, we must bear in mind that the resemblance extends no further than to proportion in point of quantity. The mechanical laws which would regulate the distribution and movements of such a film, and its adhesion to the surface, are altogether different from those which govern the phenomena of the sea.

(32.) Lastly, the greatest extent of the earth's surface which has ever been brought at once within the range of human vision was that which, but for clouds, would have been exposed to the view of Messrs. Glaisher and Coxwell, in their balloon ascent of September 5, 1863, to the enormous height of seven miles. To estimate the proportion of the area visible from this elevation to the whole earth's surface, we must have recourse to the geometry of the

sphere, which informs us that the convex surface of a spherical segment is to the whole surface to which it belongs as the thickness of the segment is to the diameter of the sphere; and further, that this thickness, in the case we are considering, is almost exactly equal to the perpendicular elevation of the point of sight above the surface. The proportion, therefore, of the visible area, in this case, to the whole earth's surface, is that of seven miles to 8000, or 1 to 1140. The portion visible from *Ætna*, the Peak of *Teneriffe*, or *Mowna Roa*, is about one 4000th.

(33.) When we ascend to any very considerable elevation above the surface of the earth, either in a balloon, or on mountains, we are made aware, by many uneasy sensations, of an insufficient supply of *air*. The barometer, an instrument which informs us of the weight of air incumbent on a given horizontal surface, confirms this impression, and affords a direct measure of the rate of diminution of the quantity of air which a given space includes as we recede from the surface. From its indications we learn, that when we have ascended to the height of 1000 feet, we have left below us about one-thirtieth of the whole mass of the atmosphere—that at 10,600 feet of perpendicular elevation (which is rather less than that of the summit of *Ætna*⁵) we have ascended through about one-third; and at 18,000 feet (which is nearly that of *Cotopaxi*) through one-half the material, or, at least, the ponderable body of air incumbent on the earth's surface. From the progression of these numbers, as well as, *a priori*, from the nature of the air itself, which is *compressible*, *i.e.* capable of being condensed or crowded

⁵ The height of *Ætna* above the Mediterranean (as it results from a barometrical measurement of my own, made in July, 1824, under very favorable circumstances) is 10,872 English feet.—*Author*.

into a smaller space in proportion to the incumbent pressure, it is easy to see that, although by rising still higher, we should continually get above more and more of the air, and so relieve ourselves more and more from the pressure with which it weighs upon us, yet the amount of this additional relief, or the *ponderable quantity* of air surmounted, would be by no means in proportion to the additional height ascended, but in a constantly decreasing ratio. An easy calculation, however, founded on our experimental knowledge of the properties of air, and the mechanical laws which regulate its dilatation and compression, is sufficient to show that, at an altitude above the surface of the earth not exceeding the hundredth part of its diameter, the tenuity, or rarefaction, of the air must be so excessive, that not only animal life could not subsist, or combustion be maintained in it, but that the most delicate means we possess of ascertaining the existence of *any air at all* would fail to afford the slightest perceptible indications of its presence.

(34.) Laying out of consideration, therefore, at present, all nice questions as to the probable existence of a definite limit to the atmosphere, beyond which there is, absolutely and rigorously speaking, *no air*, it is clear, that, for all practical purposes, we may speak of those regions which are more distant above the earth's surface than the hundredth part of its diameter as void of air, and of course of clouds (which are nothing but visible vapors, diffused and *floating* in the air, sustained by it, and rendering it *turbid* as mud does water). It seems probable, from many indications, that the greatest height at which visible clouds *ever exist* does not exceed ten miles; at which height the density of the air is about an eighth part of what it is at the level of the sea.

(35.) We are thus led to regard the atmosphere of air, with the clouds it supports, as constituting a coating of equable or nearly equable thickness, enveloping our globe on all sides; or rather as an aërial ocean, of which the surface of the sea and land constitutes the bed, and whose inferior portions or strata, within a few miles of the earth, contain by far the greater part of the whole mass, the density diminishing with extreme rapidity as we recede upward, till, within a very moderate distance (such as would be represented by the sixth of an inch on the model we have before spoken of, and which is not more in proportion to the globe on which it rests, than the downy skin of a peach in comparison with the fruit within it), all sensible trace of the existence of air disappears.

(36.) Arguments, however, are not wanting to render it, if not absolutely certain, at least in the highest degree probable, that the surface of the aërial, like that of the aqueous ocean, has a real and definite limit, as above hinted at; beyond which there is positively *no* air, and above which a fresh quantity of air, could it be added from without, or carried aloft from below, instead of dilating itself indefinitely upward, would, after a certain very enormous but still finite enlargement of volume, sink and merge, as water poured into the sea, and distribute itself among the mass beneath. With the truth of this conclusion, however, astronomy has little concern; all the effects of the atmosphere in modifying astronomical phenomena being the same, whether it be supposed of definite extent or not.

(37.) Moreover, whichever idea we adopt, within those limits in which it possesses any appreciable density its constitution is the same over all points of the earth's surface; that is to say, on the great scale, and leaving out of con-

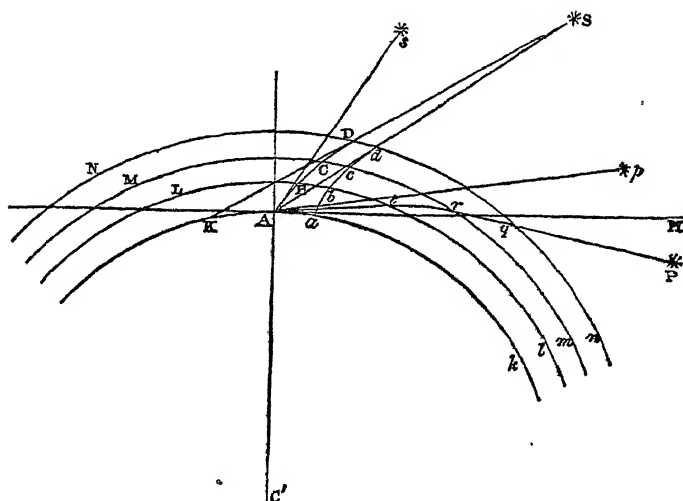
sideration temporary and local causes of derangement, such as winds, and great fluctuations, of the nature of waves, which prevail in it to an immense extent. In other words, the law of diminution of the air's density as we recede upward *from the level of the sea* is the same in every column into which we may conceive it divided, or from whatever point of the surface we may set out. It may therefore be considered as consisting of successively superposed strata or layers, each of the form of a spherical shell, concentric with the general surface of the sea and land, and each of which is *rarer*, or specifically lighter, than that immediately beneath it; and *denser*, or specifically heavier, than that immediately above it. This, at least, is the kind of distribution which alone would be consistent with the laws of the equilibrium of fluids. Inasmuch, however, as the atmosphere is not in perfect equilibrium, being always kept in a state of circulation, owing to the excess of heat in its equatorial regions over that at the poles, some slight deviation from the rigorous expression of this law takes place, and in peculiar localities there is reason to believe that even considerable permanent depressions of the contours of these strata, below their general or spherical level, subsist. But these are points of consideration rather for the meteorologist than the astronomer. It must be observed, moreover, that with this distribution of its strata the inequalities of mountains and valleys have little concern. These exercise hardly more influence in modifying their general spherical figure than the inequalities at the bottom of the sea interfere with the general sphericity of its surface. They would exercise absolutely none were it not for their effect in giving another than horizontal direction to the currents of air constituting winds, as shoals in the ocean throw up the currents which

sweep over them toward the surface, and so in some small degree tend to disturb the perfect level of that surface.

(38.) It is the power which air possesses, in common with all transparent media, of *refracting* the rays of light, or bending them out of their straight course, which renders a knowledge of the constitution of the atmosphere important to the astronomer. Owing to this property, objects seen obliquely through it appear otherwise situated than they would to the same spectator, had the atmosphere no existence. It thus produces a false impression respecting their places, which must be rectified by ascertaining the amount and direction of the displacement so apparently produced on each, before we can come at a knowledge of the true directions in which they are situated from us at any assigned moment.

(39.) Suppose a spectator placed at A, any point of the earth's surface K A *k*; and let L *l*, M *m*, N *n*, represent the successive strata or layers, of decreasing density, into which we may conceive the atmosphere to be divided, and which are spherical surfaces concentric with K *k*, the earth's surface. Let S represent a star, or other heavenly body, beyond the utmost limit of the atmosphere. Then, if the air were away, the spectator would see it in the direction of the straight line A S. But, in reality, when the ray of light S A reaches the atmosphere, suppose at *d*, it will, by the laws of optics, begin to bend *downward*, and take a more inclined direction, as *d c*. This bending will at first be imperceptible, owing to the extreme tenuity of the uppermost strata; but as it advances downward, the strata continually increasing in density, it will continually undergo greater and greater *refraction* in the same direction; and thus, instead of pursuing the straight line S *d* A, it will describe a

curve $S d c b a$, continually more and more concave downward, and will reach the earth, not at A , but at a certain point a , nearer to S . *This* ray, consequently, will not reach the spectator's eye. The ray by which he will see the star is, therefore, not $S d A$, but another ray which, had there been no atmosphere, would have struck the earth at K , a point *behind* the spectator; but which, being bent by the air into the curve $S D C B A$, actually strikes on A . Now, it



is a law of optics, that an object is seen in the direction which the visual ray has at the instant of *arriving at the eye*, without regard to what may have been otherwise its course between the object and the eye. Hence the star S will be seen, not in the direction $A S$, but in that of $A s$, a *tangent* to the curve $S D C B A$, at A . But because the curve described by the refracted ray is concave downward, the tangent $A s$ will lie *above* $A S$, the unrefracted ray: consequently the object S will appear more elevated above

the horizon AH , when seen through the refracting atmosphere, than it would appear were there no such atmosphere. Since, however, the disposition of the strata is the same in all directions around A , the visual ray will not be made to deviate *laterally*, but will remain constantly in the same vertical plane, SAC' , passing through the eye, the object, and the earth's centre.

(40.) The effect of the air's refraction, then, is to *raise* all the heavenly bodies higher above the horizon in appearance than they are in reality. Any such body, situated actually *in* the true horizon, will appear *above* it, or will have some certain apparent *altitude* (as it is called). Nay, even some of those actually below the horizon, and which would therefore be invisible but for the effect of refraction, are, by that effect, raised above it and brought into sight. Thus, the sun, when situated at P below the true horizon, AH , of the spectator, becomes visible to him, as if it stood at p , by the refracted ray $Pqr t A$, to which $A p$ is a tangent.

(41.) The exact estimation of the amount of atmospheric refraction, or the strict determination of the angle SAs , by which a celestial object at any assigned altitude, HAS , is raised in appearance above its true place, is, unfortunately, a very difficult subject of physical inquiry, and one on which geometers (from whom alone we can look for any information on the subject) are not yet entirely agreed. The difficulty arises from this, that the *density* of any stratum of air (on which its refracting power depends) is affected not *merely* by the superincumbent pressure, but also by its *temperature* or degree of heat. Now, although we know that as we recede from the earth's surface the temperature of the air is constantly diminishing, yet the

law, or amount of this diminution at different heights, is not yet fully ascertained. Moreover, the refracting power of air is perceptibly affected by its *moisture*; and this, too, is not the same in every part of an aerial column; neither are we acquainted with the laws of its distribution. The consequence of our ignorance on these points is to introduce a corresponding degree of uncertainty into the determination of the amount of refraction, which affects, to a certain appreciable extent, our knowledge of several of the most important *data* of astronomy. The uncertainty thus induced is, however, confined within such very narrow limits as to be no cause of embarrassment, except in the most delicate inquiries, and to call for no further allusion in a treatise like the present.

(42.) A "Table of Refractions," as it is called, or a statement of the amount of apparent displacement arising from this cause, at all altitudes, or in every situation of a heavenly body, from the horizon to the *zenith*⁶ (or point of the sky vertically above the spectator), and under all the circumstances in which astronomical observations are usually performed which may influence the result, is one of the most important and indispensable of all astronomical tables, since it is only by the use of such a table we are enabled to get rid of an illusion which must otherwise pervert all our notions respecting the celestial motions. Such have been, accordingly, constructed with great care, and are to be found in every collection of astronomical tables. Our design, in the present treatise, will not admit of the introduction of tables; and we must, therefore, content ourselves

⁶ From an Arabic word of this signification. See this term technically defined in Chap. II.

here, and in similar cases, with referring the reader to works especially destined to furnish these useful aids to calculation. It is, however, desirable that he should bear in mind the following general notions of its amount, and law of variations.

(43.) 1st. In the *zenith* there is no refraction. A celestial object, situated vertically overhead, is seen in its true direction, as if there were no atmosphere, at least if the air be tranquil.

2dly. In descending from the *zenith* to the horizon, the refraction continually increases. Objects near the horizon appear more elevated by it above their true directions than those at a high altitude.

3dly. The *rate* of its increase is nearly in proportion to the tangent of the apparent angular distance of the object from the zenith. But this rule, which is not far from the truth, at moderate *zenith distances*, ceases to give correct results in the vicinity of the horizon, where the law becomes much more complicated in its expression.

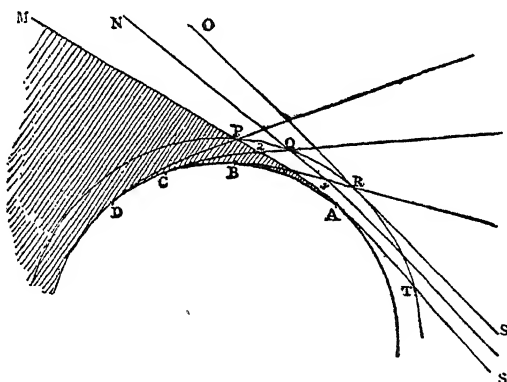
4thly. The average amount of refraction, for an object half-way between the zenith and horizon, or at an apparent altitude of 45° , is about $1'$ (more exactly, $57''$), a quantity hardly sensible to the naked eye; but at the visible horizon it amounts to no less a quantity than $33'$, which is rather more than the greatest apparent diameter of either the sun or the moon. Hence it follows, that when we see the lower edge of the sun or moon just *apparently* resting on the horizon, its whole disk is in reality below it, and would be entirely out of sight and concealed by the convexity of the earth, but for the bending round it, which the rays of light have undergone in their passage through the air, as alluded to in art. 40.

5thly. That when the barometer is higher than its average or mean state, the amount of refraction is greater than its mean amount; when lower, less: and,

6thly. That for one and the same reading of the barometer the refraction is greater, the colder the air. The variations, owing to these two causes, from its mean amount (at temp. 55° , pressure 30 inches), are about one 420th part of that amount for each degree of the thermometer of Fahrenheit, and one 300th for each tenth of an inch in the height of the barometer.

(44.) It follows from this, that one obvious effect of refraction must be to shorten the duration of night and darkness, by actually prolonging the stay of the sun and moon above the horizon. But even after they are set, the influence of the atmosphere still continues to send us a portion of their light; not, indeed, by direct transmission, but by *reflection* upon the vapors and minute solid particles which float in it, and, perhaps, also on the actual material atoms of the air itself. To understand how this takes place, we must recollect, that it is not only by the direct light of a luminous object that we see, but that whatever portion of its light which would not otherwise reach our eyes is intercepted in its course, and thrown back, or laterally, upon us, becomes to us a means of illumination. Such reflective obstacles always exist floating in the air. The whole course of a sunbeam penetrating through the chink of a window-shutter into a dark room is *visible* as a bright line in the air: and even if it be stifled, or *let out* through an opposite crevice, the light scattered through the apartment from this source is sufficient to prevent entire darkness in the room. The luminous lines occasionally seen in the air, in a sky full of partially broken clouds,

which the vulgar term "the sun drawing water," are similarly caused. They are sunbeams, through apertures in clouds, partially intercepted and reflected on the dust and vapors of the air below. Thus it is with those solar rays which, after the sun is itself concealed by the convexity of the earth, continue to traverse the higher regions of the atmosphere above our heads, and pass through and out of it, without directly striking on the earth at all. Some portion of them is intercepted and reflected by the floating particles above mentioned, and thrown back, or laterally, so as to



reach us, and afford us that secondary illumination, which is twilight. The course of such rays will be immediately understood from the above figure, in which A B C D is the earth; A a point on its surface, where the sun S is in the act of setting: its last lower ray S A M just grazing the surface at A, while its superior rays S N, S O, traverse the atmosphere above A without striking the earth, leaving it finally at the points P Q R, after being more or less bent in passing through it, the lower most, the higher less, and that which, like S R O, merely grazes the exterior limit of the

atmosphere, not at all. Let us consider several points, A, B, C, D, each more remote than the last from A, and each more deeply involved in the *earth's shadow*, which occupies the whole space from A beneath the line A M. Now, A just receives the sun's last direct ray, and, besides, is illuminated by the whole reflective atmosphere P Q R T. It therefore receives twilight from the whole sky. The point B, to which the sun has set, receives no direct solar light, nor any, direct or reflected, from all that part of *its* visible atmosphere which is below A P M; but from the lenticular portion P R α , which is traversed by the sun's rays, and which lies above the visible horizon B R of B, it receives a twilight, which is strongest at R, the point immediately below which the sun is, and fades away gradually toward P, as the luminous part of the atmosphere thins off. At C, only the last or thinnest portion, P Q α of the lenticular segment, thus illuminated, lies above the horizon, C Q, of that place; here, then, the twilight is feeble, and confined to a small space in and near the horizon, which the sun has quitted, while at D the twilight has ceased altogether.

(45.) When the sun is above the horizon, it illuminates the atmosphere and clouds, and these again disperse and scatter a portion of its light in all directions, so as to send some of its rays to every exposed point, from every point of the sky. The generally diffused light, therefore, which we enjoy in the daytime, is a phenomenon originating in the very same causes as the twilight. Were it not for the reflective and scattering power of the atmosphere, no objects would be visible to us out of direct sunshine; every shadow of a passing cloud would be pitchy darkness; the stars would be visible all day, and every apartment, into which

the sun had not direct admission, would be involved in nocturnal obscurity. This scattering action of the atmosphere on the solar light, it should be observed, is increased by the irregularity of temperature caused by the same luminary in its different parts, which, during the daytime, throws it into a constant state of undulation, and, by thus bringing together masses of air of very unequal temperatures, produces partial reflections and refractions at their common boundaries, by which some portion of the light is turned aside from the direct course, and diverted to the purposes of general illumination. A secondary twilight, however, may be traced even beyond the point D, consequent on a re-reflection of the rays dispersed through the atmosphere in the primary one. The phenomenon seen in the clear atmosphere of the Nubian desert, described by travellers under the name of the "After-glow," would seem to arise from this cause.

(46.) From the explanation we have given, in arts. 39 and 40, of the nature of atmospheric refraction, and the mode in which it is produced in the progress of a ray of light through successive strata, or layers, of the atmosphere, it will be evident, that whenever a ray passes *obliquely* from a higher level to a lower one, or *vice versâ*, its course is not rectilinear, but concave downward; and of course any object seen by means of such a ray, must appear deviated from its true place, whether that object be, like the celestial bodies, entirely beyond the atmosphere, or, like the summits of mountains seen from the plains, or other terrestrial stations at different levels seen from each other, immersed in it. Every difference of level, accompanied, as it must be, with a difference of density in the aerial strata, must also have, corresponding to it, a certain amount of refrac-

tion; less, indeed, than what would be produced by the *whole* atmosphere, but still often of very appreciable, and even considerable, amount. This refraction between terrestrial stations is termed *terrestrial refraction*, to distinguish it from that total effect which is only produced on celestial objects, or such as are beyond the atmosphere, and which is called celestial or astronomical refraction.

(47.) Another effect of refraction is to distort the visible forms and proportions of objects seen near the horizon. The sun, for instance, which at a considerable altitude always appears round, assumes, as it approaches the horizon, a flattened or oval outline; its horizontal diameter being visibly greater than that in a vertical direction. When very near the horizon, this flattening is evidently more considerable on the lower side than on the upper; so that the apparent form is neither circular nor elliptic, but a species of oval, which deviates more from a circle below than above. This singular effect, which any one may notice in a fine sunset, arises from the rapid rate at which the refraction increases in approaching the horizon. Were every visible point in the sun's circumference equally raised by refraction, it would still appear circular, though displaced; but the lower portions being *more* raised than the upper, the vertical diameter is thereby shortened, while the two extremities of its horizontal diameter are equally raised, and in parallel directions, so that its apparent length remains the same. The dilated size (generally) of the sun or moon, when seen near the horizon, beyond what they appear to have when high up in the sky, has nothing to do with refraction. It is an illusion of the judgment, arising from the terrestrial objects interposed, or placed in close comparison with them. In that situation we view and judge of

them as we do of terrestrial objects—in detail, and with an acquired habit of attention to parts. Aloft we have no associations to guide us, and their insulation in the expanse of sky leads us rather to undervalue than to overrate their apparent magnitudes. Actual measurement with a proper instrument corrects our error, without, however, dispelling our illusion. By this we learn, that the sun, when just on the horizon, subtends at our eyes almost exactly the same, and the moon a materially *less* angle, than when seen at a great altitude in the sky, owing to its greater distance from us in the former situation as compared with the latter, as will be explained further on.

(48.) After what has been said of the small extent of the atmosphere in comparison with the mass of the earth, we shall have little hesitation in admitting those luminaries which people and adorn the sky, and which, while they obviously form no part of the earth, and receive no support from it, are yet not borne along at random like clouds upon the air, nor drifted by the winds, to be external to our atmosphere. As such we have considered them while speaking of their refractions—as existing in the immensity of space beyond, and situated, perhaps, for anything we can perceive to the contrary, at enormous distances from us and from each other.

(49.) Could a spectator exist unsustained by the earth, or any solid support, he would see around him at one view the whole contents of space—the visible constituents of the universe: and, in the absence of any means of judging of their distances from him, would refer them, in the directions in which they were seen from his station, to the concave surface of an imaginary sphere, having his eye for a centre, and its surface at some vast indeterminate distance. Per-

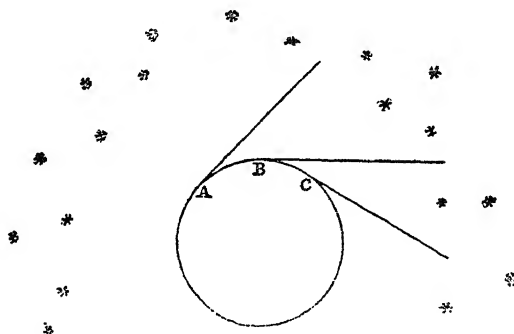
haps he might judge those which appear to him large and bright, to be nearer to him than the smaller and less brilliant; but, independent of other means of judging, he would have no warrant for this opinion, any more than for the idea that all were equidistant from him, and *really* arranged on such a spherical surface. Nevertheless, there would be no impropriety in his referring their places, geometrically speaking, to those points of such a purely imaginary sphere, which their respective visual rays intersect; and there would be much advantage in so doing, as by that means their appearance and relative situation could be accurately measured, recorded, and mapped down. The objects in a landscape are at every variety of distance from the eye, yet we lay them all down in a picture on one plane, and at one distance, in their actual *apparent proportions*, and the likeness is not taxed with incorrectness, though a man in the foreground should be represented larger than a mountain in the distance. So it is to a spectator of the heavenly bodies pictured, *projected*, or mapped down on that imaginary sphere we call the *sky* or *heaven*. Thus, we may easily conceive that the moon, which appears to us as large as the sun, though less bright, *may* owe that apparent equality to its greater proximity, and *may* be really much less; while both the moon and sun may only appear larger and brighter than the stars, on account of the remoteness of the latter.

(50.) A spectator on the earth's surface is prevented, by the great mass on which he stands, from seeing into all that portion of space which is below him, or to see which he must look in any degree downward. It is true that, if his place of observation be at a great elevation, the dip of the horizon will bring within the scope of vision a little more than a hemisphere, and refraction, wherever he may be

situated, will enable him to look, as it were, a little round the corner; but the zone thus added to his visual range can hardly ever, unless in very extraordinary circumstances, exceed a couple of degrees in breadth, and is always ill seen on account of the vapors near the horizon. Unless, then, by a change of his geographical situation, he should shift his horizon (which is always a plane passing through his eye, and touching the spherical convexity of the earth); or unless, by some movements proper to the heavenly bodies, they should of themselves come above his horizon; or, lastly, unless, by some rotation of the earth itself on its centre, the point of its surface which he occupies should be carried round, and presented toward a different region of space; he would never obtain a sight of almost one-half the objects external to our atmosphere. But if any of these cases be supposed, more, or all, may come into view according to the circumstances.

(51.) A traveller, for example, shifting his locality on our globe, will obtain a view of celestial objects invisible from his original station, in a way which may be not inaptly illustrated by comparing him to a person standing in a park close to a large tree. The massive obstacle presented by its trunk cuts off his view of all those parts of the landscape which it occupies as an object; but by walking round it a complete successive view of the whole panorama may be obtained. Just in the same way, if we set off from any station, as London, and travel southward, we shall not fail to notice that many celestial objects which are never seen from London come successively into view, as if rising up above the horizon, night after night, from the south, although it is in reality our horizon, which, travelling with us southward round the sphere, sinks in succession beneath

them. The novelty and splendor of fresh constellations thus gradually brought into view in the clear calm nights of tropical climates, in long voyages to the south, is dwelt upon by all who have enjoyed this spectacle, and never fails to impress itself on the recollection among the most



delightful and interesting of the associations connected with extensive travel. A glance at the accompanying figure, exhibiting three successive stations of a traveller, A, B, C, with the horizon corresponding to each, will place this process in clearer evidence than any description.

(52.) Again: suppose the earth itself to have a motion of rotation on its centre. It is evident that a spectator at rest (as it appears to him) on any part of it will, unperceived by himself, be carried round with it: unperceived, we say, because his horizon will constantly contain, and be limited by, the same terrestrial objects. He will have the same landscape constantly before his eyes, in which all the familiar objects in it, which serve him for landmarks and directions, retain, with respect to himself or to each other, the same invariable situations. The perfect smoothness and equality of the motion of so vast a mass, in which every object he sees around him participates alike, will (art.

15) prevent his entertaining any suspicion of his actual change of place. Yet, with respect to, external objects—that is to say, all celestial ones which do not participate in the supposed rotation of the earth—his horizon will have been all the while shifting in its relation to them, precisely as in the case of our traveller in the foregoing article. Recurring to the figure of that article, it is evidently the same thing, so far as their visibility is concerned, whether he has been carried by the earth's rotation successively into the situations A, B, C; or whether, the earth remaining at rest, he has transferred himself personally along its surface to those stations. Our spectator in the park will obtain precisely the same view of the landscape, whether he walk round the tree, or whether we suppose it sawed off, and made to turn on an upright pivot, while he stands on a projecting step attached to it, and allows himself to be carried round by its motion. The only difference will be in his view of the tree itself, of which, in the former case, he will see every part, but, in the latter, only that portion of it which remains constantly opposite to him, and immediately under his eye.

(53.) By such a rotation of the earth, then, as we have supposed, the horizon of a stationary spectator will be constantly depressing itself below those objects which lie in that region of space toward which the rotation is carrying him, and elevating itself above those in the opposite quarter, admitting into view the former, and successively hiding the latter. As the horizon of every such spectator, however, appears to *him* motionless, all such changes will be referred by him to a motion in the objects themselves so successively disclosed and concealed. In place of his horizon approaching the stars, therefore, he will judge the stars

to approach his horizon; and when it passes over and hides any of them, he will consider them as having sunk below it, or *set*; while those it has just disclosed, and from which it is receding, will seem to be rising above it.

(54.) If we suppose this rotation of the earth to continue in one and the same direction—that is to say, to be performed round one and the same *axis*, till it has completed an entire revolution, and come back to the position from which it set out when the spectator began his observations—it is manifest that everything will then be in precisely the same relative position as at the outset: all the heavenly bodies will appear to occupy the same places in the concave of the sky which they did at that instant, except such as may have actually moved in the interim; and if the rotation still continue, the same phenomena of their successive rising and setting, and return to the same places, will continue to be repeated in the same order, and (if the velocity of rotation be uniform) in equal intervals of time, *ad infinitum*.

(55.) Now, in this we have a lively picture of that grand phenomenon, the most important beyond all comparison which nature presents, the daily rising and setting of the sun and stars, their progress through the vault of the heavens, and their return to the same apparent places at the same hours of the day and night. The accomplishment of this restoration in the regular interval of twenty-four hours is the first instance we encounter of that great law of *periodicity*,¹ which, as we shall see, pervades all astronomy; by which expression we understand the continual reproduction of the same phenomena, in the same order, at equal intervals of time.

¹ Περίοδος, a *going round*, a circulation or revolution.

(56.) A free rotation of the earth round its centre, if it exist and be performed in consonance with the same mechanical laws which obtain in the motions of masses of matter under our immediate control, and within our ordinary experience, must be such as to satisfy two essential conditions. It must be invariable in its direction *with respect to the sphere itself*, and uniform in its velocity. The rotation must be performed *round an axis* or diameter of the sphere, whose *poles* or extremities, where it meets the surface, correspond always to the same points on the sphere. Modes of rotation of a solid body under the influence of external agency are conceivable, in which the poles of the imaginary line or axis about which it is at any moment revolving shall hold no fixed places on the surface, but shift upon it every moment. Such changes, however, are inconsistent with the idea of a rotation of a body of regular figure about its axis of symmetry, performed in free space, and without resistance or obstruction from any surrounding medium, or disturbing influences. The complete absence of such obstructions draws with it, of necessity, the strict fulfilment of the two conditions above mentioned.

(57.) Now, these conditions are in perfect accordance with what we observe, and what recorded observation teaches us, in respect of the diurnal motions of the heavenly bodies. We have no reason to believe, from history, that any sensible change has taken place since the earliest ages in the interval of time elapsing between two successive returns of the same star to the same point of the sky; or, rather, it is demonstrable from astronomical records that no such change *has* taken place. And with respect to the other condition—the *permanence of the axis of rotation*—the appearances which any alteration in that respect must produce,

would be marked, as we shall presently show, by a corresponding change of a very obvious kind in the apparent motions of the stars; which, again, history decidedly declares them *not* to have undergone.

(58.) But, before we proceed to examine more in detail how the hypothesis of the rotation of the earth about an axis accords with the phenomena which the diurnal motion of the heavenly bodies offers to our notice, it will be proper to describe, with precision, in what that diurnal motion consists, and how far it is participated in by them all; or whether any of them form exceptions, wholly or partially, to the common analogy of the rest. We will, therefore, suppose the reader to station himself, on a clear evening, just after sunset, when the first stars begin to appear, in some open situation whence a good general view of the heavens can be obtained. He will then perceive, above and around him, as it were, a vast concave hemispherical vault, beset with stars of various magnitudes, of which the brightest only will first catch his attention in the twilight; and more and more will appear as the darkness increases, till the whole sky is overspangled with them. When he has a while admired the calm magnificence of this glorious spectacle, the theme of so much song, and of so much thought—a spectacle which no one can view without emotion, and without a longing desire to know something of its nature and purport—let him fix his attention more particularly on a few of the most brilliant stars, such as he cannot fail to recognize again without mistake after looking away from them for some time, and let him refer their apparent situations to some surrounding objects, as buildings, trees, etc., selecting purposely such as are in different quarters of his horizon. On comparing them again with their respec-

tive points of reference, after a moderate interval, as the night advances, he will not fail to perceive that they have changed their places, and advanced, as by a general movement, in a westward direction; those toward the eastern quarter appearing to rise or recede from the horizon, while those which lie toward the west will be seen to approach it; and, if watched long enough, will, for the most part, finally sink beneath it, and disappear; while others, in the eastern quarter, will be seen to rise as if out of the earth, and, joining in the general procession, will take their course with the rest toward the opposite quarter.

(59.) If he persist for a considerable time in watching their motions, on the same or on several successive nights, he will perceive that each star appears to describe, as far as its course lies above the horizon, a circle in the sky; that the circles so described are not of the same magnitude for all the stars; and that those described by different stars differ greatly in respect of the parts of them which lie above the horizon. Some, which lie toward the quarter of the horizon which is denominated the SOUTH,⁸ only remain for a short time above it, and disappear, after describing in sight only the small upper segment of their diurnal circle; others, which rise between the south and east, describe larger segments of their circles above the horizon, remain proportionally longer in sight, and set precisely as far to the westward of south as they rose to the eastward; while such as rise exactly in the east remain just twelve hours visible, describe a semicircle, and set exactly in the west. With those, again, which rise between the east and north,

⁸ We suppose our observer to be stationed in some northern latitude; somewhere in Europe, for example.

the same law obtains; at least, as far as regards the time of their remaining above the horizon and the proportion of the visible segment of their diurnal circles to their whole circumferences. Both go on increasing; they remain in view more than twelve hours, and their visible diurnal arcs are more than semicircles. But the magnitudes of the circles themselves diminish, as we go from the east, northward; the greatest of all the circles being described by those which rise exactly in the east point. Carrying his eye further northward, he will notice, at length, stars which, in their diurnal motion, just graze the horizon at its north point, or only dip below it for a moment; while others never reach it at all, but continue always above it, revolving in entire circles round ONE POINT called the POLE, which appears to be the common centre of all their motions, and which alone, in the whole heavens, may be considered immovable. Not that this point is marked by any star. It is a purely imaginary centre; but there is near it one considerably bright star, called the Pole Star, which is easily recognized by the very small circle it describes; so small, indeed, that, without paying particular attention, and referring its position very nicely to some fixed mark, it may easily be supposed at rest, and be, itself, mistaken for the common centre about which all the others in that region describe their circles; or it may be known by its configuration with a very splendid and remarkable *constellation* or group of stars, called by astronomers the GREAT BEAR.

(60.) He will further observe, that the apparent relative situations of all the stars among one another, is not changed by their diurnal motion. In whatever parts of their circles they are observed, or at whatever hour of the night, they form with each other the same identical groups or configu-

rations, to which the name of CONSTELLATIONS has been given. It is true, that, in different parts of their course, these groups stand differently with respect to the horizon; and those toward the north, when in the course of their diurnal movement they pass alternately above and below that common centre of motion described in the last article, become actually inverted with respect to the horizon, while, on the other hand, they always turn the same points toward the pole. In short, he will perceive that the whole assemblage of stars visible at once, or in succession, in the heavens, may be regarded as one great constellation, which seems to revolve with a uniform motion, as if it formed one coherent mass; or as if it were attached to the internal surface of a vast hollow sphere, having the earth, or rather the spectator, in its centre, and turning round an axis inclined to his horizon, so as to pass through that fixed point or *pole* already mentioned.

(61.) Lastly, he will notice, if he have patience to out-watch a long winter's night, commencing at the earliest moment when the stars appear, and continuing till morning twilight, that those stars which he observed setting in the west have again risen in the east, while those which were rising when he first began to notice them have completed their course, and are now set; and that thus the hemisphere, or a great part of it, which was then above, is now beneath him, and its place supplied by that which was at first under his feet, which he will thus discover to be no less copiously furnished with stars than the other, and bespangled with groups no less permanent and distinctly recognizable. Thus he will learn that the great constellation we have above spoken of as revolving round the pole is coextensive with the whole surface of the sphere, being in reality

nothing less than a universe of luminaries surrounding the earth on all sides, and brought in succession before his view, and referred (each luminary according to its own visual ray or direction from his eye) to the imaginary spherical surface, of which he himself occupies the centre. (See art. 49.) There is always, therefore (he would justly argue), a star-bespangled canopy over his head, by day as well as by night, only that the glare of daylight (which he perceives gradually to efface the stars as the morning twilight comes on) prevents them from being seen. And such is really the case. The stars actually continue visible through telescopes in the daytime; and, in proportion to the power of the instrument, not only the largest and brightest of them, but even those of inferior lustre, such as scarcely strike the eye at night as at all conspicuous, are readily found and followed even at noonday—unless in that part of the sky which is very near the sun—by those who possess the means of pointing a telescope accurately to the proper places. Indeed, from the bottoms of deep narrow pits, such as a well, or the shaft of a mine, such bright stars as pass the zenith may even be discerned by the naked eye; and we have ourselves heard it stated by a celebrated optician, that the earliest circumstance which drew his attention to astronomy was the regular appearance, at a certain hour, for several successive days, of a considerable star, through the shaft of a chimney. Venus in our climate, and even Jupiter in the clearer skies of tropical countries, are often visible, without any artificial aid, to the naked eye of one who knows nearly where to look for them. During total eclipses of the sun, the larger stars also appear in their proper situations.

(62.) But to return to our incipient astronomer, whom

we left contemplating the sphere of the heavens, as completed in imagination beneath his feet, and as rising up from thence in its diurnal course. There is one portion or segment of this sphere of which he will not thus obtain a view. As there is a segment toward the north, adjacent to the pole above his horizon, in which the stars *never set*, so there is a corresponding segment, about which the smaller circles of the more southern stars are described, in which they *never rise*. The stars which border upon the extreme circumference of this segment just graze the southern point of his horizon, and show themselves for a few moments above it, precisely as those near the circumference of the northern segment graze his northern horizon, and dip for a moment below it, to reappear immediately. Every point in a spherical surface has, of course, another diametrically opposite to it; and as the spectator's horizon divides his sphere into two hemispheres—a superior and inferior—there must of necessity exist a depressed pole to the south, corresponding to the elevated one to the north, and a portion surrounding it, perpetually beneath, as there is another surrounding the north pole, perpetually above it.

Hic vertex nobis semper sublimis; at illum
Sub pedibus nox atra videt, manesque profundī.”—VIRGIL

One pole rides high, one plunged beneath the main,
Seeks the deep night, and Pluto's dusky reign.

(63.) To get sight of this segment, he must travel southward. In so doing, a new set of phenomena come forward. In proportion as he advances to the south, some of those constellations which, at his original station, barely grazed the northern horizon, will be observed to sink below it and set, at first remaining hid only for a very short time, but

gradually for a longer part of the twenty-four hours. They will continue, however, to circulate about the same point—that is, holding the same invariable position *with respect to them* in the concave of the heavens among the stars; but this point itself will become gradually depressed with respect to the spectator's horizon. The axis, in short, about which the diurnal motion is performed, will appear to have become continually less and less inclined to the horizon; and by the same degrees as the northern pole is depressed the southern will rise, and constellations surrounding it will come into view; at first momentarily, but by degrees for longer and longer times in each diurnal revolution—realizing, in short, what we have already stated in art. 51.

(64.) If he travel continually southward, he will at length reach a line on the earth's surface, called *the equator*, at any point of which, indifferently, if he take up his station and recommence his observations, he will find that he has both the centres of diurnal motion in his horizon, occupying opposite points, the northern pole having been depressed, and the southern raised, so that, in this geographical position, the diurnal rotation of the heavens will appear to him to be performed about a horizontal axis, every star describing half its diurnal circle above and half beneath his horizon, remaining alternately visible for twelve hours, and concealed during the same interval. In this situation, *no* part of the heavens is concealed from his *successive* view. In a night of twelve hours (supposing such a continuance of darkness possible at the equator) the whole sphere will have passed in review over him—the whole hemisphere with which he began his night's observation will have been carried down beneath him, and the entire opposite one brought up from below.

(65). If he pass the equator, and travel still further southward, the southern pole of the heavens will become elevated above his horizon, and the northern will sink below it; and the more, the further he advances southward; and when arrived at a station as far as to the south of the equator as that from which he started was to the north, he will find the whole phenomena of the heavens reversed. The stars which at his original station described their whole diurnal circles above his horizon, and never set, now describe them entirely below it, and never *rise*, but remain constantly invisible to him; and *vice versâ*, those stars which at his former station he never saw, he will now never cease to see.

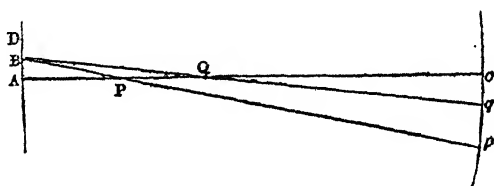
(66.) Finally, if, instead of advancing southward from his first station, he travel northward, he will observe the northern pole of the heavens to become more elevated above his horizon, and the southern more depressed below it. In consequence, his hemisphere will present a less variety of stars, because a greater proportion of the whole surface of the heavens remains constantly visible or constantly invisible: the circle described by each star, too, becomes more nearly parallel to the horizon; and, in short, every appearance leads to suppose that could he travel far enough to the north, he would at length attain a point *vertically under* the northern pole of the heavens, at which none of the stars would either rise or set, but each would circulate round the horizon in circles parallel to it. Many endeavors have been made to reach this point, which is called the north pole of the earth, but hitherto without success; a barrier of almost insurmountable difficulty being presented by the increasing rigor of the climate: but a very near approach to it has been made; and the phenomena of those

regions, though not precisely such as we have described as what must subsist *at* the pole itself, have proved to be in exact correspondence with its near proximity. A similar remark applies to the south pole of the earth, which, however, is more unapproachable, or, at least, has been less nearly approached, than the north.

(67.) The above is an account of the phenomena of the diurnal motion of the stars, as modified by different geographical situations, not grounded on any speculation, but actually observed and recorded by travellers and voyagers. It is, however, in complete accordance with the hypothesis of a rotation of the earth round a fixed axis. In order to show this, however, it will be necessary to premise a few observations on *parallactic motion* in general, and on the appearances presented by an assemblage of remote objects, when viewed from different parts of a small and circumscribed station.

(68.) It has been shown (art. 16) that a spectator in smooth motion, and surrounded by, and forming part of, a great system partaking of the same motion, is unconscious of his own movement, and transfers it in idea to objects external and unconnected, in a contrary direction; those which he leaves behind appearing to recede from, and those which he advances toward to approach, him. Not only, however, do external objects at rest appear in motion generally, with respect to ourselves when we are in motion among them, but they appear to move *one* among the other—they shift their *relative* apparent places. Let any one travelling rapidly along a highroad fix his eye steadily on any object, but at the same time not entirely withdraw his attention from the general landscape—he will see, or think he sees, the whole landscape thrown into *rotation*,

and moving round that object as a centre; all objects between it and himself appearing to move *backward*, or the contrary way to his own motion; and all beyond it, forward, or in the direction in which he moves: but let him withdraw his eye from that object, and fix it on another—a nearer one, for instance—immediately the appearance of rotation shifts also, and the apparent centre about which this illusive circulation is performed is transferred to the new object, which, for the moment, appears to rest. This apparent change of situation of objects with respect to one another, arising from a motion of the spectator, is called a *parallactic* motion. To see the reason of it we must con-



sider that the position of every object is referred by us to the surface of an imaginary sphere of an indefinite radius, having our eye for its centre; and, as we advance in any direction, A B, carrying this imaginary sphere along with us, the visual rays A P, A Q, by which objects are referred to its surface (at *c*, for instance) shift their positions with respect to the line in which we move, A B, which serves as an axis or line of reference, and assume new positions, B P *p*, B Q *q*, revolving round their respective objects as centres. Their intersections, therefore, *p*, *q*, with our visual sphere, will appear to recede on its surface, but with different degrees of angular velocity in proportion to their proximity; the same distance of advance A B subtending a

greater angle, $\angle A P B = \angle C P p$, at the near object P than at the remote one Q.

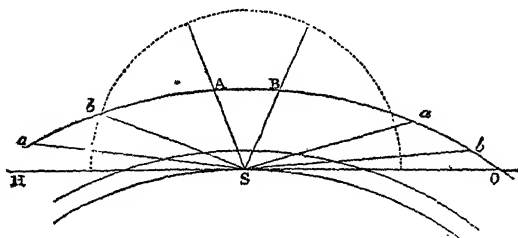
(69.) A consequence of the familiar appearance we have adduced in illustration of these principles is worth noticing, as we shall have occasion to refer to it hereafter. We observe that every object nearer to us than that on which our eye is fixed appears to recede, and those further from us to advance in relation to one another. If then we did not know, or could not judge by any other appearances, which of two objects were nearer to us, this apparent advance or recess of one of them, when the eye is kept steadily fixed on the other, would furnish a criterion. In a dark night, for instance, when all intermediate objects are unseen, the apparent relative movement of two lights which we are assured are themselves fixed, will decide as to their relative proximities. That which seems to advance with us and gain upon the other, or leave it behind it, is the furthest from us.

(70.) The apparent angular motion of an object, arising from a change of our point of view, is called in general *parallax*, and it is always expressed by the angle $\angle A P B$ subtended at the object P (see fig. of art. 68) by a line joining the two points of view A B under consideration. For it is evident that the difference of angular position of P, with respect to the invariable direction A B D, when viewed from A and from B, is the difference of the two angles $\angle D B P$ and $\angle D A P$; now, $\angle D B P$ being the exterior angle of the triangle A B P, is equal to the sum of the interior and opposite, $\angle D B P = \angle D A P + \angle A P B$, whence $\angle D B P - \angle D A P = \angle A P B$.

(71.) It follows from what has been said that the amount of parallactic motion arising from any given change of our

point of view is, *cæteris paribus*, less, as the distance of an object viewed is greater; and when that distance is extremely great in comparison with the change in our point of view, the parallax becomes insensible; or, in other words, objects do not appear to vary in situation at all. It is on this principle, that in alpine regions visited for the first time we are surprised and confounded at the little progress we appear to make by a considerable change of place. An hour's walk, for instance, produces but a small parallactic change in the relative situations of the vast and distant masses which surround us. Whether we walk round a circle of a hundred yards in diameter, or merely turn ourselves round in its centre, the distant panorama presents almost exactly the same aspect—we hardly seem to have changed our point of view.

(72.) Whatever notion, in other respects, we may form of the stars, it is quite clear they must be immensely distant. Were it not so, the apparent angular interval between any two of them seen overhead would be much greater than when seen near the horizon, and the constellations, instead

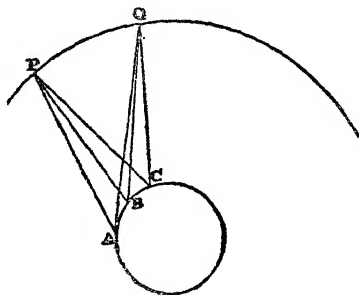


of preserving the same appearances and dimensions during their whole diurnal course, would appear to enlarge as they rise higher in the sky, as we see a small cloud in the horizon swell into a great overshadowing canopy when drifted by the wind across our zenith, or as may be seen in the above

figure, where $a b$, $A B$, $a b$, are three different positions of the same stars, as they would, if near the earth, be seen from a spectator S , under the visual angles $a S b$, $A S B$. No such change of apparent dimension, however, is observed. The nicest measurements of the apparent angular distance of any two stars *inter se*, taken in any parts of their diurnal course (after allowing for the unequal effects of refraction, or when taken at such times that this cause of distortion shall act equally on both), manifest *not the slightest* perceptible variation. Not only this, but at whatever point of the earth's surface the measurement is performed, the results are *absolutely identical*. No instruments ever yet invented by man are delicate enough to indicate, by an increase or diminution of the angle subtended, that one point of the earth is nearer to or further from the stars than another.

(73.) The necessary conclusion from this is, that the dimensions of the earth, large as it is, are comparatively *nothing*, absolutely imperceptible, when compared with the interval which separates the stars from the earth. If an observer walk round a circle not more than a few yards in diameter, and from different points in its circumference measure with a sextant or other more exact instrument adapted for the purpose, the angles $P A Q$, $P B Q$, $P C Q$, subtended at those stations by two well-defined points in his visible horizon, $P Q$, he will at once be advertised, by the difference of the results, of his change of distance from them arising from his change of place, although that difference may be so small as to produce no change in their *general* aspect to his unassisted sight. This is one of the innumerable instances where accurate measurement obtained by instrumental means places us in a totally different situa-

tion in respect to matters of fact, and conclusions thence deducible, from what we should hold, were we to rely in all cases on the mere judgment of the eye. To so great a nicety have such observations been carried by the aid of an instrument called a theodolite, that a circle even a few inches in diameter may thus be rendered *sensible*, may thus be detected to have a *size*, and an ascertainable *place*, by reference to objects distant by fully 100,000 times its own dimensions. Observations, differing, it is true, somewhat



in method, but identical in principle, and executed with quite as much exactness, have been applied to the stars, and with a result such as has been already stated. Hence it follows, incontrovertibly, that the distance of the stars from the earth cannot be *so small* as 100,000 of the earth's diameters. It is, indeed, incomparably greater; for we shall hereafter find it fully demonstrated that the distance just named, immense as it may appear, is yet much underrated.

(74.) From such a distance, to a spectator with our faculties, and furnished with our instruments, the earth would be imperceptible; and, reciprocally, an object of the earth's size, placed at the distance of the stars, would be equally undiscernible. If, therefore, at the point on which a spectator stands, we draw a plane touching the globe, and pro-

long it in imagination till it attain the region of the stars, and through the centre of the earth conceive another plane parallel to the former, and coextensive with it, to pass; these, although separated throughout their whole extent by the same interval, viz., a semidiameter of the earth, will yet, on account of the vast distance at which that interval is seen, be confounded together, and indistinguishable from each other in the region of the stars, when viewed by a spectator on the earth. The zone they there include will be of evanescent breadth to his eye, and will only mark out a great circle in the heavens, one and the same for both the stations. This great circle, when spoken of as a circle of the sphere, is called the *celestial horizon* or simply *the horizon*, and the two planes just described are also spoken of as the *sensible* and the *rational* horizon of the observer's station.

(75.) From what has been said (art. 73) of the distance of the stars, it follows, that if we suppose a spectator at the centre of the earth to have his view bounded by the *rational* horizon, in exactly the same manner as that of a corresponding spectator on the surface is by his *sensible* horizon, the two observers will see the same stars in the same relative situations, each beholding that entire hemisphere of the heavens which is above the celestial horizon, corresponding to their common zenith. Now, so far as appearances go, it is clearly the same thing whether the heavens, that is, all space with its contents, revolve round a spectator at rest in the earth's centre, or whether that spectator simply turn round in the opposite direction in his place, and view them in succession. The aspect of the heavens, at every instant, as referred to his horizon (which must be supposed to turn with him), will be the same in

both suppositions. And since, as has been shown, appearances are also, so far as the stars are concerned, the same to a spectator on the surface as to one at the centre, it follows that, whether we suppose the heavens to revolve without the earth, or the earth within the heavens, *in the opposite direction*, the diurnal phenomena, to all its inhabitants, will be no way different.

(76.) The Copernican astronomy adopts the latter as the true explanation of these phenomena, avoiding thereby the necessity of otherwise resorting to the cumbrous mechanism of a solid but invisible sphere, to which the stars must be supposed attached, in order that they may be carried round the earth without derangement of their relative situations *inter se*. Such a contrivance would, indeed, suffice to explain the diurnal revolution of the stars, so as to "save appearances"; but the movements of the sun and moon, as well as those of the planets, are incompatible with such a supposition, as will appear when we come to treat of these bodies. On the other hand, that a spherical mass of moderate dimensions (or, rather, when compared with the surrounding and visible universe, of evanescent magnitude), held by no tie, and free to move and to revolve, should do so, in conformity with those general laws which, so far as we know, regulate the motions of all material bodies, is so far from being a postulate difficult to be conceded, that the wonder would rather be should the fact prove otherwise. As a postulate, therefore, we shall henceforth regard it; and as, in the progress of our work, analogies offer themselves in its support from what we observe of other celestial bodies, we shall not fail to point them out to the reader's notice.

(77.) The earth's rotation on its axis so admitted, ex-

plaining, as it evidently does, the apparent motion of the stars in a completely satisfactory manner, prepares us for the further admission of its motion, bodily, in space, should such a motion enable us to explain, in a manner equally so, the apparently complex and enigmatical motions of the sun, moon, and planets. The Copernican astronomy adopts this idea in its full extent, ascribing to the earth, in addition to its motion of *rotation* about an axis, also one of *translation* or transference through space, in such a course or *orbit*, and so regulated in direction and celerity, as, taken in conjunction with the motions of the other bodies of the universe, shall render a rational account of the appearances they successively present—that is to say, an account of which the several parts, postulates, propositions, deductions, intelligibly cohere, without contradicting each other or the nature of things as concluded from experience. In this view of the Copernican doctrine it is rather a geometrical conception than a physical theory, inasmuch as it simply assumes the requisite motions, without attempting to explain their mechanical origin, or assign them any dependence on physical causes. The Newtonian theory of gravitation supplies this deficiency, and, by showing that all the motions required by the Copernican conception *must*, and that no others can, result from a single, intelligible, and very simple dynamical law, has given a degree of certainty to this conception, as a matter of fact, which attaches to no other creation of the human mind.

(78.) To understand this conception in its further developments, the reader must bear steadily in mind the distinction between *relative* and *absolute* motion. Nothing is easier to perceive than that, if a spectator at rest view a certain number of moving objects, they will group and

arrange themselves *to his eye*, at each successive moment, in a very different way from what they would do were he in active motion among them—if he formed one of them, for instance, and joined in their dance. This is evident from what has been said before of parallax motion; but it will be asked, How is such a spectator to disentangle from each other the two parts of the apparent motions of these external objects—that which arises from the effect of his own change of place, and which is therefore only apparent (or, as a German metaphysician would say, *subjective*—having reference only to him as perceiving it)—and that which is real (or *objective*—having a positive existence, whether perceived by him or not)? By what rule is he to ascertain, from the appearances presented to him while himself in motion, what *would be* the appearances were he at rest? It by no means follows, indeed, that he would even then at once obtain a clear conception of all the motions of all the objects. The appearances so presented to him would have still something *subjective* about them. They would be still *appearances*, not geometrical realities. They would still have a reference to the point of view, which might be very unfavorably situated (as, indeed, is the case in our system) for affording a clear notion of the real movement of each object. No geometrical figure, or curve, is seen by the eye as it is conceived by the mind to exist in reality. The laws of perspective interfere and alter the apparent directions and foreshorten the dimensions of its several parts. If the spectator be unfavorably situated, as, for instance, nearly in the plane of the figure (which is the case we have to deal with), they may do so to such an extent, as to make a considerable effort of imagination necessary to pass from the sensible to the real form.

(79.) Still, preparatory to this ultimate step, it is first necessary that the spectator should free or clear the appearances from the disturbing influence of his own change of place. And this he can always do by the following general rule or proposition:

*The relative motion of two bodies is the same as if either of them were at rest, and all its motion communicated to the other in an opposite direction.**

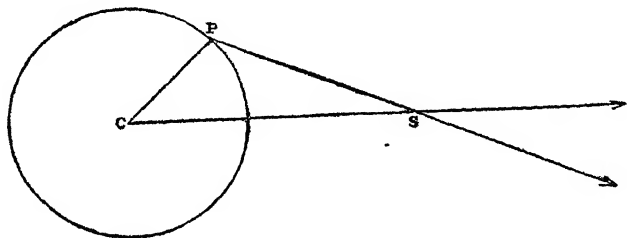
Hence, if two bodies move alike, they will, when seen from each other (without reference to other near bodies, but only to the starry sphere), appear at rest. Hence, also, if the absolute motions of two bodies be uniform and rectilinear, their relative motion is so also.

(80.) The stars are so distant, that as we have seen it is absolutely indifferent from what point of the earth's surface we view them. Their configurations *inter se* are identically the same. It is otherwise with the sun, moon, and planets, which are near enough (especially the moon) to be *parallaxically* displaced by change of station from place to place on our globe. In order that astronomers residing on different points of the earth's surface should be able to compare their observations with effect, it is necessary that they should clearly understand and take account of this effect of the difference of their stations on the appearance of the outward universe as seen from each. As an exterior object seen from one would appear to have shifted its place were

* This proposition is equivalent to the following, which precisely meets the case proposed, but requires somewhat more thought for its clear apprehension than can perhaps be expected from a beginner:

PROP.—*If two bodies, A and B, be in motion independently of each other, the motion which B seen from A would appear to have if A were at rest is the same with that which it would appear to have, A being in motion, if, in addition to its own motion, a motion equal to A's and in the same direction were communicated to it.*

the spectator suddenly transported to the other, so two spectators, viewing it from the two stations at the same instant, do not see it in the same *direction*. Hence arises a necessity for the adoption of a conventional centre of reference, or imaginary station of observation common to all the world, to which each observer, wherever situated, may refer (or, as it is called, *reduce*) his observations, by calculating and allowing for the effect of his local position with respect to that common centre (supposing him to possess the necessary data). If there were only two observers, in



fixed stations, one might agree to refer his observations to the other station; but, as every locality on the globe may be a station of observation, it is far more convenient and natural to fix upon a point equally related to all, as the common point of reference; and this can be no other than the centre of the globe itself. The parallaxic change of apparent place which would arise in an object, could any observer suddenly transport himself to the centre of the earth, is evidently the angle CSP , subtended at the object S by that radius CP of the earth which joins its centre and the place P of observation.

CHAPTER II

Terminology and Elementary Geometrical Conceptions and Relations—Terminology relating to the Globe of the Earth—To the Celestial Sphere—Celestial Perspective

(81.) SEVERAL of the terms in use among astronomers have been explained in the preceding chapter, and others used anticipatively. But the technical language of every subject requires to be formally stated, both for consistency of usage and definiteness of conception. We shall therefore proceed, in the first place, to define a number of terms in perpetual use, having relation to the globe of the earth and the celestial sphere.

(82.) DEFINITION 1. The *axis* of the earth is that diameter about which it revolves, with a uniform motion, *from west to east*; performing one revolution in the interval which elapses between any star leaving a certain point in the heavens, and returning to the same point again.

(83.) DEF. 2. The *poles* of the earth are the points where its axis meets its surface. The North Pole is that nearest to Europe; the South Pole that most remote from it.

(84.) DEF. 3. The *earth's equator* is a great circle on its surface, equidistant from its poles, dividing it into two hemispheres—a northern and a southern; in the midst of which are situated the respective poles of the earth of those names. The *plane* of the equator is, therefore, a plane perpendicular to the earth's axis, and passing through its centre.

(85.) DEF. 4. The *terrestrial meridian* of a station on the earth's surface, is a great circle of the globe passing through both poles and through the place. The plane of the meridian is the plane in which that circle lies.

(86.) DEF. 5. The *sensible* and the *rational horizon* of any station have been already defined in art. 74.

(87.) DEF. 6. A *meridian line* is the line of intersection of the plane of the meridian of any station with the plane of the sensible horizon, and therefore marks the north and south points of the horizon, or the directions in which a spectator must set out if he would travel directly toward the north or south pole.

(88.) DEF. 7. The *latitude* of a place on the earth's surface is its angular distance from the equator, measured on its own terrestrial meridian: it is reckoned in degrees, minutes, and seconds, from 0 up to 90° , and northward or southward, according to the hemisphere the place lies in. Thus, the observatory at Greenwich is situated in $51^{\circ} 28' 40''$ north latitude. This definition of latitude, it will be observed, is to be considered as only temporary. A more exact knowledge of the physical structure and figure of the earth, and a better acquaintance with the niceties of astronomy, will render some modification of its terms, or a different manner of considering it, necessary.

(89.) DEF. 8. *Parallels of latitude* are small circles on the earth's surface parallel to the equator. Every point in such a circle has the same latitude. Thus, Greenwich is said to be situated *in the parallel of* $51^{\circ} 28' 40''$.

(90.) DEF. 9. The *longitude* of a place on the earth's surface is the inclination of its meridian to that of some fixed station referred to as a point to reckon from. English astronomers and geographers use the observatory at Green-

wich for this station; foreigners, the principal observatories of their respective nations. Some geographers have adopted the island of Ferro. Hereafter, when we speak of longitude, we reckon from Greenwich. The longitude of a place is, therefore, measured by the arc of the equator intercepted between the meridian of the place and that of Greenwich; or, which is the same thing, by the spherical angle at the pole included between these meridians.

(91.) As *latitude* is reckoned north or south, so *longitude* is usually said to be reckoned west or east. It would add greatly, however, to systematic regularity, and tend much to avoid confusion and ambiguity in computations, were this mode of expression abandoned, and longitudes reckoned invariably *westward* from their origin round the whole circle from 0 to 360°. Thus, the longitude of Paris is, in common parlance, either 2° 20' 22" east, or 357° 39' 38" west of Greenwich. But, in the sense in which we shall henceforth use and recommend others to use the term, the latter is its proper designation. Longitude is also reckoned in time at the rate of 24h. for 360°, or 15° per hour. In this system the longitude of Paris is 23h. 50m. 39½s.¹

(92.) Knowing the longitude and latitude of a place, it may be laid down on an artificial globe; and thus a map of the earth may be constructed. Maps of particular countries are detached portions of this general map, extended into planes; or, rather, they are representations on planes of such portions, executed according to certain conventional systems of rules, called *projections*, the object of which is either to

¹ To distinguish minutes and seconds of time from those of angular measure we shall invariably adhere to the distinct system of notation here adopted (° ' ", and h. m. s.). Great confusion sometimes arises from the practice of using the same marks for both.

distort as little as possible the outlines of countries from what they are on the globe—or to establish easy means of ascertaining, by inspection or graphical measurement, the latitudes and longitudes of places which occur in them, without referring to the globe or to books—or for other peculiar uses. See Chap. IV.

(93.) DEF. 10. The *Tropics* are two parallels of latitude, one on the north and the other on the south side of the equator, over every point of which, respectively, the sun in its diurnal course passes vertically on the 21st of June and the 21st of December in every year. Their latitudes are about $23^{\circ} 28'$ respectively, north and south.

(94.) DEF. 11. The Arctic and Antarctic circles are two small circles or parallels of latitude as distant from the north and south poles as the tropics are from the equator, that is to say, about $23^{\circ} 28'$; their latitudes, therefore, are about $66^{\circ} 32'$. We say *about*, for the places of these circles and of the tropics are continually shifting on the earth's surface, though with extreme slowness, as will be explained in its proper place.

(95.) DEF. 12. The sphere of the heavens or of the stars is an imaginary spherical surface of infinite radius, having the eye of any spectator for its centre, and which may be conceived as a ground on which the stars, planets, etc., the visible contents of the universe, are seen projected as in a vast picture.²

² The ideal sphere without us, to which we refer the places of objects, and which we carry along with us wherever we go, is no doubt intimately connected by association with, if not entirely dependent on that obscure perception of sensation in the retinae of our eyes, of which, even when closed and unexcited, we cannot entirely divest them. We have a real spherical surface within our eyes, the seat of sensation and vision, corresponding, point for point, to the external sphere. On this the stars, etc., are really mapped down, as we have supposed them in the text to be, on the imaginary concave of the heavens. When the

(96.) DEF. 13. The *poles* of the celestial sphere are the points of that imaginary sphere toward which the earth's axis is directed.

(97.) DEF. 14. The celestial equator, or, as it is often called by astronomers, the *equinoctial*, is a great circle of the celestial sphere, marked out by the indefinite extension of the plane of the terrestrial equator.

(98.) DEF. 15. The celestial horizon of any place is a great circle of the sphere marked out by the indefinite extension of the plane of any spectator's *sensible* or (which comes to the same thing, as will presently be shown), his *rational* horizon, as in the case of the equator.

(99.) DEF. 16. The *zenith* and *nadir*³ of a spectator are the two points of the sphere of the heavens, vertically over his head, and vertically under his feet, or the *poles* of the celestial horizon; that is to say, points 90° distant from every point in it.

(100.) DEF. 17. *Vertical circles* of the sphere are great circles passing through the zenith and nadir, or great circles perpendicular to the horizon. On these are measured the *altitudes* of objects above the horizon—the complements to which are their *zenith distances*.

(101.) DEF. 18. The *celestial meridian* of a spectator is the great circle marked out on the sphere by the prolongation of the plane of his terrestrial meridian. If the earth

whole surface of the retina is excited by light, habit leads us to associate it with the idea of a real surface existing without us. Thus we become impressed with the notion of a *sky* and a *heaven*, but the concave surface of the retina itself is the true seat of all *visible* angular dimension and angular motion. The substitution of the *retina* for the *heavens* would be awkward and inconvenient in language, but it may always be mentally made. (See Schiller's pretty enigma on the eye in his *Turandot*.)

³ From Arabic words, *semî*, vertex, and *almadhîr*, corresponding or opposite to; nadir corresponds evidently to the German *nieder* (down), whence our *nether*.

be supposed at rest, this is a fixed circle, and all the stars are carried across it in their diurnal courses from east to west. If the stars rest and the earth rotate, the spectator's meridian, like his horizon (art. 52), sweeps daily across the stars from west to east. Whenever in future we speak of the meridian of a spectator or observer, we intend the celestial meridian, which being a circle passing through the poles of the heavens and the zenith of the observer, is necessarily a vertical circle, and passes through the north and south points of the horizon.

(102.) DEF. 19. The *prime vertical* is a vertical circle perpendicular to the meridian, and which therefore passes through the east and west points of the horizon.

(103.) DEF. 20. *Azimuth* is the angular distance of a celestial object from the north or south point of the horizon (according as it is the north or south pole which is *elevated*), when the object is referred to the horizon by a vertical circle; or it is the angle comprised between two vertical planes—one passing through the elevated pole, the other through the object. *Azimuth may* be reckoned eastward or westward, from the north or south point, and is usually so reckoned only to 180° either way. But to avoid confusion, and to preserve continuity of interpretation when algebraic symbols are used (a point of essential importance, hitherto too little insisted on), we shall always reckon azimuth *from* the point of the horizon *most remote* from the *elevated* pole, *westward* (so as to agree in general directions with the apparent diurnal motion of the stars), and carry its reckoning from 0° to 360° if always reckoned positive, considering the eastward reckoning as negative.

(104.) DEF. 21. The *altitude* of a heavenly body is its apparent angular elevation above the horizon. It is the

complement to 90° , therefore, of its zenith distance. The altitude and azimuth of an object being known, its place in the visible heavens is determined.

(105.) DEF. 22. The *declination* of a heavenly body is its angular distance from the equinoctial or celestial equator, or the complement to 90° of its angular distance from the nearest pole, which latter distance is called its *Polar distance*. Declinations are reckoned *plus* or *minus*, according as the object is situated in the northern or southern celestial hemisphere. *Polar distances* are always reckoned from the North Pole, from 0° up to 180° , by which all doubt or ambiguity of expression with respect to sign is avoided.

(106.) DEF. 23. Hour circles of the sphere, or circles of declination, are great circles passing through the poles, and of course perpendicular to the equinoctial. The hour circle, passing through any particular heavenly body, serves to refer it to a point in the equinoctial, as a vertical circle does to a point in the horizon.

(107.) DEF. 24. The hour angle of a heavenly body is the angle at the pole included between the hour circle passing through the body, and the celestial meridian of the place of observation. We shall always reckon it *positively* from the *upper* culmination (art. 125) *westward*, or in conformity with the *apparent* diurnal motion, completely round the circle from 0° to 360° . *Hour angles*, generally, are angles included at the pole between different hour circles.

(108.) DEF. 25. The *right ascension* of a heavenly body is the arc of the equinoctial included between a certain point in that circle called the *Vernal Equinox*, and the point in the same circle to which it is referred by the circle of declination passing through it. Or it is the angle included between two hour circles, one of which passes

through the vernal equinox (and is called the equinoctial colure), the other through the body. How the place of this initial point on the equinoctial is determined, will be explained further on.

(109.) The right ascensions of celestial objects are always reckoned *eastward* from the equinox, and are estimated either in degrees, minutes and seconds, as in the case of terrestrial longitudes, from 0° to 360° , which completes the circle; or, in time, in hours, minutes and seconds, from 0h. to 24h. The apparent diurnal motion of the heavens being contrary to the real motion of the earth, this is in conformity with the westward reckoning of longitudes. (Art. 91.)

(110.) *Sidereal time* is reckoned by the diurnal motion of the stars, or rather of that point in the equinoctial from which right ascensions are reckoned. This point may be considered as a star, though no star is, in fact, there; and, moreover, the point itself is liable to a certain slow variation—so slow, however, as not to affect, perceptibly, the interval, of any two of its successive returns to the meridian. This interval is called a sidereal day, and is divided into 24 sidereal hours, and these again into minutes and seconds. A clock which marks sidereal time, *i.e.* which goes at such a rate as always to show 0h. 0m. 0s. when the equinox comes on the meridian, is called a sidereal clock, and is an indispensable piece of furniture in every observatory. Hence the hour angle of an object reduced to time at the rate of 15° per hour, expresses the interval of sidereal time by which (if its reckoning be positive) it has passed the meridian; or if negative, the time it wants of arriving at the meridian of the place of observation. So also the right ascension of an object, if converted into time at the same

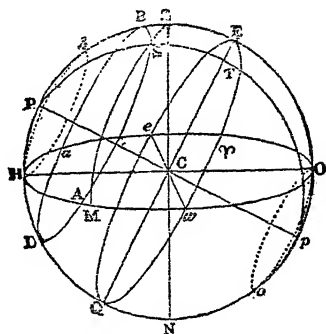
rate (since 360° being described uniformly in 24 hours, 15° must be so described in 1 hour), will express the interval of sidereal time which elapses from the passage of the vernal equinox across the meridian to that of the object next subsequent.

(111.) As a globe or maps may be made of the whole or particular regions of the surface of the earth, so also a globe, or general map of the heavens, as well as charts of particular parts, may be constructed, and the stars laid down in their proper situations relative to each other, and to the poles of the heavens and the celestial equator. Such a representation, once made, will exhibit a true appearance of the stars as they present themselves in succession to every spectator on the surface, or as they may be conceived to be seen at once by one at the centre of the globe. It is, therefore, independent of all *geographical* localities. There will occur in such a representation neither zenith, nadir, nor horizon—neither east nor west points; and although great circles may be drawn on it from pole to pole, corresponding to terrestrial meridians, they can no longer, in this point of view, be regarded as the celestial meridians of fixed points on the earth's surface, since, in the course of one diurnal revolution, every point in it passes beneath each of them. It is on account of this change of conception, and with a view to establish a complete distinction between the two branches of *Geography* and *Uranography*,^{*} that astronomers have adopted different terms (viz., *declination* and *right ascension*) to represent those arcs in the heavens which correspond to *latitudes* and *longitudes* on the earth. It is for this reason that they term the equator of the heavens the *equinoctial*; that

^{*} Γη, the earth; γράφειν, to describe or represent; ουρανός, the heaven.

allel to S C N, the direction in which an observer at A will see the *elevated* pole of the heavens; and A Z, the prolongation of the terrestrial radius C A, that of his zenith. N A E S will be his meridian; N G S that of some fixed station, as Greenwich; and G E, or the spherical angle G N E, his longitude, and E A his latitude. Moreover, if *n s* be a plane touching the surface in A, this will be his sensible horizon; *n A s* marked on that plane by its intersection with his meridian will be his meridian line, and *n* and *s* the north and south points of his horizon.

(113.) Again, neglecting the size of the earth, or conceiving him stationed at its centre, and referring everything to his *rational* horizon; let the annexed figure represent the sphere of the heavens; C the spectator; Z his zenith; and N his nadir: then will H A .O, a great circle of the sphere, whose poles are Z N, be his *celestial* hori-



zon; P *p* the *elevated* and *depressed* POLES of the heavens; H P the *altitude of the pole*, and H P Z E O his *meridian*; E T Q, a great circle perpendicular to P *p*, will be the *equinoctial*; and if *r* represent the equinox, *r T* will be the *right ascension*, *T S* the *declination*, and P S the *polar distance* of any star or object S, referred to the equi-

noctial by the *hour circle* $PSTp$; and BSD will be the diurnal circle it will appear to describe about the pole. Again, if we refer it to the horizon by the *vertical circle* ZSM , OM will be its azimuth, MS its altitude, and ZS its zenith distance. H and O are the north and south, e w the east and west points of his horizon, or of the heavens. Moreover, if Hh , Oo , be small circles, or *parallels of declination*, touching the horizon in its north and south points, Hh will be the circle of *perpetual apparition*, between which and the elevated pole the stars never set; Oo that of *perpetual occultation*, between which and the depressed pole they *never rise*. In all the zone of the heavens between Hh and Oo , they rise and set; any one of them, as S , remaining above the horizon in that part of its diurnal circle represented by aBA , and below it throughout all the part represented by ADa . It will exercise the reader to construct this figure for several different *elevations of the pole*, and for a variety of positions of the star S in each.

(114.) Celestial perspective is that branch of the general science of perspective which teaches us to conclude, from a knowledge of the real situation and forms of objects, lines, angles, motions, etc., with respect to the spectator, their apparent aspects, as seen by him projected on the imaginary concave of the heavens; and, *vice versâ*, from the apparent configurations and movements of objects so seen projected, to conclude, so far as they can be thence concluded, their real geometrical relations to each other and to the spectator. It agrees with ordinary perspective when only a small visual area is contemplated, because the concave ground of the celestial sphere, for a small extent, may be regarded as a plane surface, on which objects are seen projected or depicted as in common perspective. But

when large amplitudes of the visual area are considered, or when the whole contents of space are regarded as projected on the whole interior surface of the sphere, it becomes necessary to use a different phraseology, and to resort to a different form of conception. In common perspective there is a single "point of sight," or "centre of the picture," the visual line from the eye to which is perpendicular to the "plane of the picture," and all straight lines are represented by straight lines. In celestial perspective, every point to which the view is for the moment directed, is equally entitled to be considered as the "centre of the picture," every portion of the surface of the sphere being similarly related to the eye. Moreover, every straight line (supposed to be indefinitely prolonged) is projected into a semicircle of the sphere, that, namely, in which a plane passing through the line and the eye cuts its surface. And every system of parallel straight lines, in whatever direction, is projected into a system of semicircles of the sphere, meeting in two common apexes, or vanishing points, diametrically opposite to each other, one of which corresponds to the vanishing point of parallels in ordinary perspective; the other in such perspective has no existence. In other words, every point in the sphere to which the eye is directed may be regarded as one of the vanishing points, or one apex of a system of straight lines parallel to that radius of the sphere which passes through it or to the direction of the line of sight, seen in perspective from the earth, and the point diametrically opposite, or that from which he is looking, as the other. And any great circle of the sphere may similarly be regarded as the *vanishing circle* of a system of planes, parallel to its own.

(115.) A familiar illustration of this is often to be had

by attending to the lines of light seen in the air, when the sun's rays are darted through apertures in clouds, the sun itself being at the time obscured behind them. These lines which, marking the course of rays emanating from a point almost infinitely distant, are to be considered as parallel straight lines, are thrown into great circles of the sphere, having two apexes or points of common intersection—one in the place where the sun itself (if not obscured) would be seen, the other diametrically opposite. The first only is most commonly suggested when the spectator's view is toward the sun. But in mountainous countries, the phenomenon of sunbeams converging toward a point diametrically opposite to the sun, and as much depressed below the horizon as the sun is elevated above it, is not infrequently noticed, the back of the spectator being turned to the sun's place. Occasionally, but much more rarely, the whole course of such a system of sunbeams, stretching in semicircles across the hemisphere from horizon to horizon (the sun being near setting), may be seen.* Thus again, the streamers of the Aurora Borealis, which are doubtless electrical rays, parallel, or nearly parallel to each other, and to the dipping-needle, usually appear to diverge from the point toward which the needle, freely suspended, would dip northward (*i.e.* about 70° below the

* It is in such cases only that we conceive them as circles, the ordinary conventions of plane perspective becoming untenable. The author had the good fortune to witness on one occasion the phenomenon described in the text under circumstances of more than usual grandeur. Approaching Lyons from the south on September 30, 1826, about $5\frac{1}{2}$ h. P.M., the sun was seen nearly setting behind broken masses of stormy cloud, from whose apertures streamed forth beams of rose-colored light, traceable all across the hemisphere almost to their opposite point of convergence behind the snowy precipices of Mont Blanc, conspicuously visible at nearly 100 miles to the eastward. The impression produced was that of another but feebler sun about to rise from behind the mountain, and darting forth precursory beams to meet those of the real one opposite.

horizon and 23° west of north from London), and in their upward progress pursue the course of great circles till they again converge (in appearance) toward the point diametrically opposite (*i.e.* 70° above the horizon and 23° to the eastward of south), forming a sort of canopy overhead, having that point for its centre. So also in the phenomenon of shooting stars, the lines of direction which they appear to take on certain remarkable occasions of periodical recurrence, are observed, if prolonged backward, apparently to meet nearly in one point of the sphere; a certain indication of a general near approach to parallelism in the real directions of their motions on those occasions. On which subject more hereafter.

(116.) In relation to this idea of celestial perspective, we may conceive the north and south poles of the sphere as the two vanishing points of a system of lines parallel to the axis of the earth; and the zenith and nadir of those of a system of perpendiculars to its surface at the place of observation, etc. It will be shown that the direction of a *plumb-line* at every place is perpendicular to the surface of still water at that place, which is the true horizon; and though mathematically speaking no two plumb-lines are exactly parallel (since they converge to the earth's centre), yet over very small tracts, such as the area of a building—in one and the same town, etc., the difference from exact parallelism is so small that it may be practically disregarded.* To a spectator looking upward such a system of plumb-lines will appear to converge to his zenith; downward, to his nadir.

* An interval of a mile corresponds to a convergence of plumb-lines amounting to somewhat less space than a minute.

(117.) So also the celestial equator, or the equinoctial, must be conceived as the vanishing circle of a system of planes parallel to the earth's equator, or perpendicular to its axis. The celestial horizon of any spectator is in like manner the vanishing circle of all planes parallel to his true horizon, of which planes his *rational* horizon (passing through the earth's centre) is one, and his *sensible* horizon (the tangent plane of his station) another.

(118.) Owing, however, to the absence of all the ordinary indications of distance which influence our judgment in respect of terrestrial objects; owing to the want of determinate figure and magnitude in the stars and planets as commonly seen—the projection of the celestial bodies on the ground of the heavenly concave is not usually regarded in this its true light of a *perspective representation or picture*, and it even requires an effort of imagination to conceive them in their true relations, as at vastly different distances, one behind the other, and forming with one another lines of junction violently foreshortened, and including angles altogether differing from those which their projected representations appear to make. To do so at all with effect presupposes a knowledge of their actual situations in space, which it is the business of astronomy to arrive at by appropriate considerations. But the connections which subsist among the several parts of the *picture*, the purely geometrical relations among the angles and sides of the spherical triangles of which it consists, constitute, under the name of *Uranometry*,¹ a preliminary and subordinate branch of the general science, with which it is necessary to be familiar before any further progress can be made. Some of the most elementary

¹ Ουρανός, the heavens; μετρεῖν, to measure; the measurement of the heavens.

and frequently occurring of these relations we proceed to explain. And first, as immediate consequences of the above definitions, the following propositions will be borne in mind.

(119.) *The altitude of the elevated pole is equal to the latitude of the spectator's geographical station.*

For it appears, see *fig.* art. 112, that the angle $P A Z$ between the pole and the zenith is equal to $N C A$, and the angles $Z A n$ and $N C E$ being right angles, we have $P A n = A C E$. Now the former of these is the elevation of the pole as seen from E , the latter is the angle at the earth's centre subtended by the arc $E A$, or the latitude of the place.

(120.) Hence to a spectator at the north pole of the earth, the north pole of the heavens is in his zenith. As he travels southward it becomes less and less elevated till he reaches the equator, when both poles are in his horizon—south of the equator the north pole becomes depressed below, while the south rises above his horizon, and continues to do so till the south pole of the globe is reached, when that of the heavens will be in the zenith.

(121.) The same stars, in their diurnal revolution, come to the meridian, *successively*, of every place on the globe once in twenty-four sidereal hours. And, since the diurnal rotation is uniform, the interval, in sidereal time, which elapses between the same star coming upon the meridians of two different places is measured by the difference of longitudes of the places.

(122.) *Vice versâ*—the interval elapsing between two different stars coming on the meridian of one and the same place, expressed in sidereal time, is the measure of the difference of right ascensions of the stars.

(123.) The equinoctial intersects the horizon in the east

and west points, and the meridian in a point whose altitude is equal to the co-latitude of the place. Thus, at Greenwich, of which the latitude is $51^{\circ} 28' 40''$, the altitude of the intersection of the equinoctial and meridian is $38^{\circ} 31' 20''$. The north and south poles of the heavens are the poles of the equinoctial. The east and west points of the horizon of a spectator are the poles of his celestial meridian. The north and south points of his horizon are the poles of his prime vertical, and his zenith and nadir are the poles of his horizon.

(124.) All the heavenly bodies *culminate* (*i.e.* come to their greatest altitudes) on the meridian; which is, therefore, the best situation to observe them, being least confused by the inequalities and vapors of the atmosphere, as well as least displaced by refraction.

(125.) All celestial objects within the circle of perpetual apparition come twice on the meridian, above the horizon, in every diurnal revolution; once *above* and once *below* the pole. These are called their *upper* and *lower culminations*.

(126.) The problems of uranometry, as we have described it, consist in the solution of a variety of spherical triangles, both right and oblique angled, according to the rules, and by the formulæ of spherical trigonometry, which we suppose known to the reader, or for which he will consult appropriate treatises. We shall only here observe generally, that in all problems in which spherical geometry is concerned, the student will find it a useful practical maxim rather to consider the poles of the great circles which the question before him refers to than the circles themselves. To use, for example, in the relations he has to consider, polar distances rather than declinations, zenith distances rather than altitudes, etc. Bearing this in mind, there are few prob-

lems in uranometry which will offer any difficulty. The following are the combinations which most commonly occur for solution *when the place of one celestial object only on the sphere is concerned.*

(127.) In the triangle ZPS , Z is the zenith, P the elevated pole, and S the star, sun, or other celestial object. In this triangle occur, 1st, PZ , which being the complement of PH (the altitude of the pole), is obviously the complement of the latitude (or the *co-latitude*, as it is called) of the place; 2d, PS , the *polar distance*, or the complement of the declination (*co-declination*) of the star; 3d, ZS , the *zenith distance* or *co-altitude* of the star. If PS be greater than 90° , the object is situated on the side of the equinoctial opposite to that of the elevated pole. If ZS be so, the object is below the horizon.

In the same triangle the angles are, 1st, ZPS the hour angle; 2d, PZS (the supplement of SZO , which latter is the azimuth of the star or other heavenly body); 3d, PSZ , an angle which, from the infrequency of any practical reference to it, has not acquired a name.*

The following five astronomical magnitudes, then, occur among the sides and angles of this most useful triangle: viz., 1st, the co-latitude of the place of observation; 2d, the polar distance; 3d, the zenith distance; 4th, the hour angle; and 5th, the sub-azimuth (supplement of azimuth) of a given celestial object; and by its solution therefore may all problems be resolved, in which three of these magnitudes are directly or indirectly given, and the other two required to be found.

* In the practical discussion of the measures of double stars and other objects by the aid of the position micrometer, this angle is sometimes required to be known; and, when so required, it will be not inconveniently referred to as "the angle of position of the zenith."

the moment of observation. Hence we may find PZ , the co-latitude of the place. Moreover, the hour angle of the star being known, and also its right ascension, the point of the equinoctial is known, which is on the meridian at the moment of observation; and, therefore, the local sidereal time at that moment. This is a very useful observation for determining the latitude and time at an unknown station.

CHAPTER III¹

Of the Nature of Astronomical Instruments and Observations in General
—Of Sidereal and Solar Time—Of the Measurements of Time—
Clocks, Chronometers—Of Astronomical Measurements—Principle of
Telescopic Sights to Increase the Accuracy of Pointing—Simplest
Application of this Principle—The Transit Instrument—Of the Meas-
urement of Angular Intervals—Methods of Increasing the Accuracy of
Reading—The Vernier—The Microscope—Of the Mural Circle—The
Meridian Circle—Fixation of Polar and Horizontal Points—The Level,
Plumb-line, Artificial Horizon—Principle of Collimation—Collimators
of Rittenhouse, Kater and Bohnenberger—Of Compound Instruments
with Co-ordinate Circles—The Equatorial, Altitude and Azimuth In-
strument—Theodolite—Of the Sextant and Reflecting Circle—Principle
of Repetition—Of Micrometers—Parallel Wire Micrometer—Principle
of the Duplication of Images—The Heliometer—Double Refracting
Eye-piece—Variable Prism Micrometer—Of the Position Micrometer
—Illumination of Wires—Solar Telescope and Eye-piece—Helioscopy
—Collimation of large Reflectors

(130.) OUR first chapters have been devoted to the acquisition chiefly of preliminary notions respecting the globe we inhabit, its relation to the celestial objects which surround it, and the physical circumstances under which all

¹ The student who is anxious to become acquainted with the chief subject matter of this work, may defer the reading of that part of this chapter which is devoted to the description of particular instruments, or content himself with a cursory perusal of it, until further advanced, when it will be necessary to return to it.

astronomical observations must be made, as well as to provide ourselves with a stock of *technical words* and elementary ideas of most frequent and familiar use in the sequel. We might now proceed to a more exact and detailed statement of the facts and theories of astronomy; but, in order to do this with full effect, it will be desirable that the reader be made acquainted with the principal means which astronomers possess, of determining, with the degree of nicety their theories require, the data on which they ground their conclusions; in other words, of ascertaining by measurement the apparent and real magnitudes with which they are conversant. It is only when in possession of this knowledge that he can fully appreciate either the truth of the theories themselves, or the degree of reliance to be placed on any of their conclusions antecedent to trial: since it is only by knowing what amount of error can certainly be perceived and distinctly measured, that he can satisfy himself whether any theory offers so close an approximation, in its numerical results, to actual phenomena, as will justify him in receiving it as a true representation of nature.

(131.) Astronomical instrument-making may be justly regarded as the most refined of the mechanical arts, and that in which the nearest approach to geometrical precision is required, and has been attained. It may be thought an easy thing, by one unacquainted with the niceties required, to turn a circle in metal, to divide its circumference into 360 equal parts, and these again into smaller subdivisions—to place it accurately on its centre, and to adjust it in a given position; but practically it is found to be one of the most difficult. Nor will this appear extraordinary, when it is considered that, owing to the application of telescopes to the purposes of angular measurement, every imperfection

of structure or division becomes magnified by the whole optical power of that instrument; and that thus, not only direct errors of workmanship, arising from unsteadiness of hand or imperfection of tools, but those inaccuracies which originate in far more uncontrollable causes, such as the unequal expansion and contraction of metallic masses by a change of temperature, and their unavoidable flexure or bending by their own weight, become perceptible and measurable. An angle of one minute occupies, on the circumference of a circle of 10 inches in radius, only about $\frac{1}{360}$ th part of an inch, a quantity too small to be *certainly* dealt with without the use of magnifying glasses; yet one minute is a gross quantity in the astronomical measurement of an angle. With the instruments now employed in observatories, a single second, or the 60th part of a minute, is rendered a distinctly visible and appreciable quantity. Now the arc of a circle, subtended by one second, is less than the 200,000th part of the radius, so that on a circle of 6 feet in diameter it would occupy no greater linear extent than $\frac{1}{72000}$ th part of an inch; a quantity requiring a powerful microscope to be *discerned* at all. Let any one figure to himself, therefore, the difficulty of placing on the circumference of a metallic circle of such dimensions (supposing the difficulty of its construction surmounted), 360 marks, dots, or cognizable divisions, which shall all be true to their places within such narrow limits; to say nothing of the subdivision of the degrees so marked off into minutes, and of these again into seconds. Such a work has probably baffled, and will probably forever continue to baffle, the utmost stretch of human skill and industry; nor, if executed, could it endure. The ever varying fluctuations of heat and cold have a tendency to produce not merely temporary and transient, but per-

manent, uncompensated changes of form in all considerable masses of those metals which alone are applicable to such uses; and their own weight, however symmetrically formed, must always be unequally sustained, since it is impossible to apply the sustaining power to *every part* separately: even could this be done, at all events force must be used to move and to fix them; which can never be done without producing temporary and risking permanent change of form. It is true, by dividing them on their centres, and in the identical places they are destined to occupy, and by a thousand ingenious and delicate contrivances, wonders have been accomplished in this department of art, and a degree of perfection has been given, not merely to *chefs d'œuvre*, but to instruments of moderate prices and dimensions, and in ordinary use, which, on due consideration, must appear very surprising. But though we are entitled to look for *wonders* at the hands of scientific artists, we are not to expect *miracles*. The demands of the astronomer will always surpass the power of the artist; and it must, therefore, be constantly the aim of the former to make himself, as far as possible, independent of the imperfections incident to every work the latter can place in his hands. He must, therefore, endeavor so to combine his observations, so to choose his opportunities, and so to familiarize himself with all the causes which may produce instrumental derangement, and with all the peculiarities of structure and material of each instrument he possesses, as not to allow himself to be misled by their errors, but to extract from their indications, as far as possible, all that is *true*, and reject all that is erroneous. It is in this that the art of the practical astronomer consists—an art of itself of a curious and intricate nature, and of which we can here only notice some of the leading and general features.

(132.) The great aim of the practical astronomer being numerical correctness in the results of instrumental measurement, his constant care and vigilance must be directed to the detection and compensation of errors, either by annihilating, or by taking account of, and allowing for them. Now, if we examine the sources from which errors may arise in any instrumental determination, we shall find them chiefly reducible to three principal heads:—

(133.) 1st, External or incidental causes of error; comprehending those which depend on external, uncontrollable circumstances: such as, fluctuations of weather, which disturb the amount of refraction from its tabulated value, and, being reducible to no fixed law, induce uncertainty to the extent of their own possible magnitude; such as, by varying the temperature of the air, vary also the form and position of the instruments used, by altering the relative magnitudes and the tension of their parts; and others of the like nature.

(134.) 2dly, *Errors of observation*: such as arise, for example, from inexpertness, defective vision, slowness in seizing the exact *instant* of occurrence of a phenomenon, or precipitancy in anticipating it, etc.; from atmospheric indistinctness; insufficient optical power in the instrument, and the like. Under this head may also be classed all errors arising from momentary instrumental derangement—slips in clamping, looseness of screws, etc.

(135.) 3dly, The third, and by far the most numerous class of errors to which astronomical measurements are liable, arise from causes which may be deemed instrumental, and which may be subdivided into two principal classes. The *first* comprehends those which arise from an instrument not *being* what it professes to be, which is *error of workman-*

ship. Thus, if a pivot or axis, instead of being, as it ought, exactly cylindrical, be slightly flattened, or elliptical—if it be not exactly (as it is intended it should be) concentric with the circle it carries;—if this circle (so called) be in reality *not* exactly circular, or not in one plane;—if its divisions, intended to be precisely equidistant, should be placed in reality at unequal intervals—and a hundred other things of the same sort. These are not mere speculative sources of error, but practical annoyances, which every observer has to contend with.

(136.) The *other* subdivision of instrumental errors comprehends such as arise from an instrument not being placed in the *position* it ought to have; and from those of its parts, which are made purposely movable, not being properly disposed *inter se*. These are *errors of adjustment*. Some are unavoidable, as they arise from a general unsteadiness of the soil or building in which the instruments are placed; which, though too minute to be noticed in any other way, become appreciable in delicate astronomical observations: others, again, are consequences of imperfect workmanship, as where an instrument once well adjusted will not remain so, but keeps deviating and shifting. But the most important of this class of errors arise from the non-existence of natural indications, *other* than those afforded by astronomical observations themselves, whether an instrument has or has not the exact position, with respect to the horizon and its cardinal points, the axis of the earth, or to other principal astronomical lines and circles, which it ought to have to fulfil properly its objects.

(137.) Now, with respect to the first two classes of error, it must be observed, that, in so far as they cannot be reduced to known laws, and thereby become subjects of cal-

culatation and due allowance, they actually vitiate, to their full extent, the results of any observations in which they subsist. Being, however, in their nature casual and accidental, their effects necessarily lie sometimes one way, sometimes the other; sometimes diminishing, sometimes tending to increase the results. Hence, by greatly multiplying observations, under varied circumstances, by avoiding unfavorable, and taking advantage of favorable circumstances of weather, or otherwise using opportunity to advantage—and finally, by taking the *mean or average* of the results obtained, this class of errors may be so far *subdued*, by setting them to destroy one another, as no longer sensibly to vitiate any theoretical or practical conclusion. This is the great and indeed only resource against such errors, not merely to the astronomer, but to the investigator of numerical results in every department of physical research.

(138.) With regard to errors of adjustment and workmanship, not only the *possibility*, but the *certainty* of their existence, in every imaginable form, in all instruments, must be contemplated. Human hands or machines never formed a circle, drew a straight line, or erected a perpendicular, nor ever placed an instrument in *perfect* adjustment, unless accidentally; and then only during an instant of time. This does not prevent, however, that a great approximation to all these desiderata should be attained. But it is the peculiarity of astronomical observation to be the *ultimate means of detection* of all mechanical defects which elude by their minuteness every other mode of detection. What the eye cannot discern nor the touch perceive, a course of astronomical observations will make distinctly evident. The imperfect products of man's hands are here

tested by being brought into comparison under very great magnifying powers (corresponding in effect to a great increase in acuteness of perception) with the perfect workmanship of nature; and there is none which will bear the trial. Now, it may seem like arguing in a vicious circle, to deduce theoretical conclusions and laws from observation, and then to turn round upon the instruments with which those observations were made; accuse them of imperfection, and attempt to detect and rectify their errors by means of the very laws and theories which they have helped us to a knowledge of. A little consideration, however, will suffice to show that such a course of proceeding is perfectly legitimate.

(139.) The steps by which we arrive at the laws of natural phenomena, and especially those which depend for their verification on numerical determinations, are necessarily successive. Gross results and palpable laws are arrived at by rude observation with coarse instruments, or without any instruments at all, and are expressed in language which is not to be considered as absolute, but is to be interpreted with a degree of latitude commensurate to the imperfection of the observations themselves. These results are corrected and refined by nicer scrutiny, and with more delicate means. The first rude expressions of the laws which embody them are perceived to be inexact. The language used in their expression is corrected, its terms more rigidly defined, or fresh terms introduced, until the new state of language and terminology is brought to fit the improved state of knowledge of facts. In the progress of this scrutiny subordinate laws are brought into view which still further modify, both the verbal statement and numerical results of those which first offered themselves to our notice; and when these are

traced out and reduced to certainty, others, again, subordinate to them, make their appearance, and become subjects of further inquiry. Now, it invariably happens (and the reason is evident) that the first glimpse we catch of such subordinate laws—the first form in which they are dimly shadowed out to our minds—is that of *errors*. We perceive a discordance between what we *expect*, and what we *find*. The first occurrence of such a discordance we attribute to accident. It happens again and again; and we begin to suspect our instruments. We then inquire, to what amount of error their determinations can, *by possibility*, be liable. If their *limit of possible error* exceed the observed deviation, we at once condemn the instrument, and set about improving its construction or adjustments. Still the same deviations occur, and, so far from being palliated, are more marked and better defined than before. We are now sure that we are on the traces of a law of nature, and we pursue it till we have reduced it to a definite statement, and verified it by repeated observation, under every variety of circumstances.

(140.) Now, in the course of this inquiry, it will not fail to happen that other discordances will strike us. Taught by experience, we suspect the existence of some natural law, before unknown; we tabulate (*i.e.* draw out in order) the results of our observations; and we perceive, in this synoptic statement of them, distinct indications of a regular progression. Again we improve or vary our instruments, and we now lose sight of this supposed new law of nature altogether, or find it replaced by some other, of a totally different character. Thus we are led to suspect an instrumental cause for what we have noticed. We examine, therefore, the *theory* of our instrument; we suppose defects

in its structure, and, by the aid of geometry, we trace their influence in introducing *actual errors* into its indications. These errors have *their laws*, which, so long as we have no knowledge of causes to guide us, may be confounded with laws of nature, as they are mixed up with them in their effects. They are not fortuitous, like errors of observation, but, as they arise from sources inherent in the instrument, and unchangeable while it and its adjustments remain unchanged, they are reducible to fixed and ascertainable forms; each particular defect, whether of structure or adjustment, producing its own appropriate *form* of error. When these are thoroughly investigated, we recognize among them one which coincides in its nature and progression with that of our observed discordances. The mystery is at once solved. We have detected, by direct observation, an instrumental defect.

(141.) It is, therefore, a chief requisite for the practical astronomer to make himself completely familiar with the *theory* of his instruments. By this alone is he enabled at once to decide what *effect* on his observations any given imperfection of structure or adjustment will produce in any given circumstances under which an observation can be made. This alone also can place him in a condition to derive available and practical means of destroying and eliminating altogether the influence of such imperfections, by so arranging his observations, that it shall affect their results in opposite ways, and that its influence shall thus disappear from their mean, which is one of the chief modes by which precision is attained in practical astronomy. Suppose, for example, the principle of an instrument required that a circle should be concentric with the axis on which it is made to turn. As this is a condition which no workman-

ship can *exactly* fulfil, it becomes necessary to inquire what errors will be produced in observations made and registered on the faith of such an instrument, by any assigned deviation in this respect; that is to say, what would be the disagreement between observations made with it and with one absolutely perfect, could such be obtained. Now, simple geometrical considerations suffice to show—1st, that if the axis be excentric by a given fraction (say one thousandth part) of the radius of the circle, all angles read off on that part of the circle *toward* which the excentricity lies, will appear by that fractional amount too small, and all on the opposite side too large. And, 2dly, that *whatever* be the amount of the excentricity, and on *whatever* part of the circle any proposed angle is measured, the effect of the error in question on the result of observations depending on the graduation of its circumference (or limb, as it is technically called) will be completely annihilated by the very easy method of always reading off the divisions on two diametrically opposite points of the circle, and taking a mean; for the effect of excentricity is always to increase the arc representing the angle in question on one side of the circle, by just the same quantity by which it diminishes that on the other. Again, suppose that the proper use of the instrument required that this axis should be exactly parallel to that of the earth. As it never can be *placed* or remain so, it becomes a question, what amount of error will arise, in its use, from any assigned deviation, whether in a horizontal or vertical plane, from this precise position. Such inquiries constitute the theory of instrumental errors; a theory of the utmost importance to practice, and one of which a complete knowledge will enable an observer, with moderate instrumental means, often to attain a degree of precision

which might seem to belong only to the most refined and costly. This theory, as will readily be apprehended, turns almost entirely on considerations of pure geometry, and those for the most part not difficult. In the present work, however, we have no further concern with it. The astronomical instruments we propose briefly to describe in this chapter will be considered as perfect both in construction and adjustment.²

(142.) As the above remarks are very essential to a right understanding of the philosophy of our subject and the spirit of astronomical methods, we shall elucidate them by taking one or two special cases. Observant persons, before the invention of astronomical instruments, had already concluded the apparent diurnal motions of the stars to be performed in circles about fixed poles in the heavens, as shown in the foregoing chapter. In drawing this conclusion, however, refraction was entirely overlooked, or, if forced on their notice by its great magnitude in the immediate neighborhood of the horizon, was regarded as a local irregularity, and, as such, neglected or slurred over. As soon, however, as the diurnal paths of the stars were attempted to be traced by instruments, even of the coarsest kind, it became evident that the notion of exact circles described about one and the same pole would not represent the phenomena correctly, but that, owing to some cause or other, the apparent diurnal orbit of every star is distorted from a circular into an oval form, its lower segment being *flatter* than its upper; and the deviation being greater the

² The principle on which the chief adjustments of two or three of the most useful and common instruments, such as the transit, the equatorial, and the sextant, are performed, are, however, noticed, for the convenience of readers who may use such instruments without going further into the arcana of practical astronomy.

nearer the star approached the horizon, the effect being the same as if the circle had been squeezed upward from below, and the lower parts more than the higher. For such an effect, as it was soon found to arise from no casual or instrumental cause, it became necessary to seek a natural one; and refraction readily occurred, to solve the difficulty. In fact, it is a case precisely analogous to what we have already noticed (art. 47), of the apparent distortion of the sun near the horizon, only on a larger scale, and traced up to greater altitudes. This new law once established, it became necessary to modify the expression of that anciently received, by inserting in it a *salvo* for the effect of refraction, or by making a distinction between the *apparent* diurnal orbits, as affected by refraction, and the *true* ones cleared of that effect. This distinction between the *apparent* and the *true*—between the *uncorrected* and *corrected*—between the *rough* and *obvious*, and the *refined* and *ultimate*—is of perpetual occurrence in every part of astronomy.

(143.) Again. The first impression produced by a view of the diurnal movement of the heavens is that *all* the heavenly bodies perform this revolution in one common period, viz., a *day*, or 24 hours. But no sooner do we come to examine the matter *instrumentally*, i.e. by noting, by timekeepers, their successive arrivals on the meridian, than we find differences which cannot be accounted for by any error of observation. All the *stars*, it is true, occupy the same interval of time between their successive appulses to the meridian, or to any vertical circle; but *this* is a very different one from that occupied by the sun. It is palpably shorter; being, in fact, only 23^h 56' 4.09", instead of 24 hours, such hours as our common clocks mark. Here, then, we have already *two different days*, a *sidereal* and a

solar; and if, instead of the sun, we observe the moon, we find a third, much longer than either—a *lunar* day, whose average duration is $24^h 54^m$ of our ordinary time, which last is *solar* time, being of necessity conformable to the *sun's* successive reappearances, on which all the business of life depends.

(144.) Now, all the *stars* are found to be unanimous in giving the same exact duration of $23^h 56' 4.09''$, for the *sidereal day*; which, therefore, we cannot hesitate to receive as the period in which the earth makes one revolution on its axis. We are, therefore, compelled to look on the sun and moon as exceptions to the general law; as having a different nature, or at least a different relation to us, from the stars; and as having motions, real or apparent, of their own, independent of the rotation of the earth on its axis. Thus a great and most important distinction is disclosed to us.

(145.) To establish these facts, almost no apparatus is required. An observer need only station himself to the north of some well-defined vertical object, as the angle of a building, and, placing his eye exactly at a certain fixed point (such as a small hole in a plate of metal nailed to some immovable support), notice the successive disappearances of any star behind the building, by a watch.* When he observes the sun, he must shade his eye with a dark-colored or smoked glass, and notice the moments when its

* This is an excellent practical method of ascertaining the rate of a clock or watch, being exceedingly accurate if a few precautions are attended to; the chief of which is, to take care that that part of the edge behind which the star (a bright one, *not a planet*) disappears shall be quite smooth; as otherwise variable refraction may transfer the point of disappearance from a protuberance to a notch, and thus vary the moment of observation unduly. This is easily secured, by nailing up a smooth-edged board. The verticality of its edge should be insured by the use of a plumb-line.

western and eastern edges successively come up to the wall, from which, by taking half the interval, he will ascertain (what he cannot directly *observe*) the moment of disappearance of its centre.

(146.) When, in pursuing and establishing this general fact, we are led to attend more nicely to the times of the daily arrival of the sun on the meridian, irregularities (such they first seem to be) begin to make their appearance. The intervals between two successive arrivals are not the same at all times of the year. They are sometimes greater, sometimes less, than 24 hours, as shown by the clock; that is to say, the *solar day* is not always of the same length. About the 21st of December, for example, it is half a minute *longer*, and about the same day of September nearly as much *shorter*, than its *average duration*. And thus a distinction is again pressed upon our notice between the *actual* solar day, which is never two days in succession alike, and the *mean solar day* of 24 hours, which is an average of all the solar days throughout the year. Here, then, a new source of inquiry opens to us. The sun's apparent motion is not only not the same with that of the stars, but it is not (as the latter is) uniform. It is subject to fluctuations, whose laws become matter of investigation. But to pursue these laws, we require nicer means of observation than what we have described, and are obliged to call in to our aid an instrument called the *transit instrument*, especially destined for such observations, and to attend minutely to all the causes of irregularity in the going of clocks and watches which may affect our reckoning of time. Thus we become involved by degrees in more and more delicate instrumental inquiries; and we speedily find that, in proportion as we ascertain the amount and law

of one great or leading fluctuation, or inequality, as it is called, of the sun's diurnal motion, we bring into view others continually smaller and smaller, which were before obscured, or mixed up with errors of observation and instrumental imperfections. In short, we may not inaptly compare the *mean* length of the solar day to the mean or average height of water in a harbor, or the general level of the sea unagitated by tide or waves. The great annual fluctuation above noticed may be compared to the daily variations of level produced by the tides, which are nothing but enormous waves extending over the whole ocean, while the smaller subordinate inequalities may be assimilated to waves ordinarily so called, on which, when large, we perceive lesser undulations to ride, and on these, again, minuter rippings, to the series of whose subordination we can perceive no end.

(147.) With the causes of these irregularities in the solar motion we have no concern at present; their explanation belongs to a more advanced part of our subject: but the distinction between the solar and sidereal days, as it pervades every part of astronomy, requires to be early introduced, and never lost sight of. It is, as already observed, the *mean* or *average* length of the solar day, which is used in the civil reckoning of time. It commences at midnight, but astronomers, even when they use mean solar time, depart from the civil reckoning, commencing their day at noon, and reckoning the hours from 0 round to 24. Thus, 11 o'clock in the forenoon of the second of January, in the civil reckoning of time, corresponds to January 1 day 23 hours in the astronomical reckoning; and 1 o'clock in the afternoon of the former, to January 2 days 1 hour of the latter reckoning. This usage has its advantages and

disadvantages, but the latter seem to preponderate; and it would be well if, in consequence, it could be broken through, and the civil reckoning substituted. *Uniformity in nomenclature and modes of reckoning in all matters relating to time, space, weight, measure, etc., is of such vast and paramount importance in every relation of life as to outweigh every consideration of technical convenience or custom.**

(148.) Both astronomers and civilians, however, who inhabit different points of the earth's surface, differ from each other in their reckoning of time; as it is obvious they must, if we consider that, when it is noon at one place, it is midnight at a place diametrically opposite; sunrise at another; and sunset, again, at a fourth. Hence arises considerable inconvenience, especially as respects places differing very widely in situation, and which may even in some critical cases involve the mistake of a whole day. To obviate this inconvenience, there has lately been introduced a system of reckoning time by mean solar days and parts of a day counted from a fixed instant, common to all the world, and determined by no *local* circumstance, such as noon or midnight, but by the motion of the sun among the stars. Time, so reckoned, is called equinoctial time; and is numerically the same, at the same instant, in every part of the globe. Its origin will be explained more fully at a more advanced stage of our work.

* The only disadvantage to astronomers of using the civil reckoning is this—that their observations being chiefly carried on during the night, the day of their date will, in this reckoning, always have to be changed at midnight, and the former and latter portion of every night's observations will belong to two differently numbered civil days of the month. There is no denying this to be an inconvenience. Habit, however, would alleviate it; and *some* inconveniences must be cheerfully submitted to by all who resolve to act on general principles. All other classes of men, whose occupation extends to the night as well as day, submit to it, and find their advantage in doing so.

(149.) Time is an essential element in astronomical observation, in a twofold point of view:—1st, As the representative of angular motion. The earth's diurnal motion being uniform, every star describes its diurnal circle uniformly; and the time elapsing between the passage of the stars in succession across the meridian of any observer becomes, therefore, a direct measure of their differences of right ascension. 2dly, As the fundamental element (or natural *independent variable*, to use the language of geometers) in all dynamical theories. The great object of astronomy is the determination of the laws of the celestial motions, and their reference to their proximate or remote causes. Now, the statement of the *law* of any observed motion in a celestial object can be no other than a proposition declaring what has been, is, and will be, the real or apparent situation of that object *at any time*, past, present, or future. To compare such laws, therefore, with observation, we must possess a register of the observed situations of the object in question, and of the *times when* they were observed.

(150.) The measurement of time is performed by clocks, chronometers, clepsydras, and hour-glasses. The two former are alone used in modern astronomy. The hour-glass is a coarse and rude contrivance for measuring, or rather counting out, fixed portions of time, and is entirely disused. The clepsydra, which measured time by the gradual emptying of a large vessel of water through a determinate orifice, is susceptible of considerable exactness, and was the only dependence of astronomers before the invention of clocks and watches. At present it is abandoned, owing to the greater convenience and exactness of the latter instruments. In one case only has the revival of its use been proposed; viz., for the accurate measurement of very small portions

of time, by the flowing out of mercury from a small orifice in the bottom of a vessel, kept constantly full to a fixed height. The stream is intercepted at the moment of noting any event, and directed aside into a receiver, into which it continues to run, till the moment of noting any other event, when the intercepting cause is suddenly removed, the stream flows in its original course, and ceases to run into the receiver. The *weight* of mercury received, compared with the weight received in an interval of time observed by the clock, gives the interval between the events observed. This ingenious and simple method of resolving, with all possible precision, a problem of much importance in many physical inquiries, is due to the late Captain Kater.

(151.) The pendulum clock, however, and the balance watch, with those improvements and refinements in its structure which constitute it emphatically a *chronometer*,⁵ are the instruments on which the astronomer depends for his knowledge of the lapse of time. These instruments are now brought to such perfection, that a habitual irregularity in the *rate* of going, to the extent of a single second in twenty-four hours in two consecutive days, is not tolerated in one of good character; so that any interval of time less than twenty-four hours may be certainly ascertained within a few tenths of a second, by their use. In proportion as intervals are longer, the risk of error, as well as the amount of error risked, becomes greater, because the accidental errors of many days may accumulate; and causes producing a slow progressive change in the rate of going may subsist unperceived. It is not safe, therefore, to trust the determination of time to clocks, or watches, for many

⁵ Χρονος, time; μετρευν, to measure.

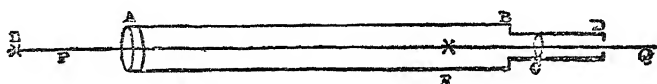
days in succession, without checking them, and ascertaining their errors by reference to natural events which we know to happen, day after day, at equal intervals. But if this be done, the longest intervals may be fixed with the same precision as the shortest; since, in fact, it is then only the times intervening between the first and the last moments of such long intervals, and such of those periodically recurring events adopted for our points of reckoning, as occur within twenty-four hours respectively of either, which we measure by artificial means. The whole days are counted out for us by nature; the fractional parts only, at either end, are measured by our clocks. To keep the reckoning of the integer days correct, so that none shall be lost or counted twice, is the object of the calendar. Chronology marks out the order of succession of events, and refers them to their proper years and days; while chronometry, grounding its determinations on the precise observation of such regularly periodical events as can be conveniently and exactly subdivided, enables us to fix the moments in which phenomena occur, with the last degree of precision.

(152.) In the *culmination* or *transit* (*i.e.* the passage across the meridian of an observer) of every star in the heavens, he is furnished with such a regularly periodical natural event as we allude to. Accordingly, it is to the *transits* of the brightest and most conveniently situated fixed stars that astronomers resort to ascertain their exact time, or, which comes to the same thing, to determine the exact amount of error of their clocks.

(153.) Before we describe the instrument destined for the purpose of observing such culminations, however, or those intended for the measurement of angular intervals in the sphere, it is requisite to place clearly before the reader

the principle on which the telescope is applied in astronomy to the precise determination of a direction in space—that, namely, of the visual ray by which we see a star or any other distant object.

(154.) The telescope most commonly used in astronomy for these purposes is the refracting telescope, which consists of an object-glass (either single, or as is now almost universal, double, forming what is called in optics, an achromatic combination) A; a tube AB, into which the brass cell of the object-glass is firmly screwed, and an eye-lens C, for which is often substituted a combination of glasses designed to in-



crease the magnifying power of the telescope, or otherwise give more distinctness of vision according to optical principles which we have no occasion here to refer to. This also is fitted into a cell, which is screwed firmly into the end B of the tube, so that object-glass, tube, and eye-glass may be considered as forming one piece, invariable in the relative position of its parts.

(155.) The line P Q joining the centres of the object and eye-glasses and produced, is called the *axis* or *line of collimation* of the telescope. And it is evident, that the situation of this line holds a fixed relation to the tube and its appendages, so long as the object and eye-glasses maintain *their* fixity in this respect.

(156.) Whatever distant object E this line is directed to, an inverted picture or image of that object F is formed (according to the principles of optics), in the focus of the object-glass, and may there be viewed as if it were a *real*

object, through the eye-lens C, which (if of short focus) enables us to *magnify* it just as such a lens would magnify a material object in the same place.

(157.) Now as this image is formed and viewed in the air, being itself immaterial and impalpable—nothing prevents our placing in that very place F in the axis of the telescope, a real, substantial object of very definite form and delicate make, such as a fine metallic point, as of a needle—or better still, a cross formed by two very fine threads (spider-lines), thin metallic wires, or lines drawn on glass intersecting each other at right angles—and whose intersection is all but a mathematical point. If such a point, wire, or cross be carefully placed and firmly fixed in the exact focus F, both of the object and eye-glass, it will be seen through the latter *at the same time*, and occupying *the same precise place* as the image of the distant star E. The magnifying power of the lens renders perceptible the smallest deviation from perfect coincidence, which, should it exist, is a proof, that the axis Q P is not directed rigorously toward E. In that case, a fine motion (by means of a screw duly applied), communicated to the telescope, will be necessary to vary the direction of the axis till the coincidence is rendered perfect. So precise is this mode of pointing found in practice, that the axis of a telescope may be directed toward a star or other definite celestial object without an error of more than a few tenths of a second of angular measure.

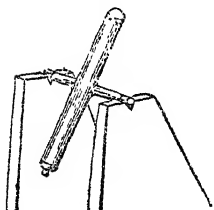
(158.) This application of the telescope may be considered as completely annihilating that part of the error of observation which might otherwise arise from an erroneous estimation of the direction in which an object lies from the observer's eye, or from the centre of the instrument. It is,

in fact, the grand source of all the precision of modern astronomy, without which all other refinements in instrumental workmanship would be thrown away; the errors capable of being committed in pointing to an object, without such assistance, being far greater than what could arise from any but the very coarsest graduation.⁶ In fact, the telescope thus applied becomes, with respect to angular, what the microscope is with respect to linear dimension. By concentrating attention on its smallest parts, and magnifying into palpable intervals the minutest differences, it enables us not only to scrutinize the form and structure of the objects to which it is pointed, but to refer their apparent places, with all but geometrical precision, to the parts of any scale with which we propose to compare them.

(159). We now return to our subject, the determination of time by the transit or culminations of celestial objects. The instrument with which such culminations are observed is called a *transit instrument*. It consists of a telescope

⁶ The honor of this capital improvement has been successfully vindicated by Derham (Phil. Trans. xxx. 603) to our young, talented and unfortunate countryman Gascoigne, from his correspondence with Crabtree and Horrockes, in his (Derham's) possession. The passages cited by Derham from these letters leave no doubt that, so early as 1640, Gascoigne had applied telescopes to his quadrants and sextants, *with threads in the common focus of the glasses*; and had even carried the invention so far as to illuminate the field of view by artificial light, which he found "*very helpful when the moon appeareth not, or it is not otherwise light enough.*" These inventions were freely communicated by him to Crabtree, and through him to his friend Horrockes, the pride and boast of British astronomy; both of whom expressed their unbounded admiration of this and many other of his delicate and admirable improvements in the art of observation. Gascoigne, however, perished at the age of twenty-three, at the battle of Marston Moor; and the premature and sudden death of Horrockes, at a yet earlier age, will account for the temporary oblivion of the invention. It was revived, or re-invented, in 1667, by Picard and Auzout (Lalande, Astron. 2310), after which its use became universal. Morin, even earlier than Gascoigne (in 1635), had proposed to substitute the telescope for plain sights; but it is the thread or wire stretched in the focus with which the image of a star can be brought to exact coincidence, which gives the telescope its advantage in practice; and the idea of this does not seem to have occurred to Morin. (See Lalande, *ubi suprà*.)

firmly fastened on a horizontal axis directed to the east and west points of the horizon, or at right angles to the plane of the meridian of the place of observation. The extremities of the axis are formed into cylindrical pivots of exactly equal diameters, which rest in notches formed in metallic supports, bedded (in the case of large instruments) on strong pieces of stone, and susceptible of nice adjustment by screws, both in a vertical and horizontal direction. By the former adjustment, the axis can be rendered precisely horizontal, by *levelling* it with a *level* made to rest on the pivots. By the latter adjustment the axis is brought

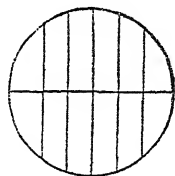


precisely into the east and west direction, the criterion of which is furnished by the observations themselves made with the instrument, in a manner presently to be explained, or by a well-defined object, called a *meridian mark*, originally deter-

mined by such observations, and then, for convenience of ready reference, permanently established, at a great distance, exactly in a *meridian line* passing through the central point of the whole instrument. It is evident, from this description, that, if the axis, or line of collimation of the telescope, be once well adjusted at right angles to the axis of the transit, it will never quit the plane of the meridian, when the instrument is turned round on its axis of rotation.

(160). In the focus of the eye-piece, and at right angles to the length of the telescope, is placed, not a single cross, as in our general explanation in art. 157, but a system of one horizontal and several equidistant vertical threads or wires (five or seven are more usually employed), as represented in the annexed figure, which always appear in the *field of view*, when properly illuminated by day by the light

of the sky, by night by that of a lamp introduced by a contrivance not necessary here to explain. The place of this system of wires may be altered by adjusting screws, giving it a lateral (horizontal) motion; and it is by this means brought to such a position, that the middle one of the vertical wires shall intersect *the line of collimation* of the telescope, where it is arrested and permanently fastened.⁷ In this situation it is evident that the middle thread will be a visible representation of that portion of the celestial meridian to which the telescope is pointed: and when a star is seen to cross this wire in the telescope, it is in the act of culminating, or passing the celestial meridian. The instant of this event is noted by the clock or chronometer, which forms an indispensable accompaniment of the transit instrument. For greater precision, the moments of its crossing all the vertical threads is noted; and a mean taken, which (since the threads are equidistant) would give exactly the same result, were all the observations perfect, and will of course, tend to subdivide and destroy their errors in an average of the whole in the contrary case.



(161.) For the mode of executing the adjustments, and allowing for the errors unavoidable in the use of this simple and elegant instrument, the reader must consult works especially devoted to this department of practical astronomy.⁸ We shall here only mention one important verification of its correctness, which consists in *reversing* the ends

⁷ There is no way of bringing the *true optic axis* of the object-glass to coincide *exactly* with the line of collimation, but, so long as the object-glass does not shift or shake in its cell, any line *holding an invariable position* with respect to that *axis*, may be taken for the *conventional* or astronomical axis with equal effect.

⁸ See Dr. Pearson's *Treatise on Practical Astronomy*. Also Bianchi *Sopra lo Stromento de' Passagi*. Ephem. di Milano, 1824.

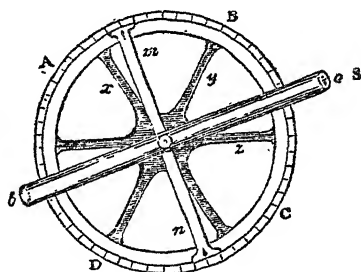
of the axis, or turning it east for west. If this be done, and it continue to give the same results, and intersect the same point on the meridian mark, we may be sure that the line of collimation of the telescope is truly at right angles to the axis, and describes strictly a plane, *i. e.* marks out in the heavens a *great circle*. In good transit observations, an error of one or two-tenths of a second of time in the moment of a star's culmination is the utmost which need be apprehended, exclusive of the error of the clock: in other words, a clock may be compared with the earth's diurnal motion by a single observation, without risk of greater error. By multiplying observations, of course, a yet greater degree of precision may be obtained.

(162.) The plane described by the line of collimation of a transit ought to be that of the meridian of the place of observation. To ascertain whether it is so or not, celestial observation must be resorted to. Now, as the meridian is a great circle passing through the pole, it necessarily bisects the diurnal circles described by all the stars, all which describe the two semicircles so arising in equal intervals of 12 sidereal hours each. Hence, if we choose a star whose whole diurnal circle is above the horizon, or which never sets, and observe the moments of its upper and lower transits across the middle wire of the telescope, if we find the two semidiurnal portions east and west of the plane described by the telescope to be described in *precisely* equal times, we may be sure that plane is the meridian.

(163.) The angular intervals measured by means of the transit instrument and clock are arcs of the equinoctial, intercepted between circles of declination passing through the objects observed; and their measurement, in this case, is performed by no artificial graduation of circles, but by the

help of the earth's diurnal motion, which carries equal arcs of the equinoctial across the meridian, in equal times, at the rate of 15° per sidereal hour. In all other cases, when we would measure angular intervals, it is necessary to have recourse to circles, or portions of circles, constructed of metal or other firm and durable material, and mechanically subdivided into equal parts, such as degrees, minutes, etc. The simplest and most obvious mode in which the measurement of the angular interval between two directions in space can be performed is as follows. Let A B C D be a circle,

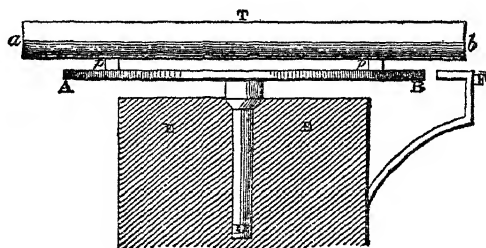
divided into 360 degrees (numbered in order from any point 0° in the circumference, round to the same point again), and connected with its centre by spokes or rays, x, y, z , firmly united to its circumference or *limb*. At the centre let a circular hole be pierced, in which shall move a pivot



exactly fitting it, carrying a tube, whose axis, $a b$, is exactly parallel to the plane of the circle, or perpendicular to the pivot; and also two arms, m, n , at right angles to it, and forming one piece with the tube and the axis; so that the motion of the axis on the centre shall carry the tube and arms smoothly round the circle, to be arrested and fixed at any point we please, by a contrivance called a *clamp*. Suppose, now, we would measure the angular interval between two fixed objects, S, T. The plane of the circle must first be adjusted so as to pass through them both, and immovably fixed and maintained in that position. This done, let the axis $a b$ of the tube be directed to one of them, S, and *clamped*. Then will a mark on the arm m point either ex-

actly to some one of the divisions on the limb, or between two of them adjacent. In the former case, the division must be noted as *the reading* of the arm *m*. In the latter, the fractional part of one whole interval between the consecutive divisions by which the mark on *m* surpasses the last inferior division must be estimated or measured by some mechanical or optical means. (See art. 165.) The division and fractional part thus noted, and reduced into degrees, minutes, and seconds, is to be set down as the *reading of the limb* corresponding to that position of the tube *a b*, where it points to the object *S*. The same must then be done for the object *T*; the tube pointed to it, and the limb "*read off*;" the position of the circle remaining meanwhile unaltered. It is manifest, then, that, if the lesser of these readings be subtracted from the greater, *their difference* will be the angular interval between *S* and *T*, as seen from the centre of the circle, at whatever point of the limb the commencement of the graduations or the point 0° be situated.

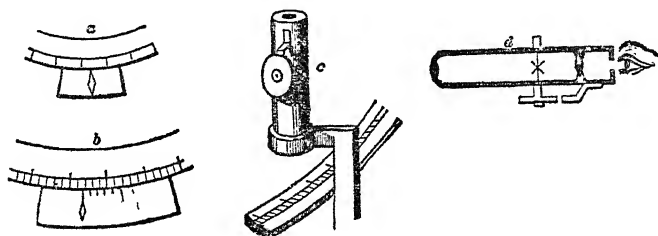
(164.) The very same result will be obtained, if, instead of making the tube movable upon the circle, we connect it



invariably with the latter, and make both revolve together on an axis concentric with the circle, and forming one piece with it, working in a hollow formed to receive and fit it in

some fixed support. Such a combination is represented in section in the above sketch. T is the tube or sight, fastened, at $p p$, on the circle A B, whose axis, D, works in the solid metallic centring E, from which originates an arm, F, carrying at its extremity an index, or other proper mark, to point out and read off the exact division of the circle at B, the point close to it. It is evident that, as the telescope and circle revolve through any angle, the part of the limb of the latter, which by such revolution is carried past the index F, will measure the angle described. This is the most usual mode of applying divided circles in astronomy.

(165.) The index F may either be a simple pointer, like a clock hand (*fig. a*); or a vernier (*fig. b*); or, lastly, a com-



pound microscope (*fig. c*), represented in section in *fig. d*, and furnished with a cross in the common focus of its object and eye-glass, movable by a fine-threaded screw, by which the intersection of the cross may be brought to exact coincidence with the image of the nearest of the divisions of the circle formed in the focus of the object lens upon the very same principle with that explained, art. 157, for the pointing of the telescope, only that here the fiducial cross is made movable; and by the turns and parts of a turn of the screw required for this purpose the distance of that division from the original or zero point of the microscope may be estimated. This simple but delicate contrivance gives to the

reading off of a circle a degree of accuracy only limited by the power of the microscope and the perfection with which a screw can be executed, and places the subdivision of angles on the same footing of optical certainty which is introduced into their measurement by the use of the telescope.

(166.) The exactness of the result thus obtained must depend, 1st, on the precision with which the tube $a\ b$ can be pointed to the objects; 2dly, on the accuracy of graduation of the limb; 3dly, on the accuracy with which the subdivision of the intervals between any two consecutive graduations can be performed. The mode of accomplishing the latter object with any required exactness has been explained in the last article. With regard to the graduation of the limb, being merely of a mechanical nature, we shall pass it without remark, further than this, that, in the present state of instrument-making, the amount of error from this source of inaccuracy is reduced within very narrow limits indeed.* With regard to the first, it must be obvious that, if the sights $a\ b$ be nothing more than simple crosses, or pin-holes at the ends of a hollow tube, or an eye-hole at one end, and a cross at the other, no greater nicety in pointing can be expected than what simple vision with the naked eye can command. But if, in place of these simple but coarse contrivances, the tube itself be converted into a telescope, having an object-glass at b , an eye-piece at a , and a fiducial cross in their common focus, as explained in art. 157; and if the motion of the tube on the limb of the circle be arrested when the object is brought just into

* In the great Ertel circle at Pulkova, the probable amount of the *accidental* error of division is stated by M. Struve not to exceed $0''.264$. Desc. de l'Obs. centrale de Pulkova, p. 147.

coincidence with the intersectional point of that cross, it is evident that a greater degree of exactness may be attained in the pointing of the tube than by the unassisted eye, in proportion to the magnifying power and distinctness of the telescope used.

(167.) The simplest mode in which the measurement of an angular interval can be executed, is what we have just described; but, in strictness, this mode is applicable only to terrestrial angles, such as those occupied on the sensible horizon by the objects which surround our station—because these only remain stationary during the interval while the telescope is shifted on the limb from one object to the other. But the diurnal motion of the heavens, by destroying this essential condition, renders the direct measurement of angular distance from *object* to *object* by this means impossible. The same objection, however, does not apply if we seek only to determine the interval between the *diurnal circles* described by any two celestial objects. Suppose every star, in its diurnal revolution, were to leave behind it a visible trace in the heavens—a fine line of light, for instance—then a telescope once pointed to a star, so as to have its image brought to coincidence with the intersection of the wires, would constantly remain pointed to some portion or other of this line, which would therefore continue to appear in its field as a luminous line, permanently intersecting the same point, till the star came round again. From one such line to another the telescope might be shifted, at leisure, without error; and then the angular interval between the two diurnal circles, *in the plane of the telescope's rotation*, might be measured. Now, though we cannot *see* the path of a star in the heavens, we can *wait* till the star itself crosses the field of view, and seize the moment of its passage to place the intersection of its wires

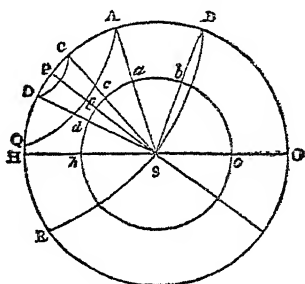
so that the star shall traverse it; by which, when the telescope is well clamped, we equally well secure the position of its diurnal circle as if we continued to *see* it ever so long. The reading off of the limb may then be performed at leisure; and when another star comes round *into the plane* of the circle, we may unclamp the telescope, and a similar observation will enable us to assign the place of *its* diurnal circle on the limb: and the observations may be repeated alternately, every day, as the stars pass, till we are satisfied with their result.

(168.) This is the principle of the mural circle, which is nothing more than such a circle as we have described in art. 163, firmly supported, in the plane of the meridian, on a long and powerful horizontal axis. This axis is let into a massive pier, or wall, of stone (whence the name of the instrument), and so secured by screws as to be capable of adjustment both in a vertical and horizontal direction; so that, like the axis of the transit, it can be maintained in the exact direction of the east and west points of the horizon, the plane of the circle being consequently truly meridional.

(169.) The meridian, being at right angles to all the diurnal circles described by the stars, its arc intercepted between any two of them will measure the least distance between these circles, and will be equal to the difference of the declinations, as also to the difference of the *meridian altitudes* of the objects—at least when corrected for refraction. These differences, then, are the angular intervals *directly* measured by the mural circle. But from these, supposing the law and amount of refraction known, it is easy to conclude, not their differences only, but the quantities themselves, as we shall now explain.

(170.) The declination of a heavenly body is the com-

plement of its distance from the pole. The pole, being a point in the meridian, might be directly observed on the limb of the circle, if any star stood *exactly* therein; and thence the *polar distances*, and, of course, the declinations of all the rest might be at once determined. But this not being the case, a bright star as near the pole as can be found is selected, and observed in its *upper* and *lower* culminations; that is, when it passes the meridian *above* and *below* the pole. Now, as its distance from the pole remains the same, the difference of reading off the circle in the two cases is, of course (when corrected for refraction), equal to twice the polar distance of the star; the arc intercepted on the limb of the circle being, in this case, equal to the angular diameter of the star's diurnal circle. In the annexed diagram, H P O represents the celestial meridian, P the pole, B R, A Q, C D the diurnal circles of stars which arrive on the meridian at B, A, and C in their upper and at R, Q, D in their lower culminations, of which D and Q happen above the horizon H O. P is the pole; and if we suppose *h p o* to be the mural circle, having S for its centre, *b a c p d* will be the points on its circumference corresponding to B A C P D in the heavens. Now the arcs *b a*, *b c*, *b d*, and *c d* are given immediately by observation; and since $C P = P D$, we have also $c p = p d$, and each of them $= \frac{1}{2} c d$, consequently the place of the *polar point*, as it is called, upon the limb of the circle becomes known, and the arcs *p b*, *p a*, *p c*, which represent on the circle the *polar distances* required, become also known.



(171.) The situation of the pole star, which is a very brilliant one, is eminently favorable for this purpose, being only about a degree and half from the pole; it is, therefore, the star usually and almost solely chosen for this important purpose; the more especially because, both its culminations taking place at great and not very different altitudes, the refractions by which they are affected are of small amount, and differ but slightly from each other, so that their correction is easily and safely applied. The brightness of the pole star, too, allows it to be easily observed in the daytime. In consequence of these peculiarities, this star is one of constant resort with astronomers for the adjustment and verification of instruments of almost every description. In the case of the transit, for instance, it furnishes an excellent object for the application of the method of testing the meridional situation of the instrument described in art. 162, in fact, the most advantageous of any for that purpose, owing to its being the most remote from the zenith, at its upper culmination, of all bright stars observable both above and below the pole.

(172.) The place of the *polar point* on the limb of the mural circle once determined, becomes an origin, or zero point, from which the polar distances of all objects, referred to other points on the same limb, reckon. It matters not whether the actual commencement 0° of the graduations stands there, or not: since it is only by the *differences* of the readings that the arcs on the limb are determined; and hence a great advantage is obtained in the power of commencing anew a fresh series of observations, in which a different part of the circumference of the circle shall be employed, and different graduations brought into use, by which inequalities of division may be detected and neutral-

ized. This is accomplished practically by detaching the telescope from its old bearings, on the circle, and fixing it afresh, by screws or clamps, on a different part of the circumference.

(173.) A point on the limb of the mural circle, not less important than the *polar point*, is the *horizontal point*, which, being once known, becomes in like manner an origin, or zero point, from which altitudes are reckoned. The principle of its determination is ultimately nearly the same with that of the polar point. As no star exists in the celestial horizon, the observer must seek to determine two points on the limb, the one of which shall be precisely as far *below* the horizontal point as the other is above it. For this purpose, a star is observed at its culmination on one night, by pointing the telescope directly to it, and the next, by pointing to *the image of the same star reflected* in the still, unruffled surface of a fluid at perfect rest. Mercury, as the most reflective fluid known, is generally chosen for that use. As the surface of a fluid at rest is necessarily horizontal, and as the angle of reflection, by the laws of optics, is equal to that of incidence, this image will be just as much depressed below the horizon as the star itself is above it (allowing for the difference of refraction at the moments of observation). The arc intercepted on the limb of the circle between the star and its reflected image thus consecutively observed, when corrected for refraction, is the double altitude of the star, and its point of bisection the horizontal point. The reflecting surface of a fluid so used for the determination of the altitudes of objects is called an *artificial horizon*.¹⁰

¹⁰ By a peculiar and delicate manipulation and management of the setting, bisection and reading off of the circle, aided by the use of a movable horizontal

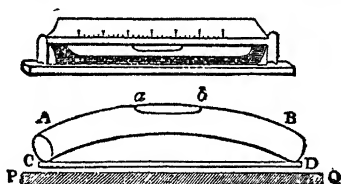
(174.) The mural circle is, in fact, at the same time, a transit instrument; and, if furnished with a proper system of vertical wires in the focus of its telescope, may be used as such. As the axis, however, is only supported at one end, it has not the strength and permanence necessary for the more delicate purposes of a transit; nor can it be verified, as a transit may, by the *reversal* of the two ends of its axis, east for west. Nothing, however, prevents a divided circle being permanently fastened on the axis of a transit instrument, either near to one of its extremities, or close to the telescope, so as to revolve with it, the reading off being performed by one or more microscopes fixed on one of its piers. Such an instrument is called a TRANSIT CIRCLE, or a MERIDIAN CIRCLE, and serves for the simultaneous determination of the right ascensions and polar distances of objects observed with it; the time of transit being noted by the clock, and the circle being read off by the lateral microscopes. There is much advantage, when extensive catalogues of small stars have to be formed, in this simultaneous determination of both their celestial co-ordinates: to which may be added the facility of applying to the meridian circle a telescope of any length and optical power. The construction of the mural circle renders this highly inconvenient, and indeed impracticable beyond very moderate limits.

(175.) The determination of the horizontal point on the limb of an instrument is of such essential importance in astronomy, that the student should be made acquainted with

micrometric wire in the focus of the object-glass, it is found practicable to observe a slow moving star (as the pole star) *on one and the same night*, both by reflection and direct vision, sufficiently near to either culmination to give the horizontal point, without risking the change of refraction in twenty-four hours; so that this source of error is thus completely eliminated.

every means employed for this purpose. These are, the artificial horizon, the plumb-line, the level, and the collimator. The artificial horizon has been already explained. The plumb-line is a fine thread or wire, to which is suspended a weight, whose oscillations are impeded and quickly reduced to rest by plunging it in water. The direction ultimately assumed by such a line, *admitting its perfect flexibility*, is that of gravity, or perpendicular to the surface of still water. Its application to the purposes of astronomy is, however, so delicate, and difficult, and liable to error, unless extraordinary precautions are taken in its use, that it is at present almost universally abandoned, for the more convenient, and equally exact instrument *the level*.

(176.) The level is a glass tube nearly filled with a liquid (sulphuric ether, or chloroform, being those now generally used, on account of their extreme *mobility*, and not being liable to freeze), the bubble in which, when the tube is

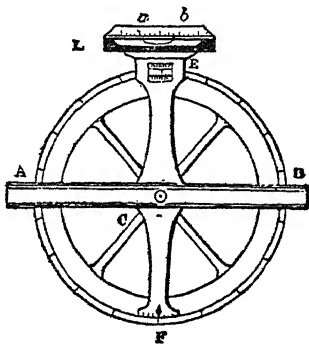


placed horizontally, would rest indifferently in any part if *the tube* could be mathematically *straight*. But that being impossible to execute, and every tube having some slight curvature; if the convex side be placed upward the bubble will occupy the higher part, as in the figure (where the curvature is purposely exaggerated). Suppose such a tube, as A B, firmly fastened on a straight bar, C D, and marked at *a b*, two points distant by the length of the bubble; then, if the instrument be so placed that the bubble shall occupy

this interval, it is clear that CD can have no other than one definite inclination to the horizon; because, were it ever so little moved one way or other, the bubble would shift its place, and run toward the elevated side. Suppose, now, that we would ascertain whether any given line PQ be horizontal; let the base of the level CD be set upon it, and note the points a b , between which the bubble is exactly contained; then turn the level end for end, so that C shall rest on Q , and D on P . If then the bubble continue to occupy the same place between a and b , it is evident that PQ can be no otherwise than horizontal. If not, the side toward which the bubble runs is highest, and must be lowered. Astronomical levels are furnished with a divided scale, by which the places of the ends of the bubble can be nicely marked; and it is said that they can be executed with such delicacy, as to indicate a single second of angular deviation from exact horizontality. In such levels accident is not trusted to to give the requisite curvature. They are ground and polished internally by peculiar mechanical processes of great delicacy.

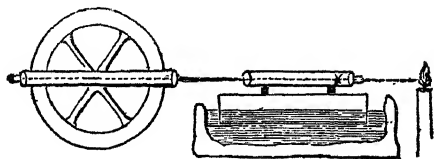
(177.) The mode in which a level may be applied to find the horizontal point on the limb of a vertical divided circle may be thus explained: Let AB be a telescope firmly fixed to such a circle, DEF , and movable in one with it on a horizontal axis C , which must be like that of a transit, susceptible of reversal (see art. 161), and with which the circle is inseparably connected. Direct the telescope on some distant well-defined object S , and bisect it by its horizontal wire, and in this position clamp it fast. Let L be a level fastened at right angles to an arm, LEF , furnished with a microscope, or vernier at F , and, if we please, another at E . Let this arm be fitted by grinding on the axis C , but capable

of moving smoothly on it without carrying it round, and also of being clamped fast on it, so as to prevent it from moving until required. While the telescope is kept fixed on the object S, let the level be set so as to bring its bubble to the marks *a b*, and clamp it there. Then will the arm L C F have some certain determinate inclination (no matter what) to the horizon. In this position let the circle be read off at F, and then let the whole apparatus be *reversed* by turning its horizontal axis end for end, *without unclamping the level arm* from the axis. This done, by the motion of the whole instrument (level and all) on its axis, restore the *level* to its horizontal position with the bubble at *a b*. Then we are sure that the telescope has now the same inclination to the horizon *the other way*, that it had when pointed to S, and the reading off at F will not have been changed. Now unclamp the level, and, keeping it nearly horizontal, turn round the circle on the axis, so as to carry back the telescope *through the zenith* to S, and in that position clamp the circle and telescope fast. Then it is evident that an angle equal to twice the zenith distance of S has been moved over by the axis of the telescope from its last position. Lastly, without unclamping the telescope and circle, let the level be once more rectified. Then will the arm L E F once more assume the same definite position with respect to the horizon: and, consequently, if the circle be again read off, the difference between this and the previous reading must measure the arc of its circumference which has passed under the



point F, which may be considered as having all the while retained an invariable position. This difference, then, will be the double zenith distance of S, and its half will be the zenith distance simply, the complement of which is its altitude. Thus the altitude corresponding to a given reading of the limb becomes known, or, in other words, the horizontal point on the limb is ascertained. Circuitous as this process may appear, there is no other mode of employing the level for this purpose which does not in the end come to the same thing. Most commonly, however, the level is used as a mere *fiducial* reference, to preserve a horizontal point once well determined by other means, which is done by adjusting it so as to stand level when the telescope is truly horizontal, and thus leaving it, depending on the permanence of its adjustment.

(178.) The last, but probably not the least exact, as it certainly is, in innumerable cases, the most convenient means of ascertaining the *horizontal point*, is that afforded by the floating collimator, an invention of Captain Kater, but of which the optical principle was first employed by Rittenhouse, in 1785, for the purpose of fixing a definite direc-

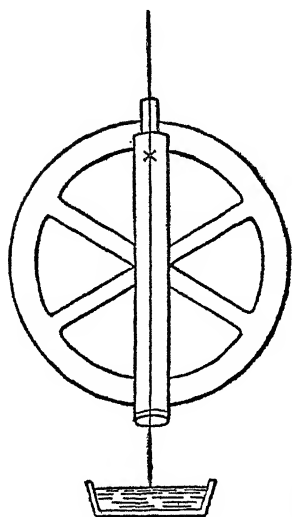


tion in space by the emergence of parallel rays from a material object placed in the focus of a fixed lens. This elegant instrument is nothing more than a small telescope furnished with a cross-wire in its focus, and fastened horizontally, or as nearly so as may be, on a flat iron *float*, which is made to swim on mercury, and which, of course, will, when left to

itself, assume always one and the same invariable inclination to the horizon. If the cross-wires of the collimator be illuminated by a lamp, being in the focus of its object-glass, the rays from them will issue parallel, and will therefore be in a fit state to be brought to a focus by the object-glass of any other telescope, in which they will form an image *as if they came from a celestial object in their direction, i.e. at an altitude equal to their inclination*. Thus the intersection of the cross of the collimator may be observed *as if it were a star*, and that, however near the two telescopes are to each other. By transferring then, the collimator *still floating* on a vessel of mercury from the one side to the other of a circle, we are furnished with two *quasi-celestial* objects, at precisely equal altitudes, on opposite sides of the centre; and if these be observed in succession with the telescope of the circle, bringing its cross to bisect the image of the cross of the collimator (for which end the wires of the latter cross are purposely set 45° inclined to the horizon), the difference of the readings on its limb will be twice the zenith distance of either; whence, as in the last article, the horizontal or zenith point is immediately determined. Another, and, in many respects, preferable form of the floating collimator, in which the telescope is *vertical*, and whereby the *zenith* point is directly ascertained, is described in the Phil. Trans. 1828, p. 257, by the same author.

(179.) By far the neatest and most delicate application of the *principle of collimation* of Rittenhouse, however, is suggested by Bohnenberger, which affords at once, and by a single observation, an exact knowledge of the *nadir* point of an astronomical circle. In this combination, the telescope of the circle is its own collimator. The object observed is the central intersectional cross of the wires in its

own focus reflected in mercury. A strong illumination being thrown upon the system of wires (art. 160) by a lateral lamp, the telescope of the instrument is directed vertically



downward toward the surface of the mercury, as in the figure annexed. The rays diverging from the wires issue in parallel pencils from the object-glass, are incident on the mercury, and are thence reflected back (without losing their parallel character) to the object-glass, which is therefore enabled to collect them again in its focus. Thus is formed a reflected image of the system of cross-wires, which, when brought by the slow motion of the telescope to exact coincidence (intersection

upon intersection) with the real system as seen in the eyepiece of the instrument, indicates the precise and rigorous verticality of the optical axis of the telescope when directed to the nadir point.

(180.) The transit and mural circle are essentially meridian instruments, being used only to observe the stars at the moment of their meridian passage. Independent of this being the most favorable moment for seeing them, it is that in which their diurnal motion is parallel to the horizon. It is therefore easier at this time than it could be at any other, to place the telescope exactly in their true direction; since their apparent course in the field of view being parallel to the horizontal thread of the system of wires therein, they may, by giving a fine motion to the telescope, be brought to exact coincidence with it, and time may be allowed to ex-

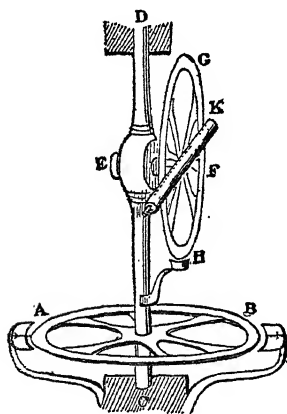
amine and correct this coincidence, if not at first accurately hit, which is the case in no other situation. Generally speaking, all angular magnitudes which it is of importance to ascertain exactly, should, if possible, be observed at their maxima or minima of increase or diminution; because at these points they remain not perceptibly changed during a time long enough to complete, and even, in many cases, to repeat and verify, our observations in a careful and leisurely manner. The angle which, in the case before us, is in this predicament, is the altitude of the star, which attains its maximum or minimum on the meridian, and which is measured on the limb of the mural circle.

(181.) The purposes of astronomy, however, require that an observer should possess the means of observing any object not directly on the meridian, but at any point of its diurnal course, or wherever it may present itself in the heavens. Now, a point in the sphere is determined by reference to two great circles at right angles to each other; or of two circles, one of which passes through the pole of the other. These, in the language of geometry, are *co-ordinates* by which its situation is ascertained: for instance—on the earth, a place is known if we know its longitude and latitude;—in the starry heavens, if we know its right ascension and declination;—in the visible hemisphere, if we know its azimuth and altitude, etc.

(182.) To observe an object at any point of its diurnal course, we must possess the means of directing a telescope to it; which, therefore, must be capable of motion in two planes at right angles to each other; and the amount of its angular motion in each must be measured on two circles *co-ordinate* to each other, whose planes must be parallel to those in which the telescope moves. The practical accom-

plishment of this condition is effected by making the axis of one of the circles penetrate that of the other at right angles. The pierced axis turns on fixed supports, while the other has no connection with any external support, but is sustained entirely by that which it penetrates, which is strengthened and enlarged at the point of penetration to receive it. The annexed figure exhibits the simplest form of such a combination, though very far indeed from the best in point of mechanism. The two circles are *read off* by verniers, or microscopes; the one attached to the fixed support which carries the principal axis, the other to an arm projecting from that axis. Both circles also are susceptible of being clamped, the clamps being attached to the same ultimate bearing with which the apparatus for reading off is connected.

(183.) It is manifest that such a combination, however its principal axis be pointed (provided that its direction be invariable), will enable us to ascertain the situation of any object with respect to the observer's station, by angles reckoned upon two great circles in the visible hemisphere, one of which has for its poles the prolongations of the principal axis or the vanishing points of a system of lines parallel to it, and the other passes always through these poles: for the former great circle is the vanishing line of all planes parallel to the circle A B, while the latter, in any position of the instrument, is the vanishing line of all the planes parallel to the circle G H; and these two planes being, by the con-



struction of the instrument, at right angles, the great circles, which are their vanishing lines, must be so too. Now, if two great circles of a sphere be at right angles to each other, the one will always pass through the other's poles.

(184.) There are, however, but two positions in which such an apparatus can be mounted so as to be of any practical utility in astronomy. The first is, when the principal axis CD is parallel to the earth's axis, and therefore points to the poles of the heavens which are the vanishing points of all lines in this system of parallels; and when, of course, the plane of the circle AB is parallel to the earth's equator, and therefore has the equinoctial for its vanishing circle, and measures, by its arcs read off, hour angles, or differences of right ascension. In this case, the great circles in the heavens, corresponding to the various positions, which the circle GH can be made to assume, by the rotation of the instrument round its axis CD , are all hour-circles; and the arcs read off on this circle will be declinations, or polar distances, or their differences.

(185.) In this position the apparatus assumes the name of an *equatorial*, or, as it was formerly called, a *parallactic* instrument. It is a most convenient instrument for all such observations as require an object to be kept long in view, because, being once set upon the object, it can be followed as long as we please by a *single motion*, *i.e.* by merely turning the whole apparatus round on its polar axis. For since, when the telescope is set on a star, the angle between its direction and that of the polar axis is equal to the polar distance of the star, it follows, that when turned about its axis, without altering the position of the telescope on the circle GH , the point to which it is directed will always lie in the small circle of the heavens coincident with the star's diurnal

path. In many observations this is an inestimable advantage, and one which belongs to no other instrument. The equatorial is also used for determining the place of an unknown by comparison with that of a known object, in a manner to be described in the fifth chapter. The adjustments of the equatorial are somewhat complicated and difficult. They are best performed in this manner:—1st, Follow the pole star round its whole diurnal course, by which it will become evident whether the polar axis is directed above or below, to the right or to the left, of the true pole—and correct it accordingly (without any attempt, during this process, to correct the errors, if any, in the position of the declination axis). 2dly, after the polar axis is thus brought into adjustment, place the plane of the declination circle in or near the meridian; and, having there secured it, observe the transits of several known stars of widely different declinations. If the intervals between these transits correspond to the known differences of right ascensions of the stars, we may be sure that the telescope describes a true meridian, and that, therefore, the declination axis is truly perpendicular to the polar one;—if not, the deviation of the intervals from this law will indicate the direction and amount of the deviation of the axis in question, and enable us to correct it.¹¹

(186.) A very great improvement has, within a few years from the present time, been introduced into the construction of the equatorial instrument. It consists in applying a clock-work movement to turn the whole instrument round

¹¹ See Littrow on the Adjustment of the Equatorial (Mem. Ast. Soc. vol. ii. p. 45), where formulæ are given for ascertaining the amount and direction of all the misadjustments simultaneously. But the practical observer, who wishes to avoid bewildering himself by doing two things at once, had better proceed as recommended in the text.

upon its polar axis, and so to follow the diurnal motion of any celestial object, without the necessity of the observer's manual intervention. The driving power is the descent of a weight which communicates motion to a train of wheel-work, and thus, ultimately, to the polar axis, while, at the same time, its *too swift* descent is controlled and regulated to the exact and uniform rate required to give that axis one turn in 24 hours, by connecting it with a regulating clock, or (which is found preferable in practice) by exhausting all the superfluous energy of the driving power, by causing it to overcome a regulated friction. Artists have thus succeeded in obtaining a perfectly smooth, uniform, and *regulable* motion, which, when so applied, serves to retain any object on which the telescope may be set, commodiously, in the centre of the field of view for whole hours in succession, leaving the attention of the observer undistracted by having a mechanical movement to direct; and with both his hands at liberty.

(187.) The other position in which such a compound apparatus as we have described in art. 182 may be advantageously mounted, is that in which the principal axis occupies a vertical position, and the one circle, A B, consequently corresponds to the celestial horizon, and the other, G H, to a vertical circle of the heavens. The angles measured on the former are therefore *azimuths*, or differences of azimuth, and those of the latter zenith distances, or altitudes, according as the graduation commences from the upper point of its limb, or from one 90° distant from it. It is therefore known by the name of an *azimuth and altitude instrument*. The vertical position of its principal axis is secured either by a plumb-line suspended from the upper end, which, however it be turned round, should continue

always to intersect one and the same fiducial mark near its lower extremity, or by a level fixed directly across it, whose bubble ought not to shift its place, on moving the instrument in azimuth. The north or south point on the horizontal circle is ascertained by bringing the vertical circle to coincide with the plane of the meridian, by the same criterion by which the azimuthal adjustment of the transit is performed (art. 162), and noting, in this position, the reading off of the lower circle; or by the following process.

(188.) Let a bright star be observed at a considerable distance to the *east* of the meridian, by bringing it on the cross wires of the telescope. In this position let the horizontal circle be read off, and the telescope securely clamped on the vertical one. When the star has passed the meridian, and is in the descending point of its daily course, let it be followed by moving the whole instrument round to the west, without, however, unclamping the telescope, until it comes into the field of view; and until, by continuing the horizontal motion, the star and the cross of the wires come once more to coincide. In this position it is evident the star must have the same precise altitude above the *western* horizon, that it had at the moment of the first observation above the *eastern*. At this point let the motion be arrested, and the horizontal circle be again read off. The difference of the readings will be the azimuthal arc described in the interval. Now, it is evident that when the altitudes of any star are equal on either side of the meridian, its *azimuths*, whether reckoned both from the north or both from the south point of the horizon, must also be equal—consequently the north or south point of the horizon must bisect the azimuthal arc thus determined, and will therefore become known.

(189.) This method of determining the north and south points of a horizontal circle is called the "method of equal altitudes," and is of great and constant use in practical astronomy. If we note, at the moments of the two observations, the time, by a clock or chronometer, the instant halfway between them will be the moment of the star's meridian passage, which may thus be determined without a transit; and, *vice versâ*, the error of a clock or chronometer may by this process be discovered. For this last purpose, it is not necessary that our instrument should be provided with a horizontal circle at all. Any means by which altitudes can be measured will enable us to determine the moments when the same star arrives at *equal* altitudes in the eastern and western halves of its diurnal course; and, these once known, the instant of meridian passage and the error of the clock become also known.

(190.) Thus also a meridian line may be drawn and a *meridian mark* erected. For the readings of the north and south points on the limb of the horizontal circle being known, the vertical circle may be brought exactly into the plane of the meridian, by setting it to that precise reading. This done, let the telescope be depressed to the north horizon, and let the point intersected there by its cross-wires be noted, and a mark erected there, and let the same be done for the south horizon. The line joining these points is a meridian line, passing through the centre of the horizontal circle. The marks may be made secure and permanent if required.

(191.) One of the chief purposes to which the altitude and azimuth circle is applicable is the investigation of the amount and laws of refraction. For, by following with it a circumpolar star which passes the zenith, and another

which grazes the horizon, through their whole diurnal course, the exact *apparent* form of their diurnal orbits, or the ovals into which their circles are distorted by refraction, can be traced; and their deviation from circles, being at every moment given by the nature of the observation *in the direction in which the refraction itself takes place* (i.e. in altitude), is made a matter of direct observation.

(192.) The *zenith sector* and the *theodolite* are peculiar modifications of the altitude and azimuth instrument. The former is adapted for the very exact observation of stars in or near the zenith, by giving a great length to the vertical axis, and suppressing all the circumference of the vertical circle, except a few degrees of its lower part, by which a great length of radius, and a consequent proportional enlargement of the divisions of its arc, is obtained. The latter is especially devoted to the measures of horizontal angles between terrestrial objects, in which the telescope never requires to be elevated more than a few degrees, and in which, therefore, the vertical circle is either dispensed with, or executed on a smaller scale, and with less delicacy; while, on the other hand, great care is bestowed on securing the exact perpendicularity of the plane of the telescope's motion, by resting its horizontal axis on two supports like the piers of a transit instrument, which themselves are firmly bedded on the spokes of the horizontal circle, and turn with it.

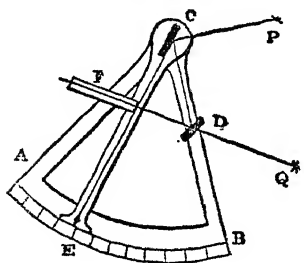
(193.) The next instrument we shall describe is one by whose aid the angular distance of any two objects may be measured, or the altitude of a single one determined, either by measuring its distance from the visible horizon (such as the sea-offing, allowing for its dip), or from its own reflection on the surface of mercury. It is the sextant, or quad-

rant, commonly called *Hadley's*, from its reputed inventor, though the priority of invention belongs undoubtedly to Newton, whose claims to the gratitude of the navigator are thus doubled, by his having furnished at once the only theory by which his vessel can be securely guided, and the only instrument which has ever been found to avail, in applying that theory to its nautical uses.¹²

(194.) The principle of this instrument is the optical property of reflected rays, thus announced: "The angle between the first and last directions of a ray which has suffered two reflections in one plane is equal to twice the inclination of the reflecting surfaces to each other." Let A B be the limb, or graduated arc, of a portion of a circle 60° in extent, but divided into 120 equal parts. On the radius C B let a silvered plane glass D be fixed, at right angles to the plane of the circle, and on the movable radius C E let another such silvered glass, C, be fixed. The glass D is permanently fixed parallel to A C, and only one half of it is silvered, the other half allowing objects to be seen through it. The glass C is wholly silvered, and its plane is parallel to the length of the movable radius C E, at the extremity E of which a vernier is placed to read off the divisions of the limb. On the radius A C is set a telescope F, through which any object, Q, may be seen by *direct* rays which pass through the unsilvered portion of the glass D, while another object, P, is seen through the same telescope by rays, which, after reflection at C, have been thrown upon

¹² Newton communicated it to Dr. Halley, who suppressed it. The description of the instrument was found, after the death of Halley, among his papers, in Newton's own handwriting, by his executor, who communicated the papers to the Royal Society, twenty-five years after Newton's death, and eleven after the publication of Hadley's invention, which might be, and probably was, independent of any knowledge of Newton's, though Hutton insinuates the contrary.

the silvered part of D, and are thence directed by a second reflection into the telescope. The two images so formed will both be seen in the field of view at once, and by moving the radius C E will (if the reflectors be truly perpendicular to the plane of the circle) meet and pass over,



without obliterating each other. The motion, however, is arrested when they meet, and at this point the angle included between the direction C P of one object, and F Q of the other, is twice the angle E C A included between the fixed and movable radii C A, C E.

Now, the graduations of the limb being purposely made only half as distant as would correspond to degrees, the arc A E, when read off, as if the graduations were *whole* degrees, will, in fact, read double its real amount, and therefore the numbers so read off will express not the angle E C A, but its double, the angle subtended by the objects.

(195.) To determine the exact distances between the stars by direct observation is comparatively of little service; but in nautical astronomy the measurement of their distances from the moon, and of their altitudes, is of essential importance; and as the sextant requires no fixed support, but can be held in the hand, and used on shipboard, the utility of the instrument becomes at once obvious. For altitudes at sea, as no level, plumb-line, or artificial horizon can be used, the sea-offing affords the only resource; and the image of the star observed, seen by reflection, is brought to coincide with the boundary of the sea seen by direct rays. Thus the altitude above the sea-line is found; and this corrected for the *dip of the horizon* (art. 23) gives the true

altitude of the star. On land, an artificial horizon may be used (art. 173), and the consideration of dip is rendered unnecessary.

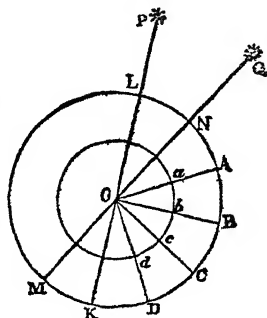
(196.) The adjustments of the sextant are simple. They consist in fixing the two reflectors, the one on the revolving radius C E, the other on the fixed one C B, so as to have their planes perpendicular to the plane of the circle, and parallel to each other, when the reading of the instrument is zero. This adjustment in the latter respect is of little moment, as its effect is to produce a *constant* error, whose amount is readily ascertained by bringing the two images of one and the same star or other distant object to coincidence; when the instrument *ought* to read zero, and if it does not, the angle which it *does* read is the zero correction and must be subtracted from *all* angles measured with the sextant. The former adjustments are essential to be maintained, and are performed by small screws, by whose aid either or both the glasses may be *tilted* a little one way or another until the direct and reflected images of a vertical line (a plumb-line) can be brought to coincidence over *their whole extent*, so as to form a single unbroken straight line, whatever be the position of the movable arm, in the middle of the field of view of the telescope, whose axis is carefully adjusted by the optician to parallelism with the plane of the limb. In practice it is usual to leave only the reflector D on the fixed radius adjustable, that on the movable being set to great nicety by the maker. In this case the best way of making the adjustment is to view a pair of lines crossing each other at right angles (one being horizontal, the other vertical) through the telescope of the instrument, holding the plane of its limb vertical—then having brought the horizontal line and its reflected image to coincidence by

the motion of the radius, the two images of the vertical arm must be brought to coincidence by tilting one way or other the fixed reflector D by means of an adjusting screw, with which every sextant is provided for that purpose. When both lines coincide *in the centre of the field* the adjustment is correct.

(197.) The reflecting circle is an instrument destined for the same uses as the sextant, but more complete, the circle being entire, and the divisions carried all round. It is usually furnished with three verniers, so as to admit of three distinct readings off, by the average of which the error of graduation and of reading is reduced. This is altogether a very refined and elegant instrument.

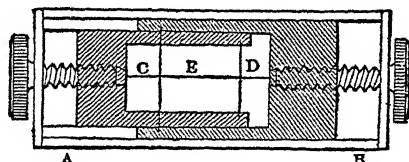
(198.) We must not conclude this part of our subject without mention of the "principle of repetition"; an invention of Borda, by which the error of graduation may be diminished to any degree, and, practically speaking, annihilated. Let P Q be two objects which we may suppose fixed, for purposes of mere explanation, and let K L be a telescope movable on O, the common axis of two circles, A M L and $a b c$, of which the former, A M L, is absolutely fixed in the plane of the objects, and carries the graduations, and the latter is freely movable on the axis. The telescope is attached permanently to the latter circle, and moves with it. An arm O a A carries the index, or vernier, which reads off the graduated limb of the fixed circle. This arm is provided with two clamps, by which it can be temporarily connected with either circle, and detached at pleasure. Suppose, now, the telescope directed to P. Clamp the index arm O A to the *inner* circle, and unclamp it from the outer, and read off. Then carry the telescope round to the other object Q. In so doing, the inner circle, and the

index-arm which is clamped to it, will also be carried round, over an arc $A B$, on the graduated limb of the outer, equal to the angle $P O Q$. Now clamp the index to the outer circle, and unclamp the inner, and read off: the difference of readings will of course measure the angle $P O Q$; but the result will be liable to two sources of error—that of *graduation* and that of *observation*, both which it is our object to get rid of. To this end transfer the telescope back to P , *without* unclamping the arm from the outer circle; *then*, having made the bisection of P , clamp the arm to b , and unclamp it from B , and again transfer the telescope to Q , by which the arm will now be carried with it to C , over a second arc, $B C$, equal to the angle $P O Q$. Now again read off; then will the difference between this reading and the *original* one measure *twice* the angle $P O Q$, affected with *both* errors of observation, but only with *the same error of graduation as before*. Let this process be repeated as often as we please (suppose ten times); then will the final arc $A B C D$ read off on the circle be ten times the required angle, affected by the joint errors of all the ten observations, but only by the same constant error of graduation, which depends on the initial and final readings off alone. Now the errors of observation, when numerous, tend to balance and destroy one another; so that, if sufficiently multiplied, their influence will disappear from the result. There remains, then, only the constant error of graduation, which comes to be divided in the final result by the number of observations, and is therefore diminished



in its influence to one-tenth of its possible amount, or to less if need be. The abstract beauty and advantage of this principle seem to be counterbalanced in practice by some unknown cause, which, probably, must be sought for in imperfect clamping.

(199.) Micrometers are instruments (as the name imports¹³) for measuring, with great precision, small angles, not exceeding a few minutes, or at most a whole degree. They are very various in construction and principle, nearly all, however, depending on the exceeding delicacy with which space can be subdivided by the turns and parts of a turn of fine screws. Thus—in the *parallel wire micrometer*, two parallel threads (spider's lines are generally used) stretched on sliding frames, one or both movable by

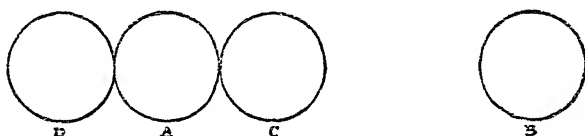


screws in a direction perpendicular to that of the threads, are placed in the common focus of the object and eye-glasses of a telescope, and brought by the motion of the screws exactly to cover the two extremities of the image of any small object seen in the telescope, as the diameter of a planet, etc., the angular distance between which it is required to measure. This done, the threads are closed up by turning one of the screws till they exactly cover each other, and the number of turns and parts of a turn

¹³ μικρος, small; μετρειν, to measure.

required gives the interval of the threads, which must be converted into angular measure, either by actual calculation from the linear measure of the threads of the screw and the focal length of the object-glass, or experimentally, by measuring the image of a known object placed at a known distance (as a foot-rule at a hundred yards, etc.) and therefore subtending a known angle.

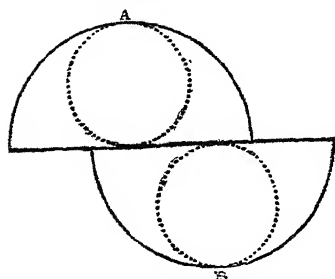
(200.) The *duplication of the image* of an object by optical means furnishes a valuable and fertile resource in micrometry. Suppose by any optical contrivance the single image A of any object can be converted into two, exactly equal and similar, A B, at a distance from one another, dependent (by some mechanical movement) on the will of the observer, and in any required direction from one another. As these



can, therefore, be made to approach to or recede from each other at pleasure, they may be brought in the first place to approach till they touch one another on one side, as at A C, and then being made by continuing the motion to cross and touch on the opposite side, as A D, it is evident that the quantity of movement required to produce the change from one contact to the other, *if uniform*, will *measure* the double diameter of the object A.

(201.) Innumerable optical combinations may be devised to operate such duplication. The chief and most important (from its recent applications), is the *heliometer*, in which the image is divided by bisecting *the object-glass of the telescope*, and making its two halves, set in separate brass frames,

slide laterally on each other, as A B, the motion being produced and measured by a screw. Each half, by the



laws of optics, forms its own image (somewhat blurred, it is true, by diffraction¹⁴), in its own axis; and thus two equal and similar images are formed side by side in the focus of the eye-piece, which may be made to approach and

recede by the motion of the screw, and thus afford the means of measurement as above described.

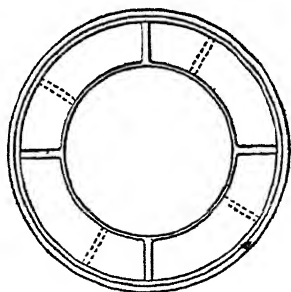
(202.) Double refraction through crystallized media affords another means of accomplishing the same end. Without going into the intricacies of this difficult branch of optics, it will suffice to state that objects viewed through certain crystals (as Iceland spar, or quartz) appear double, two images equally distinct being formed, whose angular distance from each other varies from nothing (or perfect coincidence), up to a certain limit, *according to the direction with respect to a certain fixed line in the crystal*, called its optical axis. Suppose, then, to take the simplest case, that the eye-lens of a telescope, instead of glass, were formed of such a crystal (say of quartz, which may be worked as well or better than glass), and *of a spherical form*, so as to offer no difference when turned about on its centre, other than the inclination of its optical axis to the visual ray. Then when that axis coincides with the line of collimation of the object-glass, one image only will be seen, but when made to revolve on an axis perpendicular to that line, two will arise,

¹⁴ This might be cured, though at an expense of light, by limiting each half to a circular space by diaphragms, as represented by the dotted lines.

opening gradually out from each other, and thus originating the desired duplication. In this contrivance, the angular amount of the rotation of the sphere affords the necessary datum for determining the separation of the images.

(203.) Of all methods which have been proposed, however, the simplest and most unobjectionable would appear to be the following. It is well known to every optical student, that two prisms of glass, a flint and a crown, may be opposed to each other, so as to produce a *colorless deflection* of parallel rays. An object seen through such a compound or achromatic prism will be seen simply deviated in direction, but in no way otherwise altered or distorted. Let such a prism be constructed with its surfaces so nearly parallel that the *total deviation* produced in traversing them shall not exceed a small amount

(say $5'$). Let this be cut in half, and from each half let a circular disk be formed, and cemented on a circular plate of parallel glass, or otherwise sustained, close to and concentric with the other by a framework of metal so light as to intercept but a small portion



of the light which passes on the outside (as in the above figure), where the dotted lines represent the radii sustaining one, and the undotted those carrying the other disk. The whole must be so mounted as to allow one disk to revolve in its own plane behind the other, fixed, and to allow the amount of rotation to be read off. It is evident, then, that when the deviations produced by the two disks conspire; a total deviation of $10'$ will be effected on all the light which has passed through them; that when they oppose each

other, the rays will emerge undeviated, and that in intermediate positions a deviation varying from 0 to 10', and calculable from the angular rotation of the one disk on the other, will arise. Now, let this combination be applied at such a point of the cone of rays, between the object-glass and its focus, that the disks shall occupy exactly half the area of its section. Then will half the light of the object lens pass undeviated—the other half deviated, as above described; and thus a duplication of image, variable and measureable (as required for micrometric measurement) will occur. If the object-glass be not very large, the most convenient point of its application will be externally before it, in which case the diameter of the disks will be to that of the object-glass as 707 : 1000; or (allowing for the spokes) about as 7 to 10.

(204.) The *Position Micrometer* is simply a straight thread or wire, which is carried round by a smooth revolving motion, in the common focus of the object and eye glasses, in a plane perpendicular to the axis of the telescope. It serves to determine the situation with respect to some fixed line in the field of view, of the line joining any two objects or points of an object seen in that field—as two stars, for instance, near enough to be seen at once. For this purpose the movable thread is placed so as to cover both of them, or stand, as may best be judged, parallel to their line of junction. And its angle, with the fixed one, is then read off upon a small divided circle exterior to the instrument. When such a micrometer is applied (as it most commonly is) to an equatorially mounted telescope, the *zero* of its position corresponds to a direction of the wire, such as, prolonged, will represent a circle of declination in the heavens—and the “angles of position” so read off are reckoned in-

variably from one point, and in one direction, viz., *north*, *following*, *south*, *preceding*; so that 0° position corresponds to the situation of an object exactly *north* of that assumed as a centre of reference— 90° to a situation exactly *eastward*, or *following*; 180° exactly *south*; and 270° exactly *west*, or *preceding* in the order of diurnal movement. When the relative position of two stars, very near to each other, so as to be seen at once in the same field of view, is to be determined in this way, especially if they be of unequal magnitudes, the best form of the instrument consists, not in a single thin wire to be placed centrally across both the stars, but in two thick parallel wires, *between* which both stars are brought under inspection in a symmetrical situation, by which arrangement the parallelism of the line joining their centres with the direction of the wires can be very much more accurately judged of. It gives great advantage, moreover, to the precision of such a judgment, if the position of the observer be such as to bring the principal section of his eye (that which in his upright position is vertical) into parallelism with the wires.

(204 a.) To see the fiducial threads or wires of an eyepiece or micrometer in a dark night is impossible without introducing some artificial light into the telescope, so as either to illuminate the field of view, leaving the threads dark, or *vice versa*. To illuminate the field, the light of a lamp is introduced by a lateral opening into the tube of the telescope, and dispersed by reflection on a white unpolished surface, so arranged as not to intercept any part of the cone of rays going to form the image. For illuminating the wires, direct lamp light is thrown on them from the side toward the eye; the superfluous rays being stifled by falling on a black internal coating, or suffered to pass out to the

tube through an opposite aperture opening into a dark chamber.

(204 *b.*) When the wires are seen dark on an illuminated field, the color of the illuminating light is of great importance. As a matter of experience, it is certain that a red illumination affords a far sharper and clearer view of the wires than any other.

(204 *c.*) For observing the sun, darkening glasses are necessary. In this case red glasses are inappropriate, because they transmit the solar heat freely, by which the eye would be seriously injured, and even when very deep tinted, render prolonged inspection intolerably painful. Green glasses are free from this objection. The best darkening glass, however, is a combination of a cobalt blue with a green glass, which, if the components are properly selected, transmits an almost homogeneous yellow light, and no sensible amount of heat. Both the light and heat of the sun, however, may be subdued by reflection at glass surfaces, the light returned by regular reflection on glass being only about $2\frac{1}{2}$ per cent of that incident on it. A reflecting telescope specifically adapted for viewing the sun may be constructed by making the specula of glass, the object-mirror having the form of a double concave lens, whose anterior surface (that producing the image) is worked into a paraboloid of the proper focal length, and the posterior to a sphere of considerably greater curvature to transmit and disperse outward the refracted rays into the open air behind (for which purpose the telescope should be open at both ends) and to so weaken those reflected by dispersing them as not to interfere with the distinctness of the image. Neither the quality of the glass, nor accuracy of figure in the posterior surface, is of any importance to the good per-

formance of such a reflector.¹⁵ Should the light be not sufficiently enfeebled by the first reflection, it may be still further reduced (to about 1—900th part of its original intensity) by making the small speculum of glass also in the form of a prism; the reflection being performed on one of its exterior surfaces, and the refracted portion being turned away and thrown out at the other.

(204 *d.*) Advantage may be also taken (as in Sir D. Brewster's polarizing eye-piece) of the properties of polarized light, which may be diminished in any required degree by partial reflection in a plane at right angles to that of its first oblique reflection. Or without polarization, the light may be enfeebled by successive reflections between parallel surfaces to any extent.

(204 *e.*) When the object in view is to scrutinize, under high magnifying powers, minute portions of the solar disk; the light and heat of the general surface may be intercepted by a metallic screen placed in the focus where the image is formed, and pierced with a very small hole, allowing that minute portion only to pass through and be examined with the eye-piece; the observer being thus defended from the glare. By this arrangement, Mr. Dawes, to whom the idea is due, has been enabled to observe some very extraordinary peculiarities in the constitution of the sun's surface, discernible in no other way, an account of which will be found in their proper place.

(204 *f.*) Since the use of large reflectors has become common among astronomers, the necessity of supporting the ponderous masses of their specula without constraint

¹⁵ I would take this opportunity earnestly to recommend the construction of a *helioscope* on this principle, first propounded and more fully described in my *Cape Observations* (p. 436), to the attention of the practical optician.

or undue pressure in any direction (which would distort the figure of their polished surfaces), renders the use of some ready method of verifying, from instant to instant, the adjustment of their lines of collimation (or the optical axis of the reflectors), and of readjusting it, when shifted, indispensable. For this purpose, a small collimating telescope (art. 178), illuminated by reflection from a lamp outside, is fixed within the tube of the reflector, its object-end being turned toward the speculum. *Upon the image* of the cross-wires of this telescope formed in the focus of the reflector, and seen through its eye-piece as a real object, the transits and altitudes of celestial objects may be observed as if it consisted of actual wires; for these, it is manifest, if once placed so as to bisect a star, will continue to do so, whatever amount of tilting the reflector might be subjected to, either in a lateral or vertical plane. The rays from the star and the axis of the collimator remaining parallel, the latter axis, and not that of the reflector, becomes in fact the real line of collimation or optic axis of the instrument, when objects are thus directly referred to it. Should convenience of micrometric measurement, or the observation of faint objects in a very feebly illuminated field, preclude such direct reference, the position of the speculum must from time to time be examined, and if faulty, readjusted by bringing the micrometer wires to coincidence with the image of those of the collimator by an appropriate mechanism communicating the requisite small amount of movement to the speculum in its cell.¹⁶

¹⁶ See Phil. Trans. 1833, pp. 448-9, where this application of the collimating principle used by the author since 1833, is first described. See also "Results of Astronomical Observation at the Cape of Good Hope," preface, p. xiv.

CHAPTER IV

OF GEOGRAPHY

Of the Figure of the Earth—Its Exact Dimensions—Its Form that of Equilibrium Modified by Centrifugal Force—Variation of Gravity on its Surface—Statical and Dynamical Measures of Gravity—The Pendulum—Gravity to a Spheroid—Other Effects of the Earth's Rotation—Trade Winds—Veering of the Winds—Cyclones—Foucault's Pendulum—The Gyroscope—Determination of Geographical Positions—Of Latitudes—Of Longitudes—Conduct of a Trigonometrical Survey—Of Maps—Projections of the Sphere—Measurement of Heights by the Barometer

(205.) GEOGRAPHY is not only the most important of the practical branches of knowledge to which astronomy is applied, but it is also, theoretically speaking, an essential part of the latter science. The earth being the general station from which we view the heavens, a knowledge of the local situation of particular stations on its surface is of great consequence, when we come to inquire the distances of the nearer heavenly bodies from us, as concluded from observations of their parallax as well as on all other occasions, where a difference of locality can be supposed to influence astronomical results. We propose, therefore, in this chapter, to explain the principles by which astronomical observation is applied to geographical determinations, and to give at the same time an outline of geography so far as it is to be considered a part of astronomy.

(206.) Geography, as the word imports, is a delineation or description of the earth. In its widest sense, this comprehends not only the delineation of the form of its conti-

nents and seas, its rivers and mountains, but their physical condition, climates, and products, and their appropriation by communities of men. With physical and political geography, however, we have no concern here. Astronomical geography has for its objects the exact knowledge of the form and dimensions of the earth, the parts of its surface occupied by sea and land, and the configuration of the surface of the latter, regarded as protuberant above the ocean, and broken into the various forms of mountain, tableland and valley; neither should the form of the bed of the ocean, regarded as a continuation of the surface of the land beneath the water, be left out of consideration: we know, it is true, very little of it; but this is an ignorance rather to be lamented, and, if possible, remedied, than acquiesced in, inasmuch as there are many very important branches of inquiry which would be greatly advanced by a better acquaintance with it.

(207.) With regard to the figure of the earth *as a whole*, we have already shown that, speaking loosely, it may be regarded as spherical; but the reader who has duly appreciated the remarks in art. 22 will not be at a loss to perceive that this result, concluded from observations not susceptible of much exactness, and embracing very small portions of the surface at once, can only be regarded as a first approximation, and may require to be materially modified by entering into minutiae before neglected, or by increasing the delicacy of our observations, or by including in their extent larger areas of its surface. For instance, if it should turn out (as it will), on minuter inquiry, that the true figure is somewhat elliptical, or flattened, in the manner of an orange, having the diameter which coincides with the axis about $\frac{1}{316}$ th part shorter than the diameter of its equatorial

circle;—this is so trifling a deviation from the spherical form that, if a model of such proportions were turned in wood, and laid before us on a table, the nicest eye or hand would not detect the flattening, since the difference of diameters, in a globe of fifteen inches, would amount only to $\frac{1}{20}$ th of an inch. In all common parlance, and for all ordinary purposes, then, it would still be called a globe; while, nevertheless, by careful measurement, the difference would not fail to be noticed; and, speaking strictly, it would be termed, not a globe, but an oblate ellipsoid, or spheroid, which is the name appropriated by geometers to the form above described.

(208.) The sections of such a figure by a plane are not circles, but ellipses; so that, on such a shaped earth, the horizon of a spectator would nowhere (except at the poles) be exactly circular, but somewhat elliptical. It is easy to demonstrate, however, that its deviation from the circular form, arising from so very slight an “*ellipticity*” as above supposed, would be quite imperceptible, not only to our eyesight, but to the test of the dip-sector; so that by that mode of observation we should never be led to notice so small a deviation from perfect sphericity. How we are led to this conclusion, as a practical result, will appear, when we have explained the means of determining with accuracy the dimensions of the whole, or any part of the earth.

(209.) As we cannot grasp the earth, nor recede from it far enough to view it at once as a whole, and compare it with a known standard of measure in any degree commensurate to its own size, but can only creep about upon it, and apply our diminutive measures to comparatively small parts of its vast surface in succession, it becomes necessary to supply, by geometrical reasoning, the defect of our phys-

ical powers, and from a delicate and careful measurement of such small parts to conclude the form and dimensions of the whole mass. This would present little difficulty, if we were sure the earth were strictly a sphere, for the proportion of the circumference of a circle to its diameter being known (viz., that of 3.1415026 to 1.0000000), we have only to ascertain the length of the entire circumference of any great circle, such as a meridian, in miles, feet, or any other standard units, to know the diameter in units of the same kind. Now, the circumference of the whole circle is known as soon as we know the exact length of any aliquot part of it, such as 1° or $\frac{1}{360}$ th part; and this, being not more than about seventy miles in length, is not beyond the limits of very exact measurement, and could, in fact, be measured (if we knew its exact termination at each extremity) within a very few feet, or, indeed, inches, by methods presently to be particularized.

(210.) Supposing, then, we were to begin measuring with all due nicety from any station, in the exact direction of a meridian, and go measuring on, till by some indication we were informed that we had accomplished an exact *degree* from the point we set out from, our problem would then be at once resolved. It only remains, therefore, to inquire by what indications we can be sure, 1st, that we *have* advanced *an exact degree*; and, 2dly, that we have been measuring in the *exact direction of a great circle*.

(211.) Now, the earth has no landmarks on it to indicate degrees, nor traces inscribed on its surface to guide us in such a course. The compass, though it affords a tolerable guide to the mariner or the traveller, is far too uncertain in its indications, and too little known in its laws, to be of any use in *such* an operation. We must, therefore,

look outward, and refer our situation on the surface of our globe to natural marks, *external* to it, and which are of equal permanence and stability with the earth itself. Such marks are afforded by the stars. By observations of their meridian altitudes, performed at any station, and from their known polar distances, we conclude the height of the pole; and since the altitude of the pole is equal to the latitude of the place (art. 119), the same observations give the latitudes of any stations where we may establish the requisite instruments. When our latitude, then, is found to have diminished a degree, we know that, *provided we have kept to the meridian*, we have described one three hundred and sixtieth part of the earth's circumference.

(212.) The direction of the meridian may be secured at every instant by the observations described in arts. 162, 188; and although local difficulties may oblige us to deviate in our measurement from this exact direction, yet if we keep a strict account of the amount of this deviation, a very simple calculation will enable us to *reduce* our observed measure to its *meridional* value.

(213.) Such is the principle of that most important geographical operation, the measurement of an arc of the meridian. In its detail, however, a somewhat modified course must be followed. An observatory cannot be mounted and dismounted at every step; so that we cannot identify and measure an exact degree *neither more nor less*. But this is of no consequence, provided we know with equal precision *how much*, more or less, we have measured. In place, then, of measuring this precise aliquot part, we take the more convenient method of measuring from one good observing station to another, *about* a degree, or two or three degrees, as the case may be, or indeed any deter-

minate angular interval apart, and determining by astronomical observation the precise difference of latitudes between the stations.

(214.) Again, it is of great consequence to avoid in this operation every source of uncertainty, because an error committed in the length of a single degree will be multiplied 360 times in the circumference, and nearly 115 times in the diameter of the earth concluded from it. Any error which may affect the astronomical determination of a star's altitude will be especially influential. Now, there is still too much uncertainty and fluctuation in the amount of refraction at moderate altitudes, not to make it especially desirable to avoid this source of error. To effect this, we take care to select for observation, at the extreme stations, some star which passes through or near the zeniths of both. The amount of refraction, within a few degrees of the zenith, is very small, and, its fluctuations and uncertainty, in point of quantity, so excessively minute as to be utterly inappreciable. Now, it is the same thing whether we observe the *pole* to be raised or depressed a degree, or the *zenith distance* of a star when on the meridian to have changed by the same quantity (fig. art. 128). If at one station we observe any star to pass through the zenith, and at the other to pass one degree south or north of the zenith, we are sure that the geographical latitudes, or the altitudes of the pole at the two stations, must differ by the same amount.

(215.) Granting that the terminal points of one degree can be ascertained, its *length* may be measured by the methods which will be presently described, as we have before remarked, to within a very few feet. Now, the error which may be committed in fixing each of these terminal points cannot exceed that which may be committed in the obser-

vation of the zenith distance of a star properly situated for the purpose in question. This error, with proper care, can hardly exceed half a second. Supposing we grant the possibility of ten feet of error in the length of each degree in a measured arc of five degrees, and of half a second in each of the zenith distances of one star, observed at the northern and southern stations, and, lastly, suppose all these errors to conspire, so as to tend all of them to give a result greater, or all less, than the truth, it will appear, by a very easy proportion, that the whole amount of error which would be thus entailed on an estimate of the earth's diameter, as concluded from such a measure, would not exceed 1147 yards, or about two-thirds of a mile, and this is ample allowance.

(216.) This, however, supposes that the form of the earth is that of a perfect sphere, and, in consequence, the lengths of its degrees in all parts precisely equal. But, when we come to compare the measures of meridional arcs made in various parts of the globe, the results obtained, although they agree sufficiently to show that the supposition of a spherical figure is not *very* remote from the truth, yet exhibit discordances far greater than what we have shown to be attributable to error of observation, and which render it evident that the hypothesis, in strictness of its wording, is untenable. The following table exhibits the lengths of arcs of the meridian (astronomically determined as above described), expressed in British standard feet, as resulting from actual measurement made with all possible care and precision, by commissioners of various nations, men of the first eminence, supplied by their respective governments with the best instruments, and furnished with every facility which could tend to insure a successful result of their im-

portant labors. The lengths of the degrees in the last column are derived from the numbers set down in the two preceding ones by simple proportion, a method not quite exact when the arcs are large, but sufficiently so for our purpose.

Country	Latitude of Middle of Arc	Arc measured	Measured Length in Feet	Mean Length of the De- gree at the Mid- dle Latitude in Feet
Sweden ¹ , A B . . .	+66° 20' 10''·0	1° 37' 19''·6	593277	365744
Sweden, A . . .	+66 19 37	0 57 30·4	351832	367086
Russia, A . . .	+58 17 37	3 35 5·2	1309742	365368
Russia, B . . .	+56 3 55·5	8 2 28·9	2937439	365291
Prussia, B . . .	+54 58 26·0	1 30 29·0	551073	365420
Denmark, B . . .	+54 8 13·7	1 31 53·3	559121	365087
Hanover, A B . . .	+52 32 16·6	2 0 57·4	736425	365300
England, A . . .	+52 35 45	3 57 13·1	1442953	364971
England, B . . .	+52 2 19·4	2 50 23·5	1036409	364951
France, A . . .	+46 52 2	8 20 0·3	3040605	364872
France, A B . . .	+44 51 2·5	12 22 12·7	4509832	364572
Rome, A . . .	+42 59 —	2 9 47	787919	364262
America, A . . .	+39 12 —	1 28 45·0	538100	363786
India, A B . . .	+16 8 21·5	15 57 40·7	5794598	363044
India, A B . . .	+12 32 20·8	1 34 56·4	574318	362956
Peru, A B . . .	— 1 31 0·4	3 7 3·5	1131050	362790
Cape of Good Hope, A	—33 18 30	1 13 17·5	445506	364713
Cape of Good Hope, B	—35 43 20·0	3 34 34·7	1301993	364060

It is evident from a mere inspection of the second and fifth columns of this table, that *the measured length of a degree increases with the latitude*, being greatest near the poles,

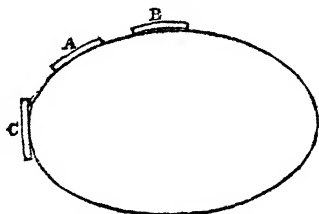
¹ The astronomers by whom these measurements were executed were as follows:—

Sweden, A B—Svanberg.
Sweden, A—Maupertuis.
Russia, A—Struve.
Russia, B—Struve, Tenner.
Prussia—Bessel, Bayer.
Denmark—Schumacher.
Hanover—Gauss.
England—Roy, Kater.
France, A—Lacaille, Cassini.

France, A B—Delambre, Mechain.
Rome—Boscovich.
America—Mason and Dixon.
India, 1st—Lambton, Everest.
India, 2d—Lambton.
Peru—Lacondamine, Bouguer.
Cape of Good Hope, A—Lacaille.
Cape of Good Hope, B—Maclear.
—Astr. Nachr. 574.

and least near the equator. Let us now consider what interpretation is to be put upon this conclusion, as regards the form of the earth.

(217.) Suppose we held in our hands a model of the earth smoothly turned in wood, it would be, as already observed, so nearly spherical, that neither by the eye nor the touch, unassisted by instruments, could we detect any deviation from that form. Suppose, too, we were debarred from measuring directly across from surface to surface in different directions with any instrument, by which we might at once ascertain whether one diameter were longer than another; how, then, we may ask, are we to ascertain whether it is a true sphere or not? It is clear that we have no resource, but to endeavor to discover, by some nicer means than simple inspection or feeling, whether the convexity of its surface is the same in every part; and if not, where it is greatest, and where least. Suppose, then, a thin plate of metal to be cut into a concavity at its edge, so as exactly to fit the surface at A: let this now be removed from A, and applied successively to several other parts of the surface, taking care to keep its plane always on a great circle of the globe, as here represented. If, then, we find any position B, in which the light can enter in the middle between the globe and plate, or any other, C, where the latter tilts by pressure, or admits the light under its edges, we are sure that the *curvature* of the surface at B is less, and at C greater, than at A.

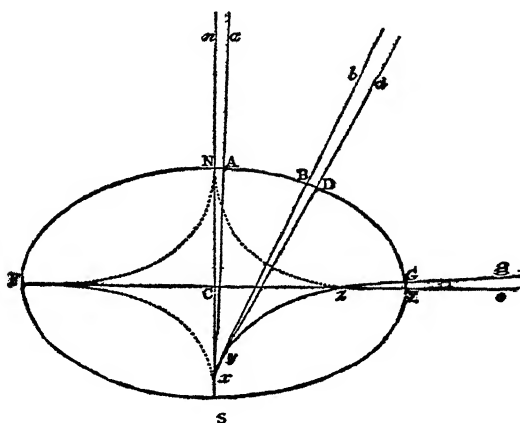


(218.) What we here do by the application of a metal plate of determinate length and curvature, we do on the

earth by the measurement of a degree of variation in the altitude of the pole. Curvature of a surface is nothing but the continual deflection of its tangent from one fixed direction as we advance along it. When, in the *same measured distance of advance* we find the tangent (which answers to our horizon) to have shifted its position with respect to a fixed direction in space (such as the axis of the heavens, or the line joining the earth's centre and some given star) *more* in one part of the earth's meridian than in another, we conclude, of necessity, that the curvature of the surface at the former spot is greater than at the latter; and *vice versâ*, when, in order to produce the same change of horizon with respect to the pole (suppose 1°) we require to travel over a *longer* measured space at one point than at another, we assign to that point a less curvature. Hence we conclude that *the curvature of a meridional section of the earth is sensibly greater at the equator than toward the poles*; or, in other words, that the earth is not spherical, but *flattened* at the poles, or, which comes to the same, protuberant at the equator.

(219.) Let $N A B D E F$ represent a meridional section of the earth, C its centre, and $N A, B D, G E$, arcs of a meridian, each corresponding to one degree of difference of latitude, or to one degree of variation in the meridian altitude of a star, as referred to the horizon of a spectator travelling along the meridian. Let $n N, a A, b B, d D, g G, e E$, be the respective directions of the *plumb-line* at the stations N, A, B, D, G, E , of which we will suppose N to be at the pole and E at the equator; then will the tangents to the surface at these points respectively be perpendicular to these directions; and, consequently, if each pair, viz. $n N$ and $a A, b B$ and $d D, g G$ and $e E$, be pro-

longed till they intersect each other (at the points x, y, z), the angles $N x A, B y D, G z E$, will each be one degree, and, therefore, all equal; so that the small curvilinear arcs $N A, B D, G E$, may be regarded as arcs of circles of one degree each, described about x, y, z , as centres. These are what in geometry are called *centres of curvature*, and the radii $x N$ or $x A, y B$ or $y D, z G$ or $z E$, represent *radii of curvature*, by which the curvatures at those points are determined and measured. Now, as the arcs of different



circles, which subtend equal angles at their respective centres, are in the direct proportion of their radii, and as the arc $N A$ is greater than $B D$, and that again than $G E$, it follows that the radius $N x$ must be greater than $B y$, and $B y$ than $E z$. Thus it appears that the mutual intersections of the plumb-lines will not, as in the sphere, all coincide in one point C , the centre, but will be arranged along a certain curve, $x y z$ (which will be rendered more evident by considering a number of intermediate stations). To this curve geometers have given the name of the *evolute*

of the curve N A B D G E, from whose centres of curvature it is constructed.

(220.) In the flattening of a round figure at two opposite points, and its protuberance at points rectangularly situated to the former, we recognize the distinguishing feature of the elliptic form. Accordingly, the next and simplest supposition that we can make respecting the nature of the meridian, since it is proved not to be a circle, is that it is an ellipse, or nearly so, having N S, the axis of the earth, for its shorter, and E F, the equatorial diameter, for its longer axis; and that the form of the earth's surface is that which would arise from making such a curve revolve about its shorter axis N S. This agrees well with the general course of the increase of the degree in going from the equator to the pole. In the ellipse, the radius of curvature at E, the extremity of the longer axis is the least, and at that of the shorter axis, the greatest it admits, and the form of its *evolute* agrees with that here represented.* Assuming, then, that it is an ellipse, the geometrical properties of that curve enable us to assign the proportion between the lengths of its axes which shall correspond to any proposed rate of variation in its curvature, as well as to fix upon their absolute lengths, corresponding to any assigned length of the degree in a given latitude. Without troubling the reader with the investigation (which may be found in any work on the conic sections), it will be sufficient to state the results which have been arrived at by the most systematic combinations of the measured arcs which have hitherto been made by geometers. The most recent is that of

* The dotted lines are the portions of the evolute belonging to the other quadrants.

Bessel,³ who by a combination of the ten arcs, marked B in our table, has concluded the dimensions of the terrestrial spheroid to be as follows:

	Feet	Miles
Greater or equatorial diameter	= 41,847,192	= 7925·604
Lesser or polar diameter	= 41,707,314	= 7899·114
Difference of diameters, or polar compression . .	= 139,768	= 26·471
Proportion of diameters as 299·15 to 298·15.		

The other combination whose result we shall state, is that of Mr. Airy,⁴ who concludes as follows:

	Feet	Miles
Equatorial diameter	= 41,847,426	= 7925·648
Polar diameter	= 41,707,620	= 7899·170
Polar compression	= 139,806	= 26·478
Proportion of diameters as 299·33 to 298·33.		

These conclusions are based on the consideration of those 13 arcs, to which the letter A is annexed,⁵ and of one other arc of $1^{\circ} 7' 31''\cdot 1$, measured in Piedmont by Plana and Carlini, whose discordance with the rest, owing to local causes hereafter to be explained, arising from the exceedingly mountainous nature of the country, render the propriety of so employing it very doubtful. Be that as it may, the strikingly near accordance of the two sets of dimensions is such as to inspire the greatest confidence in both. The measurement at the Cape of Good Hope by Lacaille, also used in this determination, has always been regarded as unsatisfactory, and has recently been demonstrated by Mr. Th. Maclear to be erroneous to a considerable extent. The omission of the former, and the substitution for the latter, of the far preferable result of Maclear's second measurement

³ Schumacher's *Astronomische Nachrichten*, Nos. 333, 334, 335, 438 (1841).

⁴ *Encyclopædia Metropolitana*, "Figure of the Earth" (1831).

⁵ In those which have both A and B, the numbers used by Mr. Airy differ slightly from Bessel's, which are those we have preferred.

would induce, however, but a trifling change in the final result.

(221.) Thus we see that the rough diameter of 8000 miles we have hitherto used, is rather too great, the excess being about 100 miles, or $\frac{1}{80}$ th part. As convenient numbers to remember, the reader may bear in mind, that in our latitude there are just as many thousands of feet in a degree of the meridian as there are days in the year (365): that, speaking loosely, a degree is about 70 British statute miles, and a second about 100 feet; that the equatorial circumference of the earth is a little less than 25,000 miles (24,899), and the ellipticity or polar flattening amounts to one 300th part of the diameter.

(222.) The two sets of results above stated are placed in juxtaposition, and the particulars given more in detail than may at first sight appear consonant, either with the general plan of this work, or the state of the reader's presumed acquaintance with the subject. But it is of importance that he should early be made to see how, in astronomy, results in admirable concordance emerge from data accumulated from totally different quarters, and how local and accidental irregularities in the data themselves become neutralized and obliterated by their impartial geometrical treatment. In the cases before us, the modes of calculation followed are widely different, and in each the mass of figures to be gone through to arrive at the result, enormous.

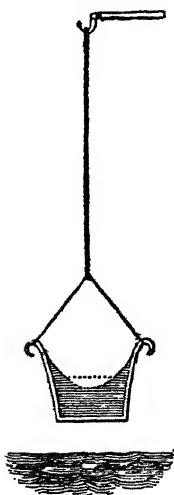
(223.) The supposition of an elliptic form of the earth's section through the axis is recommended by its simplicity, and confirmed by comparing the numerical results we have just set down with those of actual measurement. When this comparison is executed, discordances, it is true, are observed, which, although still too great to be referred to

error of measurement, are yet so small, compared to the errors which would result from the spherical hypothesis, as completely to justify our regarding the earth as an ellipsoid, and referring the observed deviations to either local or, if general, to comparatively small causes. [For § (223 a) see Note D.]

(224.) Now, it is highly satisfactory to find that the general elliptical figure thus *practically* proved to exist, is precisely what *ought theoretically* to result from the rotation of the earth on its axis. For, let us suppose the earth a sphere, at rest, of uniform materials throughout, and externally covered with an ocean of equal depth in every part. Under such circumstances it would obviously be in a state of *equilibrium*; and the water on its surface would have no tendency to run one way or the other. Suppose, now, a quantity of its materials were taken from the polar regions, and piled up all around the equator, so as to produce that difference of the polar and equatorial diameters of 26 miles which we know to exist. It is not less evident that a mountain ridge or equatorial *continent, only*, would be thus formed, down which the water would run into the excavated part at the poles. However solid matter might rest where it was placed, the liquid part, at least, would not remain there, any more than if it were thrown on the side of a hill. The consequence, therefore, would be the formation of two great polar seas, hemmed in all round by equatorial land. Now, this is by no means the case in nature. The ocean occupies, indifferently, all latitudes, with no more partiality to the polar than to the equatorial. Since, then, as we see, the water occupies an elevation above the centre no less than 13 miles greater at the equator than at the poles, and yet manifests no tendency to leave the

former and run toward the latter, it is evident that it must be *retained* in that situation by some adequate *power*. No such power, however, would exist in the case we have supposed, which is therefore not conformable to nature. In other words, the spherical form is *not* the *figure of equilibrium*; and *therefore* the earth is either not at rest, or is so internally constituted as to *attract* the water to its equatorial regions, and retain it there. For the latter supposition there is no *primâ facie* probability, nor any analogy to lead us to such an idea. The former is in accordance with all the phenomena of the apparent diurnal motion of the heavens; and therefore, if it will furnish us with the *power* in question, we can have no hesitation in adopting it as the true one.

(225.) Now, everybody knows that when a weight is whirled round, it acquires thereby a tendency to recede from the centre of its motion; which is called the centrifugal force. A stone whirled round in a sling is a common illustration; but a better, for our present purpose, will be a pail of water, suspended by a cord, and made to *spin round*, while the cord hangs perpendicularly. The surface of the water, instead of remaining horizontal, will become concave, as in the figure. The centrifugal force generates a tendency in *all* the water to leave the axis, and press toward the circumference; it is, therefore, urged against the pail, and forced up its sides, till the excess of height, and consequent increase of pressure downward, just counterbalances its centrifugal force, and a *state of equilibrium* is



attained. The experiment is a very easy and instructive one, and is admirably calculated to show how the *form of equilibrium* accommodates itself to varying circumstances. If, for example, we allow the rotation to cease by degrees, as it becomes slower we shall see the concavity of the water regularly diminish; the elevated outward portion will descend, and the depressed central rise, while all the time a perfectly *smooth* surface is maintained, till the rotation is exhausted, when the water resumes its horizontal state.

(226.) Suppose, then, a globe, of the size of the earth, at rest, and covered with a uniform ocean, were to be set in rotation about a certain axis, at first very slowly, but by degrees more rapidly, till it turned round once in twenty-four hours; a centrifugal force would be thus generated, whose general tendency would be to urge the water at every point of the surface to *recede* from the *axis*. A rotation might, indeed, be conceived so swift, as to *flirt* the whole ocean from the surface, like water from a mop. But this would require a far greater velocity than what we now speak of. In the case supposed, the *weight* of the water would still keep it *on* the earth; and the tendency to recede from the *axis* *could* only be satisfied, therefore, by the water leaving the poles, and flowing toward the equator; there heaping itself up in a ridge, just as the water in our pail accumulates against the side; and being retained in opposition to its weight, or natural tendency toward the centre, by the pressure thus caused. This, however, could not take place without laying dry the polar portions of the land in the form of immensely protuberant continents; and the difference of our supposed cases, therefore, is this:—in the former, a great equatorial continent and polar seas would be formed; in the latter, protuberant land would

appear at the poles, and a zone of ocean be disposed around the equator. This would be the first or immediate effect. Let us now see what would afterward happen, in the two cases, if things were allowed to take their natural course.

(227.) The sea is constantly beating on the land, grinding it down, and scattering its worn off particles and fragments, in the state of mud and pebbles, over its bed. Geological facts afford abundant proof that the existing continents have all of them undergone this process, even more than once, and been entirely torn in fragments, or reduced to powder, and submerged and reconstructed. Land, in this view of the subject, loses its attribute of fixity. As a mass it might hold together in opposition to forces which the water freely obeys; but in its state of successive or simultaneous degradation, when disseminated through the water, in the state of sand or mud, it is subject to all the impulses of that fluid. In the lapse of time, then, the protuberant land in both cases would be destroyed, and spread over the bottom of the ocean, filling up the lower parts, and tending continually to remodel the surface of the solid nucleus, in correspondence with the *form of equilibrium* in both cases. Thus, after a sufficient lapse of time, in the case of an earth at rest, the equatorial continent, thus forcibly constructed, would again be levelled and transferred to the polar excavations, and the spherical figure be so at length restored. In that of an earth in rotation, the polar protuberances would gradually be cut down and disappear, being transferred to the equator (as being *then* the *deepest sea*), till the earth would assume by degrees the form we observe it to have—that of a flattened or *oblate* ellipsoid.

(228.) We are far from meaning here to trace the process *by which* the earth really assumed its actual form; all we

intend is, to show that this is the form to which, under the conditions of a rotation on its axis, it must *tend*; and which it would attain, even if originally and (so to speak) perversely constituted otherwise.

(229.) But, further, the dimensions of the earth and the time of its rotation being known, it is easy thence to calculate the exact amount of the centrifugal force,⁶ which, at the equator, appears to be $\frac{1}{289}$ th part of the force or weight by which all bodies, whether solid or liquid, tend to fall toward the earth. By this fraction of its weight, then, the sea at the equator is *lightened*, and thereby rendered susceptible of being supported on a higher level, or more remote from the centre than at the poles, where no such counteracting force exists; and where, in consequence, the water may be considered as *specifically heavier*. Taking this principle as a guide, and combining it with the laws of gravity (as developed by Newton, and as hereafter to be more fully explained), mathematicians have been enabled to investigate, *a priori*, what would be the figure of equilibrium of such a body, constituted internally as we have reason to believe the earth to be; covered wholly or partially with a fluid; and revolving uniformly in twenty-four hours; and the result of this inquiry is found to agree very satisfactorily with what experience shows to be the case. From their investigations it appears that the form of equilibrium is, in fact, no other than an oblate ellipsoid, of a degree of ellipticity very nearly identical with what is observed, and which would be no doubt accurately so, did we know, with precision, the internal constitution and materials of the earth.

⁶ Newton's Principia, iii. Prop. 19.

(230.) The confirmation thus incidentally furnished, of the hypothesis of the earth's rotation on its axis, cannot fail to strike the reader. A deviation of its figure from that of a sphere was not contemplated among the original reasons for adopting that hypothesis, which was assumed solely on account of the easy explanation it offers of the apparent diurnal motion of the heavens. Yet we see that, once admitted, it draws with it, as a necessary consequence, this other remarkable phenomenon, of which no other satisfactory account could be rendered. Indeed, so direct is their connection, that the ellipticity of the earth's figure was discovered and demonstrated by Newton to be a consequence of its rotation, and its amount actually calculated by him, long before any measurement had suggested such a conclusion. As we advance with our subject, we shall find the same simple principle branching out into a whole train of singular and important consequences, some obvious enough, others which at first seem entirely unconnected with it, and which, until traced by Newton up to this their origin, had ranked among the most inscrutable arcana of astronomy, as well as among its grandest phenomena.

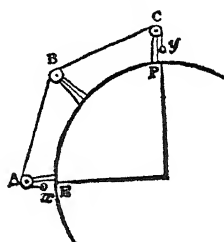
(231.) Of its more obvious consequences, we may here mention one which falls naturally within our present subject. If the earth really revolve on its axis, this rotation must generate a centrifugal force (see art. 225), the effect of which must of course be to counteract a certain portion of the *weight* of every body situated at the equator, as compared with its weight at the poles, or in any intermediate latitudes. Now, this is fully confirmed by experience. There is actually observed to exist a difference in the *gravity*, or downward tendency, of one and the same body, when conveyed successively to stations in different lati-

tudes. Experiments made with the greatest care, and in every accessible part of the globe, have fully demonstrated the fact of a regular and progressive increase in the weights of bodies corresponding to the increase of latitude, and fixed its amount and the law of its progression. From these it appears, that the extreme amount of this variation of gravity, or the difference between the equatorial and polar weights of one and the same mass of matter, is 1 part in 194 of its whole weight, the rate of increase in travelling from the equator to the pole being *as the square of the sine of the latitude*.

(232.) The reader will here naturally inquire what is *meant* by speaking of the same body as having different weights at different stations; and, how such a fact, if true, can be ascertained. When we weigh a body by a balance or a steel-yard we do but counteract its weight by the equal weight of another body under the very same circumstances; and if both the body weighed and its counterpoise be removed to another station, their gravity, if changed at all, will be changed equally, so that they will still continue to counterbalance each other. A difference in the intensity of gravity could, therefore, never be detected by these means; nor is it in *this* sense that we assert that a body weighing 194 pounds at the equator will weigh 195 at the pole. If counterbalanced in a scale or steel-yard at the former station, an additional pound placed in one or other scale at the latter would inevitably sink the beam.

(233.) The meaning of the proposition may be thus explained:—Conceive a weight x suspended at the equator by a string without weight passing over a pulley, A, and conducted (supposing such a thing possible) over other pulleys, such as B, round the earth's convexity, till the

other end hung down at the pole, and there sustained the weight y . If, then, the weights x and y were such as,

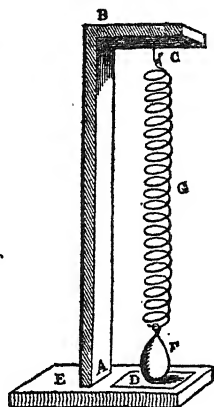


at any one station, equatorial or polar, would exactly counterpoise each other on a balance, or when suspended side by side over a single pulley, they would not counterbalance each other in this supposed situation, but the polar weight y would preponderate; and to restore

the equipoise the weight x must be increased by $\frac{1}{164}$ th part of its quantity.

(234.) The means by which this variation of gravity may be shown to exist, and its amount measured, are twofold (like all estimations of mechanical power) statical and dynamical. The former consists in putting the gravity of a weight in equilibrium, not with that of another weight, but with a natural power of a different kind not liable to be affected by local situation. Such a power is the elastic force of a spring. Let A B C be a strong support of brass standing on the foot A E D cast in one piece with it, into which is let a smooth plate of agate, D, which can be adjusted to perfect horizontality by a level. At C let a spiral spring G be attached, which carries at its lower end a weight F, polished and convex below. The length and strength of the spring must be so adjusted that the weight F shall be sustained by it just to swing clear of contact with the agate plate in the highest latitude at which it is intended to use the instrument. Then, if small weights be added cautiously, it may be made to descend till it *just grazes* the agate, a contact which can be made with the utmost imaginable delicacy. Let these weights be noted; the weight F detached; the spring G carefully lifted off its hook, and

secured, for travelling, from rust, strain, or disturbance, and the whole apparatus conveyed to a station in a lower latitude. It will then be found, on remounting it, that, although loaded with the same additional weights as before, the weight *F* will no longer have power enough to stretch the spring to the extent required for producing a similar contact. More weights will require to be added; and the additional quantity necessary will, it is evident, measure the difference of gravity between the two stations, as exerted on the whole quantity of pendent matter, *i.e.* the sum of the weight of *F* and half that of the spiral spring itself. Granting that a spiral spring can be constructed of such strength and dimensions that a weight of 10,000 grains, including its own, shall produce an elongation of 10 inches without permanently straining it,* one additional grain will produce a further extension of $\frac{1}{1000}$ th of an inch, a quantity which cannot possibly be mistaken in such a contact as that in question. Thus we should be provided with the means of measuring the power of gravity at any station to within $\frac{1}{10000}$ th of its whole quantity.



(235.) The other, or dynamical process, by which the

* Whether the process above described could ever be so far perfected and refined as to become a substitute for the use of the pendulum must depend on the degree of permanence and uniformity of action of springs, on the constancy or variability of the effect of temperature on their elastic force, on the possibility of transporting them, absolutely unaltered, from place to place, etc. The great advantages, however, which such an apparatus and mode of observation would possess, in point of convenience, cheapness, portability, and expedition, over the present laborious, tedious, and expensive process, render the attempt well worth making. [See Note E.]

force urging any given weight to the earth may be determined, consists in ascertaining the velocity imparted by it to the weight when suffered to fall freely in a given time, as one second. This velocity cannot, indeed, be directly measured; but indirectly, the principles of mechanics furnish an easy and certain means of deducing it, and, consequently, the intensity of gravity, by observing the oscillations of a pendulum. It is proved from mechanical principles,* that, if one and the same pendulum be made to oscillate at different stations, or under the influence of different forces, and the numbers of oscillations made in the same time in each case be counted, the intensities of the forces will be to each other as the squares of the numbers of oscillations made, and thus their proportion becomes known. For instance, it is found that, under the equator, a pendulum of a certain form and length makes 86,400 vibrations in a mean solar day; and that, when transported to London, the same pendulum makes 86,535 vibrations in the same time. Hence we conclude, that the intensity of the force urging the pendulum downward at the equator is to that at London as $(86,400)^2$ to $(86,535)^2$, or as 1 to 1.00315; or, in other words, that a mass of matter weighing in London 100,000 pounds, exerts the same pressure on the ground, or the same effort to crush a body placed below it, that 100,315 *of the same pounds* transported to the equator would exert there.

(236.) Experiments of this kind have been made, as above stated, with the utmost care and minutest precaution to insure exactness in all accessible latitudes; and their general and final result has been, to give $\frac{1}{144}$ for the fraction

* Newton's Principia, ii. Prop. 24. Cor. 3.

expressing the difference of gravity at the equator and poles. Now, it will not fail to be noticed by the reader, and will, probably, occur to him as an objection against the explanation here given of the fact by the earth's rotation, that this differs materially from the fraction $\frac{1}{289}$ expressing the centrifugal force at the equator. The difference by which the former fraction exceeds the latter is $\frac{1}{840}$, a small quantity in itself, but still far too large, compared with the others in question, not to be distinctly accounted for, and not to prove fatal to this explanation if it will not render a strict account of it.

(237.) The mode in which this difference arises affords a curious and instructive example of the indirect influence which mechanical causes often exercise, and of which astronomy furnishes innumerable instances. The rotation of the earth gives rise to the centrifugal force; the centrifugal force produces an ellipticity in the form of the earth itself; and this very ellipticity of form modifies its power of attraction on bodies placed at its surface, and thus gives rise to the difference in question. Here, then, we have the same cause exercising at once a direct and an indirect influence. The amount of the former is easily calculated, that of the latter with far more difficulty, by an intricate and profound application of geometry, whose steps we cannot pretend to trace in a work like the present, and can only state its nature and result.

(238.) The weight of a body (considered as undiminished by a centrifugal force) is the effect of the earth's attraction on it. This attraction, as Newton has demonstrated, consists, not in a tendency of all matter to any one particular centre, but in a disposition of every particle of matter in the universe to press toward, and if not opposed to approach

to, every other. The attraction of the earth, then, on a body placed on its surface, is not a simple but a complex force, resulting from the separate attractions of all its parts. Now, it is evident, that if the earth were a perfect sphere, the attraction exerted by it on a body placed anywhere on its surface, whether at its equator or pole, must be exactly alike—for the simple reason of the exact symmetry of the sphere in every direction. It is not less evident that, the earth being elliptical, and this symmetry or similitude of all its parts not existing, the same result cannot be expected. A body placed at the equator, and a similar one at the pole of a flattened ellipsoid, stand in a different geometrical relation to the mass as a whole. This difference, without entering further into particulars, may be expected to draw with it a difference in its forces of attraction on the two bodies. Calculation confirms this idea. It is a question of purely mathematical investigation, and has been treated with perfect clearness and precision by Newton, Maclaurin, Clairaut, and many other eminent geometers; and the result of their investigations is to show that, owing to the elliptic form of the earth alone, and independent of the centrifugal force, its attraction ought to increase the weight of a body in going from the equator to the pole by almost exactly $\frac{1}{230}$ th part; which, together with $\frac{1}{289}$ th due to the centrifugal force, make up the whole quantity, $\frac{1}{114}$ th, observed.

(239.) Another great geographical phenomenon, which owes its existence to the earth's rotation, is that of the trade-winds. These mighty currents in our atmosphere, on which so important a part of navigation depends, arise from, 1st, the unequal exposure of the earth's surface to the sun's rays, by which it is unequally heated in different latitudes; and, 2dly, from that general law in the constitution of all

fluids, in virtue of which they occupy a larger bulk, and become specifically lighter when hot than when cold. These causes, combined with the earth's rotation from west to east, afford an easy and satisfactory explanation of the magnificent phenomena in question.

(240.) It is a matter of observed fact, of which we shall give the explanation further on, that the sun is constantly vertical over some one or other part of the earth between two parallels of latitude, called the tropics, respectively $23\frac{1}{2}^{\circ}$ north, and as much south of the equator; and that the whole of that zone or belt of the earth's surface included between the tropics, and equally divided by the equator, is, in consequence of the great altitude attained by the sun in its diurnal course, maintained at a much higher temperature than those regions to the north and south which lie nearer the poles.* Now, the heat thus acquired by the earth's surface is communicated to the incumbent air, which is thereby expanded, and rendered specifically lighter than the air incumbent on the rest of the globe. It is, therefore, in obedience to the general laws of hydrostatics, displaced and buoyed up from the surface, and its place occupied by colder, and therefore heavier air, which glides in, on both sides, along the surface, from the regions beyond the tropics; while the displaced air, thus raised above its due level, and unsustained by any lateral pressure, flows over, as it were, and forms an upper current in the contrary direction, or toward the poles; which, being cooled in its course, and also sucked down to supply the deficiency in the extra-tropical regions, keeps up thus a continual circulation. That this is a real cause

* First distinctly delivered by Hadley, though often erroneously attributed to Edmund Halley, whose theory of the trade winds is altogether erroneous. (See Dove, *Meteorol. Untersuchungen*, p. 237.)

(vera causa) is placed in complete evidence by the general fact that the atmospheric pressure at the surface of the sea diminishes regularly from either tropic to the equator, where the barometer stands habitually about 0^m.2 lower than in the temperate zones.

(241.) Since the earth revolves about an axis passing through the poles, the equatorial portion of its surface has the greatest velocity of rotation, and all other parts less in the proportion of the radii of the circles of latitude to which they correspond. But as the air, when relatively and apparently at rest on any part of the earth's surface, is only so because in reality it participates in the motion of rotation proper to that part, it follows that when a mass of air near the poles is transferred to the region near the equator by any impulse urging it directly toward that circle, in every point of its progress toward its new situation it must be found deficient in rotatory velocity, and therefore unable to keep up with the speed of the new surface over which it is brought. Hence, the currents of air which set in toward the equator from the north and south must, as they glide along the surface, at the same time lag, or hang back, and *drag upon* it in the direction *opposite* to the earth's rotation, *i.e.* from east to west. Thus these currents, which but for the rotation would be simply northerly and southerly winds, acquire, from this cause, a *relative* direction toward the west, and assume the character of permanent north-easterly and southeasterly winds.

(242.) Were any considerable mass of air to be *suddenly* transferred from beyond the tropics to the equator, the difference of the rotatory velocities proper to the two situations would be so great as to produce not merely a wind, but a tempest of the most destructive violence. But this

is not the case: the advance of the air from the north and south is gradual, and all the while the earth is continually acting on, and by the friction of its surface accelerating its rotatory velocity. Supposing its progress toward the equator to cease at any point, this cause would almost immediately communicate to it the deficient motion of rotation, after which it would revolve quietly with the earth, and be at relative rest. We have only to call to mind the comparative *thinness* of the coating which the atmosphere forms around the globe (art. 35), and the immense *mass* of the latter, compared with the former (which it exceeds at least 1,200,000 times), to appreciate fully the influence of any extensive territory of the earth over the atmosphere immediately incumbent on it, in destroying any impulse once given to it, and which is not continually renewed.

(243.) It follows from this, then, that as the winds on both sides approach the equator, their easterly tendency must diminish.¹⁰ The lengths of the diurnal circles increase very slowly in the immediate vicinity of the equator, and for several degrees on either side of it hardly change at all. Thus the friction of the surface has more time to act in accelerating the velocity of the air, bringing it toward a state of *relative* rest, and diminishing thereby the relative set of the currents from east to west, which, on the other hand, is feebly, and, at length, not at all, reinforced by the cause which originally produced it. Arrived, then, at the equator, the trades must be expected to lose their easterly character altogether. But not only this, but the northern and southern currents here meeting and opposing, will

¹⁰ See Captain Hall's "Fragments of Voyages and Travels," 2d series, vol. i., p. 162, where this is very distinctly, and, so far as I am aware, for the first time, reasoned out.

mutually destroy each other, leaving only such preponderancy as may be due to a difference of local causes acting in the two hemispheres—which in some regions around the equator may lie one way, in some another.

(244.) The result, then, must be the production of two great tropical belts, in the northern of which a constant northeasterly, and in the southern a southeasterly, wind must prevail, while the winds in the equatorial belt, which separates the two former, should be comparatively calm and free from any steady prevalence of easterly character. All these consequences are agreeable to observed fact, and the system of aerial currents above described constitutes in reality what is understood by the regular *trade winds*.

(245.) The constant friction thus produced between the earth and atmosphere in the regions near the equator must (it may be objected) by degrees reduce and at length destroy the rotation of the whole mass. The laws of dynamics, however, render such a consequence, generally, impossible; and it is easy to see, in the present case, where and how the compensation takes place. The heated equatorial air, while it rises and flows over toward the poles, carries with it the rotatory velocity due to its equatorial situation into a higher latitude, where the earth's surface has less motion. Hence, as it travels northward or southward, it will *gain* continually more and more on the surface of the earth in its diurnal motion, and assume constantly more and more a *westerly* relative direction; and when at length it returns to the surface, in its circulation, which it must do more or less in all the interval between the tropics and the poles, it will act on it by its friction as a powerful southwest wind in the northern hemisphere, and a northwest in the southern, and restore to it the impulse taken up from it at the equator.

We have here the origin of the southwest and westerly gales so prevalent in our latitudes, and of the almost universal westerly winds in the North Atlantic, which are, in fact, nothing else than a part of the general system of the reaction of the trades, and of the process by which the equilibrium of the earth's motion is maintained under their action.

(245 a.) If in any region of the earth's surface, where the latitude is considerable, and where, in consequence, the circumference of the diurnal circles described by points on the same meridian a few degrees asunder differ considerably, an impulse (from whatever cause arising) from the pole toward the equator be communicated to a portion of the atmosphere covering several square degrees; an observer situated on the equatorial limit of the area so disturbed will, in the first instant of the disturbance, experience a wind blowing *directly from the pole*, i.e. a north wind in the northern hemisphere and a south in the southern. To fix our ideas, suppose him situate in north latitude and beyond the tropic. The air which reaches him in the first instant, arising from a place in his immediate vicinity, has the same diurnal rotatory velocity with himself, and will therefore have no *relative* movement westward. But the southward movement of the whole mass of air continuing, the wind which subsequently reaches his station arriving from latitudes continually more and more north, and therefore setting out with a rotatory velocity continually more and more inferior to that of the observer, will lag more and more behind the easterly motion of the earth's surface at his station, and will therefore become, relatively to him, more and more of an east wind. In other words, a wind commencing to blow from the north will not continue long to do so, but

will "draw toward the east," veering gradually round to N.N.E. and N.E.; *vice versâ* if the impulse of the mass of air be from south to north. The first impression on the observer will be that of a south wind, which in the progress of time will veer round through S.S.W. to S.W., and so *mutatis mutandis* in the other hemisphere; and thus arises a general tendency of the wind in extra-tropical latitudes to veer in a fixed direction, or "to follow the sun," which meteorological observation very decisively confirms as a matter of fact, and is therefore *pro tanto* a proof of the reality of the assigned cause."

(245 *b.*) It is, however, in those tremendous visitations called "hurricanes," which sweep across land and sea with a devastating power exceeded only by the earthquake, that we find the most striking verification of the principle above stated. Suppose that in any locality in the northern hemisphere some considerable portion of the surface, whether of sea or land, should become so much more heated by the sun's rays than that surrounding it, as to determine an upward movement in the air above it in the nature of an ascending column, thereby giving rise to a diminished barometric pressure, and as a necessary consequence to an indraught of air from all quarters toward the heated area. Those portions which arrive from the east and west, participating in the entire diurnal movement corresponding to the latitude, will simply meet and be hurried upward, without any tendency to gyrate round a centre. But the portions which arrive from the northward will all reach the heated region or its immediate confines with a modified power. Those which come from the northeasterly quadrant

¹¹ See Article *Meteorology*, Encyc. Brit. § 70.

will have their westerly force increased, and those from the northwest quadrant, their easterly force diminished, so that in arriving from the northward, the general current setting to the heated region will have assumed a tendency from east to west, and in arriving from the southward from west to east, and these portions being drawn up together into the ascending column, will necessarily assume a rotation round its general axis in the direction N.W.S.E.,¹² whereas, were the earth at rest, the air coming in from all quarters with equal force, each particle would make direct for the centre, and simply be thrown up vertically without any gyration.

(245 c.) The rotation thus given to the ascending column in the northern hemisphere is in a direction contrary to that of the hands of a watch face upward, which we may term retrograde. And by a similar reasoning, in the southern it will be seen that a contrary, or direct rotation ought to arise from the operation of the same causes. It is, moreover, obvious, that the energy of the vortex so produced must be, *cæteris paribus*, proportional to that of its efficient causes. In high latitudes there is a deficiency of solar heat to produce a powerful ascensional current. On and about the equator, on the other hand, though heat be abundant, the other efficient cause, viz. a considerable difference of diurnal rotatory velocity, is absent. Such movements, therefore, cannot exist on the equator, and their intensity will chiefly be confined to regions in moderate latitudes.

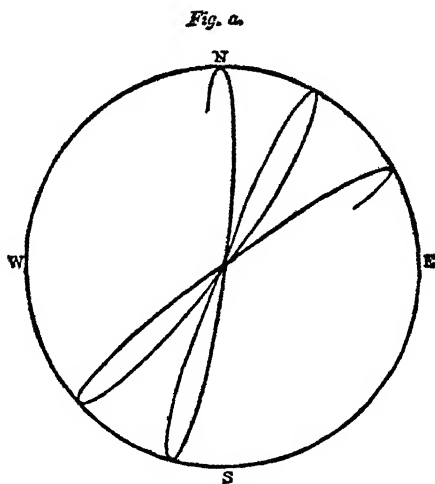
(245 d.) Now every one of these particulars is in exact

¹² See Encyclop. Brit. *Meteorology*, § 73, for the complete reasoning out of this process.

conformity with the history of those hurricanes, or cyclones, as they have been called, from their revolving character, which infest the Atlantic along the east coasts of the United States and the West Indies, the Indian Ocean, and (under the name of typhoons) the China seas. Their extent and violence are frightful; their rotation in the same hemisphere is invariably the same, and in each, that which theory indicates; and they are utterly wanting on the equator. This grand result, the establishment of which we owe to the labors of Mr. Redfield, Sir W. Reid, and Mr. Piddington, forms a capital feature in the array of evidence by which the rotation of the earth, as a physical fact, is demonstrated.

(245 e.) Another class of phenomena, inexplicable except on the hypothesis of the earth's rotation on its axis, but flowing easily and naturally from the admission of that principle, has, within a few years from the present time, been brought under our inspection by M. Foucault. If a heavy mass of metal (a globe of lead, for example) be suspended by a long wire from a solid and perfectly fixed support, over the centre of a plane table of a circular form, and, being drawn aside from the perpendicular (suppose in the direction of the meridian), be then allowed to oscillate, taking extreme care to avoid giving it any lateral motion (which may be accomplished by drawing it aside by a fine thread, and, when quite at rest, burning off the thread), it will of course commence its oscillations in the plane of the meridian. But when watched attentively, marking on the table the points of its circumference, from time to time, opposite to its points of extreme excursion, it will in a few minutes be seen to have (apparently) shifted its plane of motion; the northern extremity of its excursions to and fro *having inva-*

riably gone round in azimuth toward the east, and the southern toward the west (supposing the experiment made in the northern hemisphere—*vice versâ* in the southern). Although in a few oscillations the deviation is too small to be readily perceived, it at length becomes apparent that the path traced on the table by the projection of the centre of the globe, instead of being a rigorous straight line, as it must be according to the laws of dynamics, were the table at rest, is, in

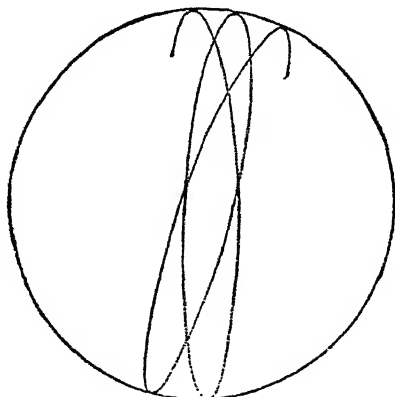


reality, a looped curve of the form here shown (*fig. a*) (the intervals of the loops being much exaggerated); all of them passing through the centre of the table.

(245 *f.*) It is evident that such a motion is quite different from that which a small lateral motion accidentally communicated to the pendulous body would produce. The effect of such an impulse would be to make the central mass describe a series of elongated ovals, a kind of elliptic spiral, the convolutions of which would pass, not *through*, but

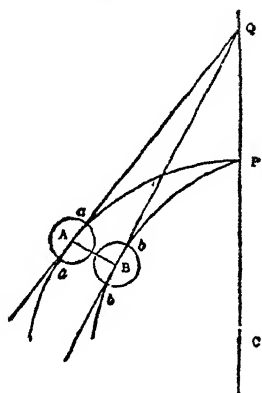
round the centre, as here represented (*fig. b*); and *that* indifferently in one direction or the other, according to the accident of the lateral impulse. On the other hand, the observed effect is precisely such as would take place, supposing the plane of oscillation to remain invariable, and the table to revolve beneath it in its own plane in a contrary direction (from north to west), with an angular motion duly adjusted. Supposing the oscillating ball to leave a trace on the table so turning, that trace would evidently be such a

Fig. 4.



one as described in the preceding article; and if we admit the rotation of the earth, it is a fact that the table, though unperceived by us, does so turn. It is not transferred by the earth's rotation bodily to the eastward by a parallel movement of all its parts. The southern extremity of its meridional diameter *S* is carried in a given time (suppose one minute) more to the eastward than the northern; so that it has virtually rotated in its own plane through an angle corresponding to the difference of these two movements of transference.

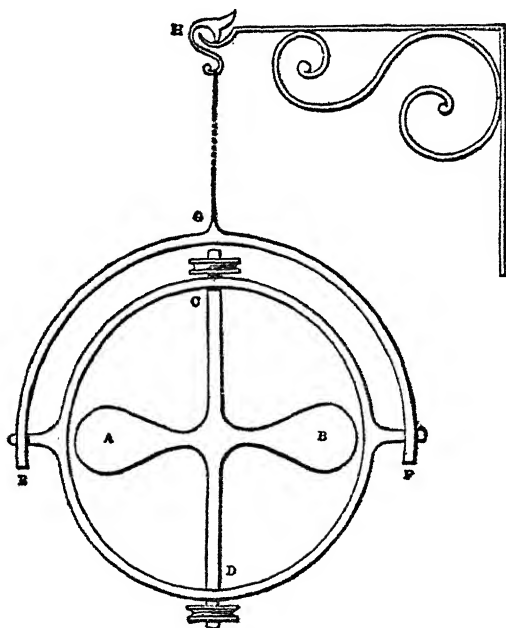
(245 g.) This difference is a maximum at the pole (where it is obvious that the table turns entirely in its own plane, as the earth's surface there does); and it is *nil* at the equator, where, in consequence, the experiment would be made in vain (the entire rotation of the table *there* being in a plane perpendicular to its own); and generally the effect will be more strikingly developed in high than in low latitudes. To show this more clearly, suppose P the north pole, C the centre of the earth, C P Q its axis prolonged, A B two successive positions of the table at an interval of one minute of time, during which the meridian A P has rotated through an angle of $0^{\circ} 15'$ round P to the position B P. The plane of the table, being a tangent to the earth's surface, will, if produced (whether it be at A or B), meet the axis at Q, the vertex of a cone having for its base the diurnal circle of the place of observation. During the small interval in question, the portion A Q B of this conical surface



may be regarded as plane, and the motion of the table will be the same as if it formed a part of that plane, and revolved round a pivot at Q, the meridional diameter *a a* being transferred into the position *b b*, making with *a a* an angle equal to A Q B. Now this angle at the equator is *nil*, the summit of the cone being there infinitely remote; whereas, on the pole it is identical with the spherical angle A P B, the table there rotating about its own centre.

(245 h.) The gyroscope is an instrument devised by M. Foucault to exhibit the same sort of effect in another manner. It depends on the very obvious principle that a body

revolving round one of its axes of permanent rotation, and free from any disturbing attachment to surrounding objects, will preserve its plane of rotation unaltered. Imagine a metallic disk, thin in the centre but very thick at the circumference so as to present in section the figure A B, to be fixed on an axis C D, perpendicular to its plane which turns



in pivot holes C D, on opposite ends of the diameter of a ring of metal, which is itself provided with exterior pivots on the extremities of a diameter at right angles to the former, and let these rest in pivot holes E F, at the lower ends of a semicircular metallic arc E G F, supported from its middle G by a torsionless suspension, such as may be formed by attaching a thread to a hook at the lower end of a steel arm, terminating in an inverted conical point resting

in a polished agate cup as at H. The whole of this apparatus is to be executed with extreme delicacy, and with every precaution to secure perfect equilibrium and freedom from friction in the pivots. Suppose now that by some sufficient mechanical means an exceedingly rapid rotation is communicated to the disk which is then abandoned to itself. It is evident then, 1st, that it may be set in rotation originally in any given plane, and, 2dly, that however that initial plane be situated, it will thenceforward continue to rotate in that plane, since the mode of suspension is such as to exercise no control over it, in that respect. If the disk be heavy, the initial rotation very rapid (and especially if suspended in vacuo) the motion will be kept up for a considerable time—quite long enough to exhibit the phenomena due to the earth's rotation.

(245 i.) There being no action exerted by either the pivots or the suspension which can affect the plane of rotation, this will necessarily continue unchanged, so that the axis C D about which it spins will remain parallel to itself, however the point of suspension may be varied *in place* by bodily transfer of the whole apparatus, or *in relative position* by change in the absolute direction of gravity consequent on the earth's diurnal rotation. Suppose then the axis C D to point at any instant to a given fixed star, then if the earth were at rest and the diurnal movement of the starry heavens real, it could not continue so to point, since the star would move away out of its line of direction, and would appear to leave it behind. The contrary however is the case. The axis of the disk continues to point to the star so long as the disk itself continues to revolve, and, could its rotation be kept up for twenty-four hours, would doubtless continue to follow it through its whole diurnal circle both

above and below the horizon, affording thus a clear ocular demonstration of the earth's rotation, since if a line, of whose fixity of direction we are *a priori* sure, appear to vary in position with respect to the visible horizon and surrounding objects it cannot be but that *that* horizon and those terrestrial points of reference have, themselves, shifted in position by a corresponding opposite movement.

(245 j.) If the conditions of suspension be such as to limit the axis of rotation of the disk to a plane holding a determinate position with respect to the horizon, as, for instance, that of the horizon itself, or of the meridian of the place, its movements are in conformity with what the principles of dynamics indicate as the result of a composition of the free rotation of the disk and that of the earth so partially communicated to it. We shall not however pursue the subject into these details. The student will find them lucidly explained by Professor Powell, in the monthly notices of the Astronomical Society for April, 1855. The mechanical fact on which the whole theory turns (the powerful resistance opposed by a rapidly revolving heavy body to a change of position in its axis of rotation) may be brought under the evidence of the senses by the following simple and elegant experiment. Let any one detach an 18-inch terrestrial globe from its wooden frame, and, holding it by the brass meridian with the plane of that circle horizontal, let a rapid rotation be given to the globe by an assistant. So long as no attempt is made to alter the position of the axis, the only sensation experienced by the holder will be the effort of sustaining the weight of the globe, just as if it were at rest. But so soon as he attempts to shift the *direction* of the axis, whether in a horizontal, a vertical, or any other plane, he will at once become aware of a resistance in the revolving

globe to such a change, quite different from the simple inertia of a globe at rest—a kind of internal struggle—an effort to twist the globe in his hands, as if some animal were inclosed within its hollow, or as if it were no longer equally balanced on its centre. If he endeavor to roll the globe on its brass meridian in a right line along the floor (which with a non-rotating globe would be easy) he will find it impracticable without perpetually and forcibly interfering, not merely to keep the meridian upright but to prevent its running out of the right line. Suppose, for instance, the brass meridian to be vertical and its plane coincident with that of the true meridian, the axis horizontal, and the globe to rotate in the direction in which the heavens appear to revolve, *i.e. from the east upward; to the west downward*, and let him attempt to roll it (lightly held by the finger and thumb by the highest point of the circle) in a northerly direction. He will find it run round to the eastward, causing the plane of the brass meridian to shift in azimuth in a direction similar to that of the hands of a watch, and *vice versa* if he try to make it roll southward. That end of the axis which rises appears to be swept along *with* the revolving motion of the globe as seen from above.

(246.) In order to construct a map or model of the earth, and obtain a knowledge of the distribution of sea and land over its surface, the forms of the outlines of its continents and islands, the courses of its rivers and mountain chains, and the relative situations, with respect to each other, of those points which chiefly interest us, as centres of human habitation, or from other causes, it is necessary to possess the means of determining correctly the situation of any proposed station on its surface. For this two elements require to be known, the latitude and longitude, the former assign-

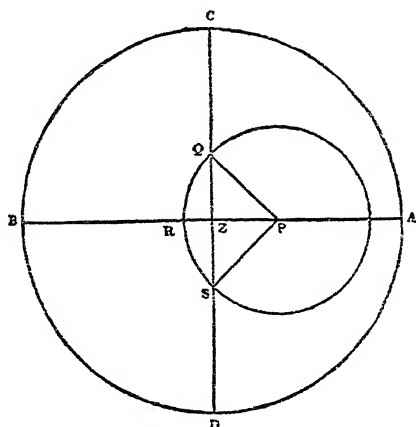
ing its distance from the poles or the equator, the latter, the meridian on which that distance is to be reckoned. To these, in strictness, should be added, its height above the sea level; but the consideration of this had better be deferred, to avoid complicating the subject.

(247.) The latitude of a station on a sphere would be merely the length of an arc of the meridian, intercepted between the station and the nearest point of the equator, reduced into degrees. (See art. 88.) But as the earth is elliptic, this mode of conceiving latitudes becomes inapplicable, and we are compelled to resort for our definition of latitude to a generalization of that property (art. 119), which affords the readiest means of determining it by observation, and which has the advantage of being independent of the figure of the earth, which, after all, is not *exactly* an ellipsoid, or any known geometrical solid. The latitude of a station, then, is the altitude of the elevated pole, and is, therefore, astronomically determined by those methods already explained for ascertaining that important element. In consequence, it will be remembered that, to make a *perfectly* correct map of the whole, or any part of the earth's surface, equal differences of latitude are not represented by exactly equal intervals of surface.

(248.) For the purposes of geodesical¹³ measurements and trigonometrical surveys, an exceedingly correct determination of the latitudes of the most important stations is required. For this purpose, therefore, the zenith sector (an instrument capable of great precision) is most commonly used to observe stars passing the meridian near the zenith, whose declinations have become known by previous long

¹³ *Γη*, the earth; *δεσις* (from *δεω*, to bind), a joining or connection (of parts).

series of observations at fixed observatories, and which are therefore called standard or fundamental stars. Recently a method¹⁴ has been employed with great success, which consists in the use of an instrument similar in every respect to the transit instrument, but having the plane of motion of the telescope not coincident with the meridian, but with the prime vertical, so that its axis of rotation prolonged passes through the north and south points of the horizon. Let A B C D be the celestial hemisphere projected on the hori-



zon, P the pole, Z the zenith, A B the meridian, C D the prime vertical, Q R S part of the diurnal circle of a star passing near the zenith, whose polar distance P R is but little greater than the co-latitude of the place, or the arc P Z, between the zenith and pole (art. 112). Then the moments of this star's arrival on the prime vertical at Q and S will, if the instrument be correctly adjusted, be those of its crossing the middle wire in the field of view of the telescope (art.

¹⁴ Devised originally by Römer. Revived or re-invented by Bessel. — *Astr. Nachr.*, No. 40.

160). Consequently the interval between these moments will be the time of the star passing from Q to S, or the measure of the diurnal arc Q R S, which corresponds to the angle Q P S at the pole. This angle, therefore, becomes known *by the mere observation of an interval of time*, in which it is not even necessary to know the error of the clock, and in which, when the star passes near the zenith, so that the interval in question is small, even the *rate* of the clock, or its gain or loss on true sidereal time, may be neglected. Now the angle Q P S, or its half Q P R, and P Q the polar distance of the star, being known, P Z the zenith distance of the pole can be calculated by the resolution of the right-angled spherical triangle P Z Q, and thus the co-latitude (and of course the latitude) of the place of observation becomes known. The advantages gained by this mode of observation are, 1st, that no readings of a divided arc are needed, so that errors of graduation and reading are avoided: 2dly, that the arc Q R S is very much greater than its versed sine R Z, so that the difference R Z between the latitude of the place and the declination of the star is given by the observation of a magnitude very much greater than itself, or is, as it were, observed on a greatly enlarged scale. In consequence, a very minute error is entailed on R Z by the commission of even a considerable one in Q R S: 3dly, that in this mode of observation all the merely instrumental errors which affect the ordinary use of the transit instrument are either uninfluential or eliminated by simply reversing the axis.

(249.) To determine the latitude of a station, then, is easy. It is otherwise with its longitude, whose exact determination is a matter of more difficulty. The reason is this:—as there are no meridians marked upon the earth, any

more than parallels of latitude, we are obliged in this case, as in the case of the latitude, to resort to marks external to the earth, *i.e.* to the heavenly bodies, for the objects of our measurement; but with this difference in the two cases—to observers situated at stations on the same *meridian* (*i.e.* differing in latitude) the heavens present different aspects at *all* moments. The portions of them which become visible in a complete diurnal rotation are not the same, and stars which are common to both describe circles differently inclined to their horizons, and differently divided by them, and attain different altitudes. On the other hand, to observers situated on the same *parallel* (*i.e.* differing only in longitude) the heavens present the same aspects. Their visible portions are the same; and the same stars describe circles equally inclined, and similarly divided by their horizons, and attain the same altitudes. In the former case there *is*, in the latter there is *not*, anything in the appearance of the heavens, watched through a whole diurnal rotation, which indicates a difference of locality in the observer.

(250.) But no two observers, at different points of the earth's surface, can have at *the same instant* the same celestial hemisphere visible. Suppose, to fix our ideas, an observer stationed at a given point of the equator, and that at the moment when he noticed some bright star to be in his zenith, and therefore on his meridian, he should be suddenly transported, in an instant of time, round one quarter of the globe in a *westerly* direction, it is evident that he will no longer have the same star vertically above him: it will now appear to him to be just rising, and he will have to wait six hours before it again comes to his zenith, *i.e.* before the earth's rotation from west to east *carries him back again* to

the line joining the star and the earth's centre from which he set out.

(251.) The difference of the cases, then, may be thus stated, so as to afford a key to the astronomical solution of the problem of the longitude. In the case of stations differing only in latitude, the same star comes to the meridian at the same *time*, but at different *altitudes*. In that of stations differing only in longitude, it comes to the meridian at the same *altitude*, but at different *times*. Supposing, then, that an observer is in possession of any means by which he can certainly ascertain the *time* of a known star's transit across his meridian, he knows his longitude; or if he knows the difference between its time of transit across his meridian and across that of any other station, he knows their difference of longitudes. For instance, if the same star pass the meridian of a place A at a certain moment, and that of B exactly one hour of sidereal time, or one twenty-fourth part of the earth's diurnal period, later, then the difference of longitude between A and B is one hour of time or 15° of arc, and B is so much west of A.

(252.) In order to have a perfectly clear understanding of the principle on which the problem of finding the longitude by astronomical observations is resolved, the reader must learn to distinguish between time, in the abstract, as common to the whole universe, and therefore reckoned from an epoch independent of local situation, and *local time*, which reckons, at each particular place, from an epoch, or initial instant, determined by local convenience. Of time reckoned in the former, or abstract manner, we have an example in what we have before defined as equinoctial time, which dates from an epoch determined by the sun's motion among the stars. Of the latter, or *local* reckoning, we have

instances in every sidereal clock in an observatory, and in every town clock for common use. Every astronomer regulates, or aims at regulating, his sidereal clock, so that it shall indicate $0^h\ 0^m\ 0^s$, when a certain point in the heavens, called the equinox, is on the meridian of his station. This is the *epoch* of his sidereal time; which is, therefore, entirely a *local* reckoning. It gives no information to say that an event happened at such and such an hour of sidereal time, unless we particularize the station to which the sidereal time meant appertains. Just so it is with mean or common time. This is also a local reckoning, having for its epoch *mean noon*, or the average of all the times throughout the year, when the sun is on the meridian *of that particular place to which it belongs*; and, therefore, in like manner, when we date any event by mean time, it is necessary to name the place, or particularize *what* mean time we intend. On the other hand, a date by equinoctial time is absolute, and requires no such explanatory addition.

(253.) The astronomer sets and regulates his sidereal clock by observing the meridian passages of the more conspicuous and well-known stars. Each of these holds in the heavens a certain determinate and known place with respect to that imaginary point called the equinox, and by noting the times of their passage in succession by his clock he knows when the equinox passed. At that moment his clock ought to have marked $0^h\ 0^m\ 0^s$; and if it did not, he knows and can correct its error, and by the agreement or disagreement of the errors assigned by each star he can ascertain whether his clock is correctly regulated to go twenty-four hours in one diurnal period, and if not, can ascertain and allow for its rate. Thus, although his clock may not, and indeed cannot, either be set correctly, or go truly, yet

by applying its *error* and *rate* (as they are technically termed), he can correct its indications, and ascertain the exact sidereal times corresponding to them, and proper to his locality. This indispensable operation is called getting his *local time*. For simplicity of explanation, however, we shall suppose the clock a perfect instrument; or, which comes to the same thing, its error and rate applied at every moment it is consulted, and included in its indications.

(254.) Suppose, now, of two observers, at distant stations, A and B, each, independently of the other, to set and regulate his clock to the true sidereal time of his station. It is evident that if one of these clocks could be taken up without deranging its going, and set down by the side of the other, they would be found, on comparison, to differ by the exact difference of their local epochs; that is, by the time occupied by the equinox, or by any star, in passing from the meridian of A to that of B; in other words, by their difference of longitude, expressed in sidereal hours, minutes, and seconds.

(255.) A pendulum clock cannot be thus taken up and transported from place to place without derangement, but a chronometer may. Suppose, then, the observer at B to use a chronometer instead of a clock, he may, by bodily transfer of the instrument to the other station, procure a direct comparison of sidereal times, and thus obtain his longitude from A. And even if he employ a clock, yet by comparing it first with a good chronometer, and then transferring the latter instrument for comparison with the other clock, the same end will be accomplished, provided the going of the chronometer can be depended on.

(256.) Were chronometers perfect, nothing more complete and convenient than this mode of ascertaining dif-

ferences of longitude could be desired. An observer, provided with such an instrument, and with a portable transit, or some equivalent method of determining the local time at any given station, might, by journeying from place to place, and observing the meridian passages of stars at each (taking care not to alter his chronometer, or let it run down), ascertain their differences of longitude with any required precision. In this case, the same time-keeper being used at every station, if, at one of them, A, it mark true sidereal time, at any other, B, it will be just so much sidereal time in error as the difference of longitudes of A and B is equivalent to: in other words, the longitude of B from A will appear as the error of the time-keeper on the local time of B. If he travel westward, then his chronometer will appear continually to gain, although it really goes correctly. Suppose, for instance, he set out from A, when the equinox was on the meridian, or his chronometer at 0^h, and in twenty-four hours (sid. time) had travelled 15° westward to B. At the moment of arrival there, his chronometer will again point to 0^h; but the equinox will be, not on his new meridian, but on that of A, and he must wait one hour more for its arrival at that of B. When it does arrive there, then his watch will point not to 0^h but to 1^h, and will therefore be 1^h *fast* on the local time of B. If he travel eastward, the reverse will happen.

(257.) Suppose an observer now to set out from any station as above described, and constantly travelling westward to make the tour of the globe, and return to the point he set out from. A singular consequence will happen; he will have lost a day in his reckoning of time. He will enter the day of his arrival in his diary, as Monday, for instance, when, in fact, it is Tuesday. The reason is obvious. Days

and nights are caused by the alternate appearance of the sun and stars, as the rotation of the earth carries the spectator round to view them in succession. So many turns as he makes absolutely round the centre, so often will he pass through the earth's shadow, and emerge into light, and so many nights and days will he experience. But if he travel once round the globe in the direction of its motion, he will, on his arrival, have really made one turn *more* round its centre; and if in the opposite direction, one turn *less* than if he had remained upon one point of its surface: in the former case, then, he will have witnessed one alternation of day and night more, in the latter one less, than if he had trusted to the rotation of the earth alone to carry him round. As the earth revolves from west to east, it follows that a westward direction of his journey, by which he counteracts its rotation, will cause him to lose a day, and an eastward direction, by which he conspires with it, to gain one. In the former case, all his days will be longer; in the latter, shorter than those of a stationary observer. This contingency has actually happened to circumnavigators. Hence, also, it must necessarily happen that distant settlements, *on the same meridian*, will differ a day in their usual reckoning of time, according as they have been colonized by settlers arriving in an eastward or in a westward direction—a circumstance which may produce strange confusion when they come to communicate with each other. The only mode of correcting the ambiguity, and settling the disputes which such a difference may give rise to, consists in having recourse to the equinoctial date, which can never be ambiguous.

(258.) Unfortunately for geography and navigation, the chronometer, though greatly and indeed wonderfully im-

proved by the skill of modern artists, is yet far too imperfect an instrument to be relied on implicitly. However such an instrument may preserve its uniformity of rate for a few hours, or even days, yet in long absences from home the chances of error and accident become so multiplied, as to destroy all security of reliance on even the best. To a certain extent this may, indeed, be remedied by carrying out several, and using them as checks on each other; but, besides the expense and trouble, this is only a palliation of the evil—the great and fundamental—as it is the only one to which *the determination of longitudes by time-keepers* is liable. It becomes necessary, therefore, to resort to other means of communicating from one station to another a knowledge of its local time, or of propagating from some principal station, as a centre, its local time as a universal standard with which the local time at any other, however situated, may be at once compared, and thus the longitudes of all places be referred to the meridian of such central point.

(259.) The simplest and most accurate method by which this object can be accomplished, when circumstances admit of its adoption, is that by telegraphic signal. Let A and B be two observatories, or other stations, provided with accurate means of determining *their respective local times*, and let us first suppose them visible from each other. Their clocks being regulated, and their errors and rates ascertained and applied, let a signal be made at A, of some sudden and definite kind, such as the flash of gunpowder, the explosion of a rocket, the sudden extinction of a bright light, or any other which admits of no mistake, and can be seen at great distances. The moment of the signal being made must be noted by *each* observer at his respective clock

or watch, as if it were the transit of a star, or any astronomical phenomenon, and the error and rate of the clock at each station being applied, the local time of the signal at each is determined. Consequently, when the observers communicate their observations of the signal to each other, since (owing to the almost instantaneous transmission of light) it must have been seen at the same *absolute* instant by both, the difference of their local times, and therefore of their longitudes, becomes known. For example, at A the signal is observed to happen at $5^h 0^m 0^s$, sid. time at A, as obtained by applying the error and rate to the time shown by the clock at A, when the signal was seen there. At B the same signal was seen at $5^h 4^m 0^s$, sid. time. at B, similarly deduced from the time noted by the clock at B, by applying *its* error and rate. Consequently, the difference of their local epochs is $4^m 0^s$, which is also their difference of longitudes in time, or $1^\circ 0' 0''$ in hour angle.

(260.) The accuracy of the final determination may be increased by making and observing several signals at stated intervals, each of which affords a comparison of times, and the mean of all which is, of course, more to be depended on than the result of any single comparison. By this means, the error introduced by the comparison of clocks may be regarded as altogether destroyed.

(261.) The distances at which signals can be rendered visible must of course depend on the nature of the interposed country. Over sea the explosion of rockets may easily be seen at fifty or sixty miles; and in mountainous countries the flash of gunpowder in an open spoon may be seen, if a proper station be chosen for its exhibition, at much greater distances.

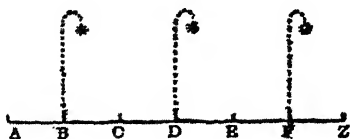
(262.) When the direct light of the flash can no longer be

perceived, either owing to the convexity of the interposed segment of the earth, or to intervening obstacles, the sudden illumination cast on the under surface of the clouds by the explosion of considerable quantities of powder may often be observed with success; and in this way signals have been made at very much greater distances. Whatever means can be devised of exciting in two distant observers the same sensation, whether of sound, light, or visible motion, at *precisely the same instant of time*, may be employed as a longitude signal. Wherever, for instance, an unbroken line of electrotelegraphic connection has been, or hereafter may be, established, the means exist of making as complete a comparison of clocks or watches as if they stood side by side, so that no method more complete for the determination of differences of longitude can be desired. Thus, the difference of longitude between the observatories of Greenwich and Paris was ascertained in 1854. The extreme deviation of the most discordant result from the mean of 29 single determinations (0 h. 9 m. 20.64 sec.), amounted barely to a quarter of a second.

(263.) Where no such electric communication exists, however, the interval between observing stations may be increased by causing the signals to be made not at one of them, but at an intermediate point; for, provided they are seen by both parties, it is a matter of indifference where they are exhibited. Still the interval which could be thus embraced would be very limited, and the method in consequence of little use, but for the following ingenious contrivance, by which it can be extended to any distance, and carried over any tract of country, however difficult.

(264.) This contrivance consists in establishing, between the extreme stations, whose difference of longitude is to be

ascertained, and at which the local times are observed, a chain of intermediate stations, alternately destined for signals and for observers. Thus, let A and Z be the extreme stations. At B let a signal station be established, at which rockets, etc., are fired at stated intervals. At C let an observer be placed, provided with a chronometer; at D, another signal station; at E, another observer and chronometer; till the whole line is occupied by stations so arranged, that the signal at B can be seen from A and C; those at D, from C and E; and so on. Matters being thus arranged, and the errors and rates of the clocks at A and Z ascertained by astronomical observation, let a signal be made at B, and observed at A and C, and the times noted.



Thus the difference between A's clock and C's chronometer becomes known. After a short interval (five minutes for instance) let a signal be made at D, and observed by C and E. Then will the difference between their respective chronometers be determined; and the difference between the former and the clock at A being already ascertained, the difference between the clock A and chronometer E is therefore known. This, however, supposes that the intermediate chronometer C has kept true sidereal time, or at least a known rate, in the interval between the signals. Now this interval is purposely made so very short, that no instrument of any pretensions to character can possibly produce an appreciable amount of error in its lapse by deviations from its usual *rate*. Thus the time propagated from

A to C may be considered as handed over, without gain or loss (save from error of observation), to E. Similarly, by the signal made at F, and observed at E and Z, the time so transmitted to E is forwarded on to Z, and thus at length the clocks at A and Z are compared. The process may be repeated as often as is necessary to destroy error by a mean of results; and when the line of stations is numerous, by keeping up a succession of signals, so as to allow each observer to note alternately those on either side, which is easily prearranged, many comparisons may be kept running along the line at once, by which time is saved, and other advantages obtained.¹⁵ In important cases the process is usually repeated on several nights in succession.

(265.) In place of artificial signals, natural ones, when they occur sufficiently definite for observation, may be equally employed. In a clear night the number of those singular meteors, called shooting stars, which may be observed, is often very great, especially on the 9th and 10th of August, and some other days, as November 12 and 13; and as they are sudden in their appearance and disappearance, and from the great height at which they have been ascertained to take place are visible over extensive regions of the earth's surface, there is no doubt that they may be resorted to with advantage, by previous concert and agreement between distant observers to watch and note them.¹⁶

¹⁵ For a complete account of this method, and the mode of deducing the most advantageous result from a combination of all the observations, see a paper on the difference of longitudes of Greenwich and Paris, *Phil. Trans.* 1826, by the Author of this volume.

¹⁶ This idea was first suggested by the late Dr. Maskelyne, to whom, however, the practically useful fact of their periodic recurrence was unknown. Mr. Cooper has thus employed the meteors of the 10th and 12th August, 1847, to determine the difference of longitudes of Markree and Mount Eagle, in Ireland. Those of the same epoch have also been used in Germany for ascertaining the longitudes of several stations, and with very satisfactory results.

Those sudden disturbances of the magnetic needle, to which the name of magnetic shocks has been given, have been satisfactorily ascertained to be, very often at least, simultaneous over whole continents, and in some, perhaps, over the whole globe. These, if observed at magnetic observatories with precise attention to astronomical time, may become the means of determining their differences of longitude with more precision, possibly, than by any other method, if a sufficient number of remarkable shocks be observed to ascertain their *identity*, about which the intervals of time between their occurrence (exactly alike at both stations) will leave no doubt.

(266.) Another species of natural signal, visible at once over a whole terrestrial hemisphere, is afforded by the eclipses of Jupiter's satellites, of which we shall speak more at large when we come to treat of those bodies. Every such eclipse is an event which possesses one great advantage in its applicability to the purpose in question, viz. that the time of its happening, at any fixed station, such as Greenwich, can be *predicted* from a long course of previous recorded observation and calculation thereon founded, and that this prediction is sufficiently precise and certain, to stand in the place of a corresponding observation. So that an observer at any other station wherever, who shall have observed one or more of these eclipses, and ascertained his local time, instead of waiting for a communication with Greenwich, to inform him at what moment the eclipse took place there, may use the *predicted Greenwich time* instead, and thence, at once, and on the spot, determine his longitude. This mode of ascertaining longitudes is, however, as will hereafter appear, not susceptible of great exactness, and should only be resorted to when others

cannot be had. The nature of the observation also is such that it cannot be made at sea;¹⁷ so that, however useful to the geographer, it is of no advantage to navigation.

(267.) But such phenomena as these are of only occasional occurrence; and in their intervals, and when cut off from all communication with any fixed station, it is indispensable to possess some means of determining longitudes, on which not only the geographer may rely for a knowledge of the exact position of important stations on land in remote regions, but on which the navigator can securely stake, at every instant of his adventurous course, the lives of himself and comrades, the interests of his country, and the fortunes of his employers. Such a method is afforded by LUNAR OBSERVATIONS. Though we have not yet introduced the reader to the phenomena of the moon's motion, this will not prevent us from giving here the exposition of the principle of the lunar method; on the contrary, it will be highly advantageous to do so, since by this course we shall have to deal with the naked principle, apart from all the peculiar sources of difficulty with which the lunar theory is encumbered, but which are, in fact, completely extraneous to the *principle* of its application to the problem of the longitudes, which is quite elementary.

(268.) If there were in the heavens a clock furnished with a dial-plate and hands, which always marked Greenwich

¹⁷ To accomplish this is still a desideratum. Observing chairs, suspended with studious precaution for insuring freedom of motion, have been resorted to, under the vain hope of mitigating the effect of the ship's oscillation. The opposite course seems more promising, viz. to merely deaden the motion by a somewhat stiff suspension (as by a coarse and rough cable), and by friction strings attached to weights running through loops (not pulleys) fixed in the woodwork of the vessel. At least, such means have been found by the author of singular efficacy in increasing personal comfort in the suspension of a cot. [Vide Journal of the Society of Arts, January 4, 1861.]

time, the longitude of any station would be at once determined, so soon as the *local time* was known, by comparing it with this clock. Now, the offices of the dial-plate and hands of a clock are these:—the former carries a set of marks upon it, whose position is known; the latter, by passing over and among these marks, informs us, by the place it holds with respect to them, what it is o'clock, or what time has elapsed since a certain moment when it stood at one particular spot.

(269.) In a clock the marks on the dial-plate are uniformly distributed all around the circumference of a circle, whose centre is that on which the hands revolve with a uniform motion. But it is clear that we should, with equal certainty, though with much more trouble, tell what o'clock it were, if the marks on the dial-plate were *unequally* distributed—if the hands were *excentric*, and their motion *not* uniform—provided we knew, 1st, the exact intervals round the circle at which the hour and minute marks were placed; which would be the case if we had them all registered in a table, from the results of previous careful measurement;—2dly, if we knew the exact amount and direction of *excentricity* of the centre of motion of the hands;—and, 3dly, if we were fully acquainted with all the mechanism which put the hands in motion, so as to be able to say at every instant what were their velocity of movement, and so as to be able to calculate, without fear of error, *HOW MUCH time* should correspond to *SO MUCH angular movement*.

(270.) The visible surface of the starry heavens is the dial-plate of our clock, the stars are the fixed marks distributed around its circuit, the moon is the movable hand, which, with a motion that, superficially considered, seems uniform, but which, when carefully examined, is found to

be far otherwise, and which, regulated by mechanical laws of astonishing complexity and intricacy in result, though beautifully simple in principle and design, performs a monthly circuit among them, passing visibly over and hiding, or, as it is called, occulting some, and gliding beside and between others; and whose position among them can, at any moment when it is visible, be exactly measured by the help of a sextant, just as we might measure the place of our clock-hand among the marks on its dial-plate with a pair of compasses, and thence, from the known and calculated laws of its motion, deduce the time. That the moon *does* so move *among the stars*, while the latter hold constantly, with respect to each other, the same relative position, the notice of a few nights, or even hours, will satisfy the commencing student, and this is all that at present we require.

(271.) There is only one circumstance wanting to make our analogy complete. Suppose the hands of our clock, instead of moving *quite close* to the dial-plate, were considerably elevated above, or distant in front of it. Unless, then, in viewing it, we kept our eye just in the line of their centre, we should not see them exactly thrown or *projected* upon their proper places on the dial. And if we were either unaware of this cause of optical change of place, this *parallax*—or negligent in not taking it into account—we might make great mistakes in reading the time, by referring the hand to the wrong mark, or incorrectly appreciating its distance from the right. On the other hand, if we took care to note, in every case when we had occasion to observe the time, the exact position of the eye, there would be no difficulty in ascertaining and allowing for the precise influence of this cause of apparent displacement. Now, this is just what

obtains with the apparent motion of the moon among the stars. The former (as will appear) is comparatively near to the earth—the latter immensely distant; and in consequence of our not occupying the centre of the earth, but being carried about on its surface, and constantly changing place, there arises a *parallax*, which displaces the moon apparently among the stars, and must be allowed for before we can tell the true place she would occupy if seen from the centre.

(272.) Such a clock as we have described might, no doubt, be considered a very bad one; but if it were our *only* one, and if incalculable interests were at stake on a perfect knowledge of time, we should justly regard it as most precious, and think no pains ill bestowed in studying the laws of its movements, or in facilitating the means of *reading* it correctly. Such, in the parallel we are drawing, is the lunar theory, whose object is to reduce to regularity the indications of this strangely irregular-going clock, to enable us to predict, long beforehand, and with absolute certainty, *whereabouts* among the stars, at every hour, minute, and second, in every day of every year, in Greenwich local time, the moon *would* be seen from the earth's centre, and *will* be seen from every accessible point of its surface; and such is the *lunar method* of longitudes. The moon's apparent angular distance from all those principal and conspicuous stars which lie in its course, as seen from the earth's centre, are computed and tabulated with the utmost care and precision in almanacs published under national control. No sooner does an observer, in any part of the globe, at sea or on land, measure its actual distance from any one of those *standard stars* (whose places in the heavens have been ascertained for the purpose with the most anxious solicitude), than he has, in fact, performed that com-

parison of his local time with the local times of every observatory in the world, which enables him to ascertain his difference of longitude from one or all of them.

(273.) The latitudes and longitudes of any number of points on the earth's surface may be ascertained by the methods above described; and by thus laying down a sufficient number of principal points, and filling in the intermediate spaces by local surveys, might maps of countries be constructed. In practice, however, it is found simpler and easier to divide each particular nation into a series of great triangles, the angles of which are stations conspicuously visible from each other. Of these triangles, the *angles* only are measured by means of the *theodolite*, with the exception of *one side* only of *one triangle*, which is called a *base*, and which is measured with every refinement which ingenuity can devise or expense command. This *base* is of moderate extent, rarely surpassing six or seven miles, and purposely selected in a perfectly horizontal plane, otherwise conveniently adapted to the purposes of measurement. Its length between its two extreme points (which are dots on plates of gold or platina let into massive blocks of stone, and which are, or at least *ought to be*, in all cases preserved with almost religious care, as monumental records of the highest importance), is then measured, with every precaution to insure precision,¹⁸ and its position with respect to the meridian, as well as the geographical positions of its extremities, carefully ascertained.

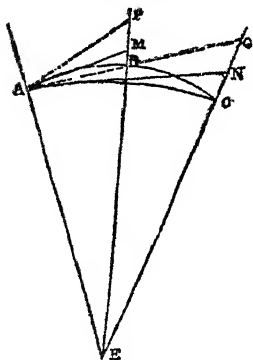
(274.) The annexed figure represents such a chain of triangles. A B is the base, O, C, stations visible from

¹⁸ The greatest *possible* error in the Irish base of between seven and eight miles, near Londonderry, is supposed not to exceed two inches.

free from any *very* great inequality in their angles. For instance, the triangle K B F would be a very improper one to determine the situation of F from observations at B and K, because the angle F being very acute, a small error in the angle K would produce a great one in the place of F *upon the line B F*. Such *ill-conditioned* triangles, therefore, must be avoided. But if this be attended to, the accuracy of the determination of the calculated sides will not be much short of that which would be obtained by actual measurement (were it practicable); and, therefore, as we recede from the base on all sides as a centre, it will speedily become practicable to use *as bases*, the sides of *much larger* triangles, such as G F, G H, H K, etc.; by which means the next step of the operation will come to be carried on on a much larger scale, and embrace far greater intervals, than it would have been safe to do (for the above reason) in the immediate neighborhood of the base. Thus it becomes easy to divide the whole face of a country into *great triangles* of from 30 to 100 miles in their sides (according to the nature of the ground), which, being once well determined, may be afterward, by a second series of subordinate operations, broken up into smaller ones, and these again into others of a still minuter order, till the final filling in is brought within the limits of personal survey and draughtsmanship, and till a map is constructed, with any required degree of detail.

(276.) The next remark we have to make is, that all the triangles in question are not, rigorously speaking, *plane*, but *spherical*—existing on the surface of a sphere, or rather, to speak correctly, of an ellipsoid. In very small triangles of six or seven miles in the side, this may be neglected, as the difference is imperceptible; but in the larger ones it must be taken into consideration. It is evident that, as

every object used for pointing the telescope of a theodolite has some certain *elevation*, not only above the *soil*, but above the level of the *sea*, and as, moreover, these elevations differ in every instance, a *reduction to the horizon* of all the measured angles would appear to be required. But, in fact, by the construction of the theodolite (art. 192), which is nothing more than an altitude and azimuth instrument, this reduction is *made* in the very act of reading off the horizontal angles. Let E be the centre of the earth; A, B, C, the places on its *spherical surface*, to which three stations, A, P, Q, in a country are referred by radii E A, E B P, E C Q.



If a theodolite be stationed at A, the axis of its horizontal circle will point to E when truly adjusted, and its plane will be a tangent to the sphere at A, intersecting the radii E B P, E C Q, at M and N, *above* the spherical surface. The telescope of the theodolite, it is true, is pointed in succession to P, and Q; but the readings off of its azimuth circle give—*not* the angle

P A Q between the directions of the telescope, or between the objects P, Q, as seen from A; *but the azimuthal angle* M A N, which is the measure of the angle A of the spherical triangle B A C. Hence arises this remarkable circumstance—that the sum of the three observed angles of any of the great triangles in geodesical operations is always found to be rather *more* than 180° . Were the earth's surface a *plane*, it ought to be exactly 180° ; and this *excess*, which is called the *spherical excess*, is so far from being a proof of incorrectness in the work, that it is essential to its accuracy, and offers at the same time another palpable proof of the earth's sphericity.

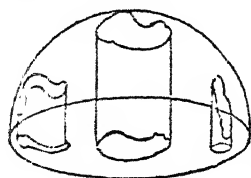
(277.) The true way, then, of conceiving the subject of a trigonometrical survey, when the spherical form of the earth is taken into consideration, is to regard the network of triangles with which the country is covered, as the bases of an assemblage of pyramids converging to the centre of the earth. The theodolite gives us *the true measures of the angles included by the planes of these pyramids*; and the surface of an imaginary sphere on the level of the sea intersects them in an assemblage of spherical triangles; above whose angles, in the radii prolonged, the real stations of observation are raised, by the superficial inequalities of mountain and valley. The operose calculations of spherical trigonometry which this consideration would seem to render necessary for the reductions of a survey, are dispensed with in practice by a very simple and easy rule, called the *rule for the spherical excess*, which is to be found in most works on trigonometry. If we would take into account the ellipticity of the earth, it may also be done by appropriate processes of calculation, which, however, are too abstruse to dwell upon in a work like the present.

(278.) Whatever process of calculation we adopt, the result will be a reduction to the level of the sea, of all the triangles, and the consequent determination of the geographical latitude and longitude of every station observed. Thus we are at length enabled to construct maps of countries; to lay down the outlines of continents and islands; the courses of rivers; the places of cities, towns and villages; the direction of mountain ridges, and the places of their principal summits; and all those details which, as they belong to physical and statistical, rather than to astronomical geography, we need not here dilate on. A few words, however, will be necessary

respecting maps, which are used as well in astronomy as in geography.

(279.) A map is nothing more than a representation, upon a plane, of some portion of the surface of a sphere, on which are traced the particulars intended to be expressed, whether they be continuous outlines or points. Now, as a spherical surface¹⁹ can by no contrivance be extended or projected into a plane, without undue enlargement or contraction of some parts in proportion to others; and as the system adopted in so extending or projecting it will decide *what* parts shall be enlarged or relatively contracted, and in what proportions; it follows, that when large portions of the sphere are to be mapped down, a great difference in their representations may subsist, according to the system of projection adopted.

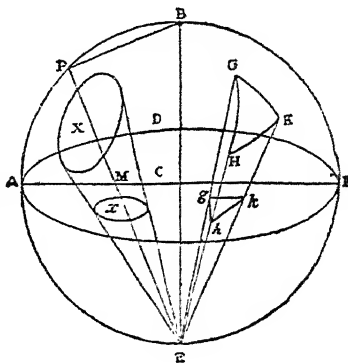
(280.) The projections chiefly used in maps, are the *orthographic*, *stereographic*, and *Mercator's*. In the *orthographic* projection, every point of the hemisphere is referred



to its diametral plane or base, by a perpendicular let fall on it, so that the representation of the hemisphere thus mapped on its base, is such as would actually appear to an eye placed at an infinite distance from it. It is obvious, from the annexed figure, that in this projection only the central portions are represented of their true forms, while all the exterior is more and more distorted and crowded together as we approach the edges of the map. Owing to this cause, the orthographic projection, though very good for small portions of the globe, is of little service for large ones.

¹⁹ We here neglect the ellipticity of the earth, which, for such purpose as map-making, is too trifling to have any material influence.

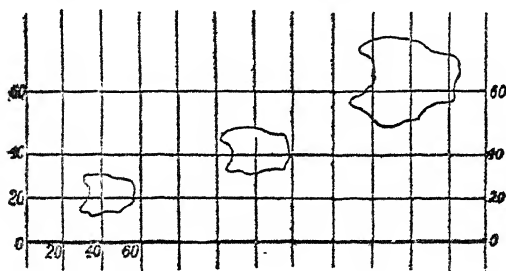
(281.) The *stereographic* projection is in great measure free from this defect. To understand this projection we must conceive an eye to be placed at E, one extremity of a diameter, E C B, of the sphere, and to view the concave surface of the sphere, every point of which, as P, is referred to the diametral plane A D F, perpendicular to E B by the visual line P M E. The stereographic projection of a sphere, then, is a true perspective representation of its concavity on a diametral plane; and, as such, it possesses some singularly elegant geometrical properties, of which we shall state one or two of the principal.



(282.) And first, then, all circles on the sphere are represented by circles in the projection. Thus the circle X is projected into x . Only great circles passing through the vertex B are projected into straight lines traversing the centre C: thus, B P A is projected into C A.

2dly. Every very small triangle, G H K, on the sphere, is represented by a *similar* triangle $g h k$, in the projection. This is a very valuable property, as it insures a general similarity of appearance in the map to the reality in all its smaller parts, and enables us to project at least a hemisphere in a single map, without any violent distortion of the configurations on the surface from their real forms. As in the orthographic projection, the borders of the hemisphere are unduly crowded together; in the stereographic, their projected dimensions are, on the contrary, somewhat enlarged in receding from the centre.

(283.) Both these projections may be considered *natural* ones, inasmuch as they are really perspective representations of the surface on a plane. Mercator's is entirely an artificial one, representing the sphere as it cannot be seen from any one point, but as it might be seen by an eye carried successively over every part of it. In it, the degrees of *longitude*, and those of *latitude*, bear always to each other their due proportion: the equator is conceived to be extended out into a straight line, and the meridians are straight lines at right angles to it, as in the figure. Altogether, the



general character of maps on this projection is not very dissimilar to what would be produced by referring every point in the globe to a circumscribing cylinder, by lines drawn from the centre, and then unrolling the cylinder into a plane. Like the stereographic projection, it gives a true representation, as to *form*, of every particular small part, but varies greatly in point of *scale* in its different regions; the polar portions in particular being extravagantly enlarged; and the whole map, even of a single hemisphere, not being comprisable within any finite limits.

(283 a.) A very convenient projection, at once simple in principle, and remarkable for the facility with which places on the earth's surface may be laid down from a knowledge of their latitudes and longitudes, or stars from that of their

right ascensions and polar distances; or read off from the chart when projected, is one in which (the radius of a circle being divided into ninety equal parts, representing degrees of polar distance) parallels of latitude or of declination are expressed by concentric circles, described through each of the points of division, and circles of longitude or of declination are represented by the radii. In a planisphere constructed on this principle, the proportions of the spaces occupied on the chart by equal areas differently situated, are better preserved than in any of those already described, and with an amount of distortion of shape, on the whole, as little offensive as the nature of a planisphere chart allows. This projection (as does also one recently proposed by Sir H. James, which takes in two-thirds of the sphere) admits of being extended considerably beyond a hemisphere, without producing a very intolerable distortion.

(283 *b.*) The following projection, in which equal areas on the projection correspond *precisely* to equal areas on the spherical surface projected, is also occasionally employed.²⁹

Take out, upon any scale, from a table of natural sines, the sines of $30'$, 1° , $1^\circ 30'$, . . . up to 45° , and from any centre with these as radii describe circles. These will represent the projections of small circles of the sphere about a pole, whose projection is their common centre, having the respective polar distances 1° , 2° , 3° , . . . 90° .

(284.) We shall not, of course, enter here into any geographical details; but one result of maritime discovery on the great scale is, so to speak, *massive* enough to call for mention as an astronomical feature. When the continents

²⁹ See "Results of Astronomical Observations at the Cape of Good Hope," by the Author, Plate XI., where this projection is used to exhibit the law of distribution of the Nebulæ.

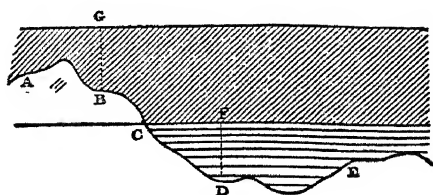
and seas are laid down on a globe (and since the discovery of Australia and the recent addition to our antarctic knowledge of Victoria Land by Sir J. C. Ross, we are sure that no very extensive tracts of land remain unknown), we find that it is possible so to divide the globe into two hemispheres, that one shall contain *nearly all the land*; the other being almost entirely sea. It is a fact, not a little interesting to Englishmen, and, combined with our insular station in that great highway of nations, the Atlantic, not a little explanatory of our commercial eminence, that London²¹ occupies nearly the centre of the terrestrial hemisphere. Astronomically speaking, the fact of this divisibility of the globe into an oceanic and a terrestrial hemisphere is important, as demonstrative of a want of absolute equality in the density of the solid material of the two hemispheres. Considering the whole mass of land and water as in a state of *equilibrium*, it is evident that the half which protrudes must of necessity be *buoyant*; not, of course, that we mean to assert it to be lighter than *water*, but, as compared with the whole globe, *in a less degree heavier* than that fluid. We leave to geologists to draw from these premises their own conclusions (and we think them obvious enough) as to the internal constitution of the globe, and the immediate nature of the forces which sustain its continents at their actual elevation; but in any future investigations which may have for their object to explain the local deviations of the inten-

²¹ More exactly, Falmouth. The central point of the hemisphere which contains the maximum of land falls very nearly indeed upon this port. The land in the opposite hemisphere, with exception of the tapering extremity of South America and the slender peninsula of Malacca, is wholly insular, and were it not for Australia, would be quite insignificant in amount. This interesting feature of geography was first noticed by Colson (Phil. Tr. xxxix. p. 210). A pair of planispheres for the horizon of London has been published by Hughes (London 1839).

sity of gravity, from what the hypothesis of an exact elliptic figure would require, this, as a general fact, ought not to be lost sight of.

(285.) Our knowledge of the surface of our globe is incomplete, unless it include the heights above the sea level of every part of the land, and the depression of the bed of the ocean below the surface over all its extent. The latter object is attainable (with whatever difficulty and howsoever slowly) by direct sounding; the former by two distinct methods: the one consisting in trigonometrical measurement of the differences of level of all the stations of a survey; the other, by the use of the barometer, the principle of which is, in fact, identical with that of the sounding line. In both cases we measure the distance of the point whose level we would know from the surface of an equilibrated ocean; only in the one case it is an ocean of water; in the other, of air. In the one case our sounding line is real and tangible; in the other, an imaginary one, measured by the length of the column of quicksilver the superincumbent air is capable of counterbalancing.

(286.) Suppose that instead of air, the earth and ocean were covered with oil, and that human life could subsist under such circumstances. Let A B C D E be a continent,



of which the portion A B C projects above the water, but is covered by the oil, which also floats at a uniform depth on the whole ocean. Then if we would know the depth of any

point D below the sea level, we let down a plummet from F. But, if we would know the height of B above the same level, we have only to send up a float from B to the surface of the oil; and *having done the same at C, a point at the sea level, the difference of the two float lines gives the height in question.*

(287.) Now, though the atmosphere differs from oil in not having a positive *surface* equally definite, and in not being capable of carrying up any float adequate to such a use, yet it possesses all the properties of a fluid really essential to the purpose in view, and this in particular—that, over the whole surface of the globe, its *strata of equal density* supposed in a state of equilibrium, are parallel to the surface of equilibrium, or to what *would be* the surface of the sea, if *prolonged under the continents*, and therefore each or any of them has all the characters of a definite surface to measure from, provided it can be ascertained and identified. Now, the height at which, at any station B, the mercury in a barometer is supported, informs us *at once* how much of the atmosphere is incumbent on B, or, in other words, *in what stratum* of the general atmosphere (indicated by its density) B is situated: whence we are enabled finally to conclude, by mechanical reasoning,²² at what height above the sea level *that degree of density* is to be found over the whole surface of the globe. Such is the principle of the application of the barometer to the measurement of heights. For details, the reader is referred to other works.²³

(288.) We will content ourselves here with a general caution against an implicit dependence on barometric meas-

²² Newton's Princip. ii. Prop. 22.

²³ Biot, *Astronomie Physique*, vol. iii. For tables, see the work of Biot cited. Also those of Oltmann, annually published by the French board of longitudes in their *Annuaire*; and Mr. Baily's *Collection of Astronomical Tables and Formulæ*. See also *Encyc. Brit.*, "Meteorology," § 34.

urements, except as a differential process, at stations not too remote from each other. They rely in their application on the assumption of a state of equilibrium in the atmospheric strata over the whole globe—which is very far from being their actual state (art. 37). Winds, especially steady and general currents sweeping over extensive continents, undoubtedly *tend* to produce some degree of conformity in the curvature of these strata to the *general* form of the land-surface, and therefore to give an undue elevation to the mercurial column at some points. On the other hand, the existence of localities on the earth's surface, where a permanent depression of the barometer prevails to the astonishing extent of nearly an inch, has been clearly proved by the observations of Ermann in Siberia and of Ross in the Antarctic Seas, and is probably a result of the same cause, and may be conceived as complementary to an undue habitual elevation in other regions. The mode in which both elevations and depressions of a permanent character may be maintained in the surface of a fluid in motion, will not be enigmatical to any one who contemplates the ripple caused by a pebble in a brook.

(289.) Possessed of a knowledge of the heights of stations above the sea, we may connect all stations at the same altitude by level lines, the lowest of which will be the outline of the sea-coast; and the rest will mark out the successive coast-lines which would take place were the sea to rise by regular and equal accessions of level over the whole world, till the highest mountains were submerged. The bottoms of valleys and the ridge-lines of hills are determined by their property of intersecting all these level lines at right angles, and being, subject to that condition, the shortest and longest, that is to say, the steepest, and the most gently

sloping courses respectively which can be pursued from the summit to the sea. The former constitute "the water courses" of a country; the latter its lines of "watershed" ²⁴ by which it is divided into distinct basins of drainage. Thus originate natural districts of the most ineffaceable character, on which the distribution, limits, and peculiarities of human communities are in great measure dependent. The mean height of the continent of Europe, or that height which its surface would have were all inequalities levelled and the mountains spread equally over the plains, is, according to Humboldt, 1342 English feet; that of Asia, 2274; of North America, 1496; and of South America, 2302. ²⁵

CHAPTER V

OF URANOGRAPHY

Construction of Celestial Maps and Globes by Observations of Right Ascension and Declination—Celestial Objects Distinguished into Fixed and Erratic—Of the Constellations—Natural Regions in the Heavens—The Milky Way—The Zodiac—Of the Ecliptic—Celestial Latitudes and Longitudes—Precession of the Equinoxes—Nutation—Aberration—Refraction—Parallax—Summary View of the Uranographical Corrections

(290.) THE determination of the relative situations of objects in the heavens, and the construction of maps and globes which shall truly represent their mutual configurations as well as of catalogues which shall preserve a more precise numerical record of the position of each, is a task at once simpler and less laborious than that by which the

²⁴ *Wasser-scheide*, the separation of the waters.

²⁵ Humboldt's numbers are the halves of these, but express, not the mean heights of the *surfaces*, but the heights of the several *centres of gravity* of the continental masses above the sea level.

surface of the earth is mapped and measured. Every star in the great constellation which appears to revolve above us, constitutes, so to speak, a celestial station; and among these stations we may, as upon the earth, triangulate, by measuring with proper instruments their angular distances from each other, which, cleared of the effect of refraction, are then in a state for laying down on charts, as we would the towns and villages of a country: and this without moving from our place, at least for all the stars which rise above our horizon.

(291.) Great exactness might, no doubt, be attained by this means, and excellent celestial charts constructed; but there is a far simpler and easier, and at the same time infinitely more accurate course laid open to us if we take advantage of the earth's rotation on its axis, and by observing each celestial object as it passes our meridian, refer it separately and independently to the celestial equator, and thus ascertain its place on the surface of an imaginary sphere, which may be conceived to revolve with it, and on which it may be considered as projected.

(292.) The right ascension and declination of a point in the heavens correspond to the longitude and latitude of a station on the earth; and the place of a star on a celestial sphere is determined, when the former elements are known, just as that of a town on a map, by knowing the latter. The great advantages which the method of meridian observation possesses over that of triangulation from star to star, are, then, 1st, That in it every star is observed in that point of its diurnal course, when it is best seen and least displaced by refraction. 2dly, That the instruments required (the transit and meridian circle) are the simplest and least liable to error or derangement of any used by astronomers.

3dly, That all the observations can be made systematically, in regular succession, and with equal advantages; there being here no question about advantageous or disadvantageous triangles, etc. And, lastly, That, by adopting this course, the very quantities which we should otherwise have to calculate by long and tedious operations of spherical trigonometry, and which are essential to the formation of a catalogue, are made the objects of immediate measurement. It is almost needless to state, then, that this is the course adopted by astronomers.

(293.) To determine the right ascension of a celestial object, all that is necessary is to observe the moment of its meridian passage with a transit instrument, by a clock regulated to exact sidereal time, or reduced to such by applying its known error and rate. The *rate* may be obtained by repeated observations of the same star at its successive meridian passages. The *error*, however, requires a knowledge of the *equinox*, or initial point from which all right ascensions in the heavens reckon, as longitudes do on the earth from a first meridian.

(294.) The nature of this point will be explained presently; but for the purposes of uranography, in so far as they concern only the actual configurations of the stars *inter se*, a knowledge of the equinox is not necessary. The choice of the equinox, as a zero point of right ascensions, is purely artificial, and a matter of convenience. As on the earth, any station (as a national observatory) may be chosen for an origin of longitudes; so in uranography, any conspicuous star might be selected as an initial point from which hour angles might be reckoned, and from which, by merely observing *differences* or *intervals* of time, the situation of all others might be deduced. In practice, these in-

tervals are affected by certain minute causes of inequality, which must be allowed for, and which will be explained in their proper places.

(295.) The declinations of celestial objects are obtained, By observation of their *meridian altitudes*, with the mural or meridian circle, or other proper instruments. This requires a knowledge of the geographical latitude of the station of observation, which itself is only to be obtained by celestial observation. 2dly, And more directly, by observation of their *polar distances* on the mural circle, as explained in art. 170, which is independent of any previous determination of the latitude of the station; neither, however, in this case, does observation give directly and immediately the *exact* declinations. The observations require to be corrected, first for refraction, and moreover for those minute causes of inequality which have been just alluded to in the case of right ascensions.

(296.) In this manner, then, may the places, one among the other, of all celestial objects be ascertained, and maps and globes constructed. Now here arises a very important question. How far are these places permanent? Do these stars and the greater luminaries of heaven preserve forever one invariable connection and relation of place *inter se*, as if they formed part of a solid though invisible firmament; and, like the great natural landmarks on the earth, preserve immutably the same distances and bearings each from the other? If so, the most rational idea we could form of the universe would be that of an earth at absolute rest in the centre, and a hollow crystalline sphere circulating round it, and carrying sun, moon and stars along in its diurnal motion. If not, we must dismiss all such notions, and inquire individually into the distinct history of each

object, with a view to discovering the laws of its peculiar motions, and whether any and what other connection subsists between them.

(297.) So far is this, however, from being the case, that observations, even of the most cursory nature, are sufficient to show that some, at least, of the celestial bodies, and those the most conspicuous, are in a state of continual change of place among the rest. In the case of the moon, indeed, the change is so rapid and remarkable, that its alteration of situation with respect to such bright stars as may happen to be near it may be noticed any fine night in a few hours; and if noticed on two successive nights, cannot fail to strike the most careless observer. With the sun, too, the change of place among the stars is constant and rapid; though, from the invisibility of stars to the naked eye in the day-time, it is not so readily recognized, and requires either the use of telescopes and angular instruments to measure it, or a longer continuance of observation to be struck with it. Nevertheless, it is only necessary to call to mind its greater meridian altitude in summer than in winter, and the fact that the stars which come into view at night (and which are therefore situated in a hemisphere opposite to that occupied by the sun, and having that luminary for its centre) vary with the season of the year, to perceive that a great change must have taken place in that interval in its relative situation with respect to all the stars. Besides the sun and moon, too, there are several other bodies, called planets, which, for the most part, appear to the naked eye only as the largest and most brilliant stars, and which offer the same phenomenon of a constant change of place among the stars; now approaching, and now receding from, such of them as we may refer them to as marks; and, some in

longer, some in shorter periods, making, like the sun and moon, the complete tour of the heavens.

(298.) These, however, are exceptions to the general rule. The innumerable multitude of the stars which are distributed over the vault of the heavens form a constellation, which preserves, not only to the eye of the casual observer, but to the nice examination of the astronomer, a uniformity of aspect which, when contrasted with the perpetual change in the configurations of the sun, moon and planets, may well be termed invariable. It is true, indeed, that, by the refinement of exact measurements prosecuted from age to age, some small changes of apparent place, attributable to no illusion and to no *terrestrial* cause, have been detected in many of them. Such are called, in astronomy, the *proper motions* of the stars. But these are so excessively slow, that their accumulated amount (even in those stars for which they are greatest) has been insufficient, in the whole duration of astronomical history, to produce any obvious or material alteration in the appearance of the starry heavens.

(299.) This circumstance, then, establishes a broad distinction of the heavenly bodies into two great classes;—the fixed, among which (unless in a course of observations continued for many years) no change of mutual situation can be detected; and the erratic, or wandering—(which is implied in the word planet¹)—including the sun, moon and planets, as well as the singular class of bodies termed comets, in whose apparent places among the stars, and among each other, the observation of a few days, or even hours, is sufficient to exhibit an indisputable alteration.

¹ Πλανητης, a wanderer.

(300.) Uranography, then, as it concerns the fixed celestial bodies (or, as they are usually called, the *fixed stars*), is reduced to a simple marking down of their relative places on a globe or on maps; to the insertion on that globe, in its due place in the great constellation of the stars, of the pole of the heavens, or the vanishing point of parallels to the earth's axis; and of the equator and place of the equinox: points and circles these, which, though artificial and having reference entirely to our earth, and therefore subject to all changes (if any) to which the earth's axis may be liable, are yet so convenient in practice, that they have obtained an admission (with some other circles and lines), sanctioned by usage, in all globes and planispheres. The reader, however, will take care to keep them separate in his mind, and to familiarize himself with the idea rather of *two* or more celestial globes, superposed and fitting on each other, on one of which—a real one—are inscribed the stars; on the others those imaginary points, lines and circles, which astronomers have devised for their own uses, and to aid their calculations; and to accustom himself to conceive in the latter or artificial spheres a capability of being shifted in any manner upon the surface of the other; so that, should experience demonstrate (as it does) that these artificial points and lines are brought, by a slow motion of the earth's axis, or by other *secular variations* (as they are called), to coincide, at very distant intervals of time, with different stars, he may not be unprepared for the change, and may have no confusion to correct in his notions.

(301.) Of course we do not here speak of those uncouth figures and outlines of men and monsters, which are usually scribbled over celestial globes and maps, and serve, in a rude and barbarous way, to enable us to talk of groups of

stars, or districts in the heavens, by names which, though absurd or puerile in their origin, have obtained a currency from which it would be difficult to dislodge them. In so far as they have really (as some have) any slight resemblance to the figures called up in imagination by a view of the more splendid "constellations," they have a certain convenience; but as they are otherwise entirely arbitrary, and correspond to no *natural* subdivisions or groupings of the stars, astronomers treat them lightly, or altogether disregard them² except for briefly *naming* remarkable stars, as α Leonis, β Scorpii, etc., by letters of the Greek alphabet attached to them. The reader will find them on any celestial charts or globes, and may compare them with the heavens, and there learn for himself their position.

(302.) There are not wanting, however, *natural* districts in the heavens, which offer great peculiarities of character, and strike every observer: such is the *milky way*, that great luminous band, which stretches, every evening, all across the sky, from horizon to horizon, and which, when traced with diligence, and mapped down, is found to form a zone completely encircling *the whole sphere*, almost in a great circle, which is neither an *hour* circle, nor coincident with any other of our astronomical *grammata*. It is divided in one part of its course, sending off a kind of branch, which unites again with the main body, after remaining distinct for about 150 degrees, within which it suffers an interruption in its continuity. This remarkable belt has main-

² This disregard is neither supercilious nor causeless. The constellations seem to have been almost purposely named and delineated to cause as much confusion and inconvenience as possible. Innumerable snakes twine through long and contorted areas of the heavens, where no memory can follow them; bears, lions and fishes, large and small, northern and southern, confuse all nomenclature, etc. A better system of constellations might have been a material help as an artificial memory.

tained, from the earliest ages, the same relative situation among the stars, and, when examined through powerful telescopes, is found (wonderful to relate!) *to consist entirely of stars scattered by millions*, like glittering dust, on the black ground of the general heavens. It will be described more particularly in the subsequent portion of this work.

(303.) Another remarkable region in the heavens is the *zodiac*, not from anything peculiar in its own constitution, but from its being the area within which the apparent motions of the sun, moon, and all the greater planets are confined. To trace the path of any one of these, it is only necessary to ascertain, by continued observation, its places at successive epochs, and entering these upon our map or sphere in sufficient number to form a series, not too far disjoined, to connect them by lines from point to point, as we mark out the course of a vessel at sea by mapping down its place from day to day. Now when this is done, it is found, first, that the apparent path, or track, of the sun on the surface of the heavens, is no other than an exact great circle of the sphere which is called the *ecliptic*, and which is inclined to the equinoctial at an angle of about $23^{\circ} 28'$, intersecting it at two opposite points, called the equinoctial points, or equinoxes, and which are distinguished from each other by the epithets vernal and autumnal; the vernal being that at which the sun crosses the equinoctial from south to north; the autumnal, when it quits the northern and enters the southern hemisphere. Secondly, that the moon and all the planets pursue paths which, in like manner, encircle the whole heavens, but are not, like that of the sun, great circles exactly returning into themselves and bisecting the sphere, but rather spiral curves of much complexity, and described with very unequal velocities in their different

parts. They have all, however, this in common, that the *general direction* of their motions is the same with that of the sun, viz. from *west to east*, that is to say, the contrary to that in which both they and the stars appear to be carried by the diurnal motion of the heavens; and, moreover, that they never deviate far from the ecliptic on either side, crossing and recrossing it at regular and equal intervals of time, and confining themselves within a zone, or belt (the *zodiac* already spoken of), extending (with certain exceptions among the smaller planets) not further than 8° or 9° on either side of the ecliptic.

(304.) It would manifestly be useless to map down on globes or charts the apparent paths of any of those bodies which never retrace the same course, and which, therefore, demonstrably, must occupy at some one moment or other of their history, every point in the area of that zone of the heavens within which they are circumscribed. The apparent complication of their movements arises (that of the moon excepted) from our viewing them from a station which is itself in motion, and would disappear, could we shift our point of view and observe them from the sun. On the other hand the apparent motion of the sun is presented to us under its least involved form, and is studied, from the station we occupy, to the greatest advantage. So that, independent of the importance of that luminary to us in other respects, it is by the investigation of the laws of its motions in the first instance that we must rise to a knowledge of those of all the other bodies of our system.

(305.) The ecliptic, which is its apparent path among the stars, is traversed by it in the period called the *sidereal year*, which consists of $365^{\text{d}} 6^{\text{h}} 9^{\text{m}} 9.6^{\text{s}}$, reckoned in mean solar time, or $366^{\text{d}} 6^{\text{h}} 9^{\text{m}} 9.6^{\text{s}}$, reckoned in sidereal time. The

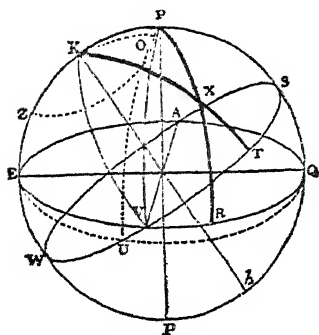
reason of this difference (and it is this which constitutes the origin of the difference between solar and sidereal time) is, that as the sun's apparent annual motion *among* the stars is performed in a contrary direction to the apparent *diurnal* motion of both sun and stars, it comes to the same thing as if the diurnal motion of the sun were so much *slower* than that of the stars, or as if the sun lagged behind them in its daily course. When this has gone on for a whole year, the sun will have fallen behind the stars by a whole circumference of the heavens—or, in other words, in a year the sun will have made fewer diurnal revolutions, by one, than the stars. So that the same interval of time which is measured by 366^d 6^h, etc., of sidereal time, will be called 365 days, 6 hours, etc., if reckoned in mean solar time. Thus, then, is the proportion between the mean solar and sidereal day established, which, reduced into a decimal fraction, is that of 1.00273791 to 1. The measurement of time by these different standards may be compared to that of space by the standard feet, or ells of two different nations; the proportion of which, once settled and borne in mind, can never become a source of error.

(306.) The position of the ecliptic among the stars may, for our present purpose, be regarded as invariable. It is true that this is not strictly the case; and on comparing together its position at present with that which it held at the most distant epoch at which we possess observations, we find evidences of a small change, which theory accounts for, and whose nature will be hereafter explained; but that change is so excessively slow, that for a great many successive years, or even for whole centuries, this circle may be regarded, for most ordinary purposes, as holding the same position in the sidereal heavens.

(307.) The *poles of the ecliptic*, like those of any other great circle of the sphere, are opposite points on its surface, equidistant from the ecliptic in every direction. They are of course not coincident with those of the equinoctial, but removed from it by an angular interval equal to the inclination of the ecliptic to the equinoctial ($23^{\circ} 28'$), which is called the *obliquity of the ecliptic*. In the next figure, if $P p$ represent the north and south poles (by which when used without qualification we always mean the poles of the *equinoctial*), and $E A Q V$ the equinoctial, $V S A W$ the ecliptic, and $K k$, its poles—the spherical angle $Q V S$ is the obliquity of the ecliptic, and is equal in angular measure to $P K$ or $S Q$. If we suppose the sun's apparent motion to be in the direction $V S A W$, V will be the *vernal* and A the *autumnal equinox*. S and W , the two points at which the ecliptic is most distant from the equinoctial, are termed *solstices*, because, when arrived there, the sun ceases to recede from the equator, and (in that sense, so far as its motion in declination is concerned) to stand still in the heavens. S , the point where the sun has the greatest *northern* declination, is called the *summer*, and W , that where it is furthest south, the *winter* solstice. These epithets obviously have their origin in the dependence of the seasons on the sun's declination, which will be explained in the next chapter. The circle $E K P Q k p$, which passes through the poles of the ecliptic and equinoctial, is called the *solstitial colure*; and a meridian drawn through the equinoxes, $P V p A$, the *equinoctial colure*.

(308.) Since the ecliptic holds a determinate situation in the starry heavens, it may be employed, like the equinoctial, to refer the positions of the stars to, by circles drawn through them from *its* poles, and therefore, perpendicular

to it. Such circles are termed, in astronomy, circles of latitude—the distance of a star from the ecliptic, reckoned on the circle of latitude passing through it, is called the *latitude* of the stars—and the arc of the ecliptic intercepted between the vernal equinox and this circle, its *longitude*. In



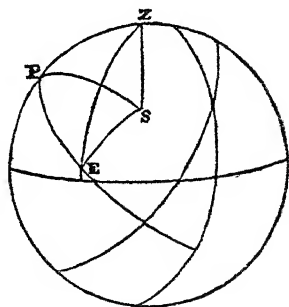
the figure, X is a star, P X R a circle of declination drawn through it, by which it is referred to the equinoctial, and K X T a circle of latitude referring it to the ecliptic—then, as V R is the right ascension, and R X the declination, of X, so also is V T its longitude, and T X its latitude.

The use of the terms longitude and latitude, in this sense, seems to have originated in considering the ecliptic as forming a kind of natural equator to the heavens, as the terrestrial equator does to the earth—the former holding an invariable position with respect to the stars, as the latter does with respect to stations on the earth's surface. The force of this observation will presently become apparent.

(309.) Knowing the right ascension and declination of an object, we may find its longitude and latitude, and *vice versa*. This is a problem of great use in physical astronomy—the following is its solution: In our last figure, E K P Q, the solstitial colure is of course 90° distant from V, the vernal equinox, which is one of its poles—so that V R (the right ascension) being given, and also V E, the arc E R, and its measure, the spherical angle E P R, or K P X, is known. In the spherical triangle K P X, then, we have given, 1st, the side P K, which, being the distance of the

poles of the ecliptic and equinoctial, is equal to the obliquity of the ecliptic; 2, the side $P X$, the *polar distance*, or the complement of the declination $R X$; and, 3, the included angle $K P X$; and therefore, by spherical trigonometry, it is easy to find the other side $K X$, and the remaining angles. Now $K X$ is the complement of the required latitude $X T$, and the angle $P K X$ being known, and $P K V$ being a right angle (because $S V$ is 90°), the angle $X K V$ becomes known. Now this is no other than the measure of the longitude $V T$ of the object. The inverse problem is resolved by the same triangle, and by a process exactly similar.

(§10.) It is often of use to know the situation of the ecliptic in the visible heavens at any instant; that is to say, the points where it cuts the horizon, and the altitude of its highest point, or, as it is sometimes called, the *nonagesimal* point of the ecliptic, as well as the longitude of this point on the ecliptic itself from the equinox. These, and all questions referable to the same data and quæsitæ, are resolved by the spherical triangle $Z P E$, formed by the zenith Z (considered as the pole of the horizon), the pole of the equinoctial P , and the pole of the ecliptic E . The sidereal time being given, and also the right ascension of the pole of the ecliptic (which is always the same, viz. $18^h 0^m 0^s$), the hour angle $Z P E$ of that point is known. Then, in this triangle we have given $P Z$, the co-latitude; $P E$, the polar distance of the pole of the ecliptic, $23^\circ 28'$, and the angle $Z P E$ from which we may find, 1st, the side $Z E$, which is



easily seen to be equal to the altitude of the nonagesimal point sought; and 2dly, the angle $P Z E$, which is the azimuth of the pole of the ecliptic, and which, therefore, being added to and subtracted from 90° , gives the azimuth of the eastern and western intersections of the ecliptic with the horizon. Lastly, the longitude of the nonagesimal point may be had, by calculating in the same triangle the angle $P E Z$, which is its complement.

(311.) The *angle of situation* of a star is the angle included between circles of latitude and of declination passing through it. To determine it in any proposed case, we must resolve the triangle $P S E$, in which are given $P S$, $P E$, and the angle $S P E$, which is the difference between the star's right ascension and 18 hours; from which it is easy to find the angle $P S E$ required. This angle is of use in many inquiries in physical astronomy. It is called in most books on astronomy, the angle of *position*, but this expression has become otherwise and more conveniently appropriated. (See art. 204.)

(312.) The same course of observations by which the path of the sun among the fixed stars is traced, and the ecliptic marked out among them, determines, of course, the place of the equinox V (fig. art. 308) upon the starry sphere, at that time—a point of great importance in practical astronomy, as it is the origin or zero point of right ascension. Now, when this process is repeated at considerably distant intervals of time, a very remarkable phenomenon is observed; viz. that the equinox does *not* preserve a constant place among the stars, but shifts its position, travelling continually and regularly, although with extreme slowness, *backward*, along the ecliptic, in the direction $V W$ from east to west, or the *contrary* to that in which the sun appears to

move in that circle. As the ecliptic and equinoctial are not very much inclined, this motion of the equinox from east to west along the former, conspires (speaking generally) with the diurnal motion, and carries it, with reference to that motion, continually in advance upon the stars: hence it has acquired the name of the *precession of the equinoxes*, because the place of the equinox among the stars, at every subsequent moment, *precedes* (with reference to the diurnal motion) that which it held the moment before. The amount of this motion by which the equinox travels backward, or retrogrades (as it is called), on the ecliptic, is $0^{\circ} 0' 50.10''$ *per annum*, an extremely minute quantity, but which, by its continual accumulation from year to year, at last makes itself very palpable, and that in a way highly inconvenient to practical astronomers, by destroying, in the lapse of a moderate number of years, the arrangement of their catalogues of stars, and making it necessary to reconstruct them. Since the formation of the earliest catalogue on record, the place of the equinox has retrograded already about 30° . The period in which it performs a complete tour of the ecliptic is 25,868 years.

(313.) The immediate uranographical effect of the precession of the equinoxes is to produce a uniform *increase of longitude* in all the heavenly bodies, whether fixed or erratic. For the vernal equinox being the initial point of longitudes, as well as of right ascension, a retreat of this point on the ecliptic *tells* upon the longitudes of all alike, whether at rest or in motion, and produces, so far as its amount extends, the *appearance* of a motion in longitude common to all, *as if* the whole heavens had a slow rotation round the poles of the ecliptic in the long period above mentioned, similar to what they have in twenty-four hours

round those of the equinoctial. This increase of longitude, the reader will of course observe and bear in mind, is, properly speaking, neither a real nor an apparent *movement of the stars*. It is a purely technical result, arising from the gradual shifting of the zero point from which longitudes are reckoned. Had a fixed star been chosen as the origin of longitudes, they would have been invariable.

(314.) To form a just idea of this curious astronomical phenomenon, however, we must abandon, for a time, the consideration of the ecliptic, as tending to produce confusion in our ideas; for this reason, that the stability of the ecliptic itself among the stars is (as already hinted, art. 306) only approximate, and that in consequence its intersection with the equinoctial is liable to a certain amount of change, arising from *its* fluctuation, which mixes itself with what is due to the principal uranographical cause of the phenomenon. This cause will become at once apparent, if, instead of regarding the equinox, we fix our attention on the pole of the equinoctial, or the vanishing point of the earth's axis.

(315.) The place of this point among the stars is easily determined at any epoch, by the most direct of all astronomical observations—those with the meridian or mural circle. By this instrument we are enabled to ascertain at every moment the exact distance of the polar point from any three or more stars, and therefore to lay it down, by triangulating from these stars, with unerring precision, on a chart or globe, without the least reference to the position of the ecliptic, or to any other circle not naturally connected with it. Now, when this is done with proper diligence and exactness, it results that, although for short intervals of time, such as a few days, the place of the pole may be regarded as not sensibly variable, yet in reality it is in a state of con-

stant, although extremely slow motion; and, what is still more remarkable, this motion is not uniform, but compounded of one principal, uniform, or nearly uniform, part, and other smaller and subordinate periodical fluctuations: the former giving rise to the phenomena of *precession*; the latter to another distinct phenomenon called *nutation*. These two phenomena, it is true, belong, theoretically speaking, to one and the same general head, and are intimately connected together, forming part of a great and complicated chain of consequences flowing from the earth's rotation on its axis: but it will be conducive to clearness at present to consider them separately.

(§16.) It is found, then, that in virtue of the uniform part of the motion of the pole, it describes a circle in the heavens around the pole of the ecliptic as a centre, keeping constantly at the same distance of $23^{\circ} 28'$ from it in a direction from east to west, and with such a velocity, that the annual angle described by it, in this its imaginary orbit, is $50.10''$; so that the whole circle would be described by it in the above-mentioned period of 25,868 years. It is easy to perceive how such a motion of the pole will give rise to the retrograde motion of the equinoxes; for in the figure, art. 808, suppose the pole P in the progress of its motion in the small circle P O Z round K to come to O, then, as the situation of the equinoctial E V Q is determined by that of the pole, this, it is evident, must cause a displacement of the equinoctial, which will take a new situation, E U Q, 90° distant in every part from the new position O of the pole. The point U, therefore, in which the displaced equinoctial will intersect the ecliptic, *i.e.* the displaced equinox, will lie on that side of V, its original position, toward which the motion of the pole is directed, or to the westward.

(317.) The precession of the equinoxes thus conceived, consists, then, in a real but very slow motion of the pole of the heavens among the stars, in a small circle round the pole of the ecliptic. Now this cannot happen without producing corresponding changes in the apparent diurnal motion of the sphere, and the aspect which the heavens must present at very remote periods of history. The pole is nothing more than the vanishing point of the earth's axis. As this point, then, has such a motion as we have described, it necessarily follows that the earth's *axis* must have a conical motion, in virtue of which it points successively to every part of the small circle in question. We may form the best idea of such a motion by noticing a child's pegtop, when it spins not upright, or that amusing toy the te-to-tum, which, when delicately executed, and nicely balanced, becomes an elegant philosophical instrument, and exhibits, in the most beautiful manner, the whole phenomenon. The reader will take care not to confound the variation of the *position of the earth's axis in space* with a mere shifting of the imaginary line about which it revolves, in its interior. The whole earth participates in the motion, and goes along with the axis as if it were really a bar of iron driven through it. That such is the case is proved by the two great facts: 1st, that the latitudes of places on the earth, or their geographical situation with respect to the poles, have undergone no perceptible change from the earliest ages. 2dly, that the sea maintains its level, which could not be the case if the motion of the axis were not accompanied with a motion of the whole mass of the earth.*

* Local changes of the sea level, arising from purely geological causes, are easily distinguished from that general and systematic alteration which a shifting of the axis of rotation would give rise to.

(318.) The visible effect of precession on the aspect of the heavens consists in the *apparent* approach of some stars and constellations to the pole and recess of others. The bright star of the Lesser Bear, which we call the pole star, has not always been, nor will always continue to be, our cynosure. At the time of the construction of the earliest catalogues it was 12° from the pole—it is now only $1^\circ 24'$, and will approach yet nearer, to within half a degree, after which it will again recede, and slowly give place to others, which will succeed in its companionship to the pole. After a lapse of about 12,000 years, the star α Lyrae, the brightest in the northern hemisphere, will occupy the remarkable situation of a pole star approaching within about 5° of the pole.

(319.) At the date of the erection of the Great Pyramid of Gizeh, which precedes by 3970 years (say 4000) the present epoch, the longitudes of all the stars were less by $55^\circ 45'$ than at present. Calculating from this datum⁴ the place of the pole of the heavens among the stars, it will be found to fall near α Draconis; its distance from that star being $3^\circ 44' 25''$. This being the *most* conspicuous star⁵ in the immediate neighborhood was therefore the pole star at that epoch. And the latitude of Gizeh being just 30° north, and consequently the altitude of the north pole there also 30° , it follows that the star in question must have had at its lower culmination, at Gizeh, an altitude of $26^\circ 15' 35''$. Now it is a remarkable fact, ascertained by the late researches of Col.

⁴ On this calculation the diminution of the obliquity of the ecliptic in the 4000 years elapsed has no influence. That diminution arises from a change in the plane of the earth's *orbit*, and has nothing to do with the change in the position of its *axis*, as referred to the starry sphere.

⁵ α Draconis is now an *inconspicuous* star of the 4th magnitude, but there is distinct evidence to show that it was formerly brighter.

Vyse, that of the nine pyramids still existing at Gizeh, six (including all the largest) have the narrow passages by which alone they can be entered (all which open out on the northern faces of their respective pyramids) inclined to the horizon downward at angles as follows:

1st, or Pyramid of Cheops	26° 41'
2d, or Pyramid of Cephren	25 55
3d, or Pyramid of Mycerinus	26 2
4th,	27 0
5th,	27 12
9th,	28 0
Mean	26 47

Of the two pyramids at Abousseir also, which alone exist in a state of sufficient preservation to admit of the inclinations of their entrance passages being determined, one has the angle $27^{\circ} 5'$, the other 26° .

(320.) At the bottom of every one of these passages, therefore, the *then* pole star must have been visible at its lower culmination, a circumstance which can hardly be supposed to have been unintentional, and was doubtless connected (perhaps superstitiously) with the astronomical observation of that star, of whose proximity to the pole at the epoch of the erection of these wonderful structures, we are thus furnished with a monumental record of the most imperishable nature.

(321.) The *nutation* of the earth's axis is a small and slow subordinate gyratory movement, by which, if subsisting alone, the pole would describe among the stars, in a period of about nineteen years, a minute ellipsis, having its longer axis equal to $18''\cdot5$, and its shorter to $13''\cdot74$; the longer being directed toward the pole of the ecliptic, and the shorter, of course, at right angles to it. The consequence of this real motion of the pole is an *apparent* approach and

recess of all the stars in the heavens to the pole in the same period. Since, also, the place of the equinox on the ecliptic is determined by the place of the pole in the heavens, the same cause will give rise to a small alternate advance and recess of the equinoctial points, by which, in the same period, both the longitudes and the right ascensions of the stars will be also alternately increased and diminished.

(322.) Both these motions, however, although here considered separately, subsist jointly; and since, while in virtue of the nutation, the pole is describing its little ellipse of $18''\cdot5$ in diameter, it is carried by the greater and regularly progressive motion of precession over so much of its circle round the pole of the ecliptic as corresponds to nineteen years—that is to say, over an angle of nineteen times $50''\cdot1$ round the centre (which, in a small circle of $23^{\circ}28'$ in diameter, corresponds to $6'20''$, as seen from the centre of the sphere): the path which it will pursue in virtue of the two motions, subsisting jointly, will be neither an ellipse nor an exact circle, but a gently undulated ring like that in the figure (where, however, the undulations are much exaggerated). (See fig. to art. 325.)

(323.) These movements of precession and nutation are common to all the celestial bodies, both fixed and erratic; and this circumstance makes it impossible to attribute them to any other cause than a real motion of the earth's axis such as we have described. Did they only affect the stars, they might, with equal plausibility, be urged to arise from a *real* rotation of the starry heavens, as a solid shell, round an axis passing through the poles of the ecliptic in 25,868 years, and a real elliptic gyration of *that* axis in nineteen years: but since they also affect the sun, moon, and planets, which, having motions independent of the general body of

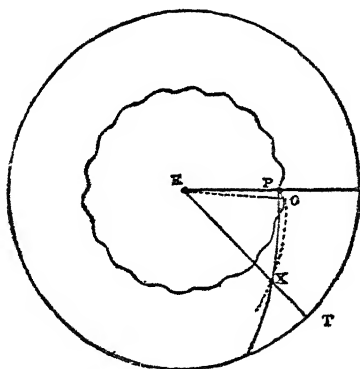
the stars, cannot without extravagance be supposed *attached* to the celestial concave,* this idea falls to the ground; and there only remains, then, a real motion in the earth by which they *can* be accounted for. It will be shown in a subsequent chapter that they are necessary consequences of the rotation of the earth, combined with its elliptical figure, and the unequal attraction of the sun and moon on its polar and equatorial regions.

(324.) Uranographically considered, as affecting the apparent places of the stars, they are of the utmost importance in practical astronomy. When we speak of the right ascension and declination of a celestial object, it becomes necessary to state what *epoch* we intend, and whether we mean the *mean* right ascension—cleared, that is, of the periodical fluctuation in its amount, which arises from nutation, or the *apparent* right ascension, which, being reckoned from the actual place of the vernal equinox, is affected by the periodical advance and recess of the equinoctial point produced by nutation—and so of the other elements. It is the practice of astronomers to *reduce*, as it is termed, all their observations, both of right ascension and declination, to some common and convenient epoch—such as the beginning of the year for temporary purposes, or of the decade, or the century for more permanent uses, by subtracting from them the whole effect of precession in the interval; and, moreover, to divest them of the influence of nutation by investigating and subducting the amount of change, both in right ascension and declination, due to the displacement of the

* This argument, cogent as it is, acquires additional and decisive force from the law of nutation, which is dependent on the position, for the time, of the lunar orbit. If we attribute it to a real motion of the celestial sphere, we must then maintain that sphere to be kept in a constant state of tremor by the motion of the moon!

pole from the centre to the circumference of the little ellipse above mentioned. This last process is technically termed *correcting* or *equating* the observation for nutation; by which latter word is always understood, in astronomy, the getting rid of a periodical cause of fluctuation, and presenting a result, not as it *was* observed, but as it would have been observed, had that cause of fluctuation had no existence.

(325.) For these purposes, in the present case, very convenient formulæ have been derived, and tables constructed. They are, however, of too technical a character for this work; we shall, however, point out the manner in which the investigation is conducted. It has been shown in art. 309 by what means the right ascension and declination of an object are derived from its longitude and latitude. Referring to the figure of that article, and supposing the triangle



K P X orthographically projected on the plane of the ecliptic as in the annexed figure: in the triangle K P X, K P is the obliquity of the ecliptic, K X the *co-latitude* (or complement of latitude), and the angle P K X the *co-longitude* of the object X. These are the *data* of our question, of which the second is constant, and the other two are varied by the effect of precession and nutation: and their variations (considering the minuteness of the latter effect generally, and the small number of years in comparison of the whole period of 25,868, for which we ever require to estimate the effect of

the former) are of that order which may be regarded as infinitesimal in geometry, and treated as such without fear of error. The whole question, then, is reduced to this:—In a spherical triangle $K P X$, in which one side $K X$ is constant, and an angle K , and adjacent side $K P$ vary by given infinitesimal changes of the position of P : required the changes thence arising in the other side $P X$, and the angle $K P X$. This is a very simple and easy problem of spherical geometry, and being resolved, it gives at once the reductions we are seeking; for $P X$ being the polar distance of the object, and the angle $K P X$ its right ascension *plus* 90° , their variations are the very quantities we seek. It only remains, then, to express in proper form the amount of the precession and nutation in *longitude* and *latitude*, when their amount in right ascension and declination will immediately be obtained.

(326.) The precession in *latitude* is zero, since the latitudes of objects are not changed by it; that in *longitude* is a quantity proportional to the time at the rate of $50''\cdot10$ per annum. With regard to the nutation in *longitude* and *latitude*, these are no other than the abscissa and ordinate of the little ellipse in which the pole moves. The law of its motion, however, therein, cannot be understood till the reader has been made acquainted with the principal features of the moon's motion on which it depends.

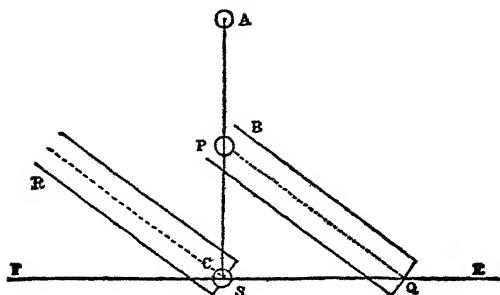
(327.) Another consequence of what has been shown respecting precession and nutation, is that *sidereal time*, as astronomers use it, *i.e.* as reckoned from the transit of the equinoctial point, is *not a mean or uniformly flowing quantity*, being affected by nutation; and, moreover, that so reckoned, even when cleared of the periodical fluctuation of nutation, it does not *strictly* correspond to the earth's

diurnal rotation. As the sun *loses* one day in the year on the stars, by its *direct* motion in longitude; so the equinox *gains* one day in 25,868 years on them by its *retrogradation*. We ought, therefore, as carefully to distinguish between mean and apparent sidereal as between mean and apparent solar time.

(328.) Neither precession nor nutation change the apparent places of celestial objects *inter se*. We see them, so far as these causes go, *as they are*, though from a station more or less unstable, as we see distant land objects correctly formed, though appearing to rise and fall when viewed from the heaving deck of a ship in the act of pitching and rolling. But there is an optical cause, independent of refraction or of perspective, which displaces them *one among the other*, and causes us to view the heavens under an aspect always to a certain slight extent false; and whose influence must be estimated and allowed for before we can obtain a precise knowledge of the place of any object. This cause is what is called the aberration of light; a singular and surprising effect arising from this, that we occupy a station not at rest but in rapid motion; and that the apparent directions of the rays of light are not the same to a spectator in motion as to one at rest. As the estimation of its effect belongs to uranography, we must explain it here, though, in so doing, we must anticipate some of the results to be detailed in subsequent chapters.

(329.) Suppose a shower of rain to fall perpendicularly in a dead calm; a person exposed to the shower, who should stand quite still and upright, would receive the drops on his hat, which would thus shelter him, but if he ran forward in any direction they would strike him in the face. The effect would be the same as if he remained still, and

a wind should arise of the same velocity, and drift them against him. Suppose a ball let fall from a point A above a horizontal line E F, and that at B were placed to receive it the open mouth of an inclined hollow tube P Q; if the tube were held immovable the ball would strike on its lower side, but if the tube were carried forward in the direction E F, with a velocity properly adjusted at every instant to that of the ball, while *preserving its inclination*



to the horizon, so that when the ball in its natural descent reached C, the tube should have been carried into the position R S, it is evident that the ball would, throughout its whole descent, be found in the axis of the tube; and a spectator referring to the tube the motion of the ball, and carried along with the former, unconscious of its motion, would fancy that the ball had been moving in the inclined direction R S of the tube's axis.

(330.) Our eyes and telescopes are such tubes. In whatever manner we consider light, whether as an advancing wave in a motionless ether, or a shower of atoms traversing space (provided that in both cases we regard it as absolutely incapable of suffering resistance or corporeal obstruction from the particles of transparent media traversed by

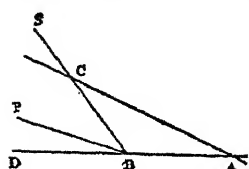
it'), if in the interval between the rays traversing the object-glass of the one or the cornea of the other (*at which moment* they acquire that convergence which directs them to a certain point *in fixed space*), and their arrival at their focus, the cross wires of the one or the retina of the other be *slipped aside*, the point of convergence (which remains unchanged) will no longer correspond to the intersection of the wires or the central point of our visual area. The object then will *appear* displaced; and the amount of this displacement is *aberration*.

(331.) The earth is moving through space with a velocity of about 19 miles per second, in an elliptic path round the sun, and is therefore changing the direction of its motion at every instant. Light travels with a velocity of 192,000 miles per second, which, although much greater than that of the earth, is yet not *infinitely so*. Time is occupied by it in traversing any space, and in that time the earth describes a space which is to the former as 19 to 192,000, or as the tangent of $20''\cdot5$ to radius. Suppose now A P S to represent a ray of light from a star at A, and let the tube P Q be that of a telescope so inclined forward that the focus *formed* by its object-glass shall be *received* upon its cross wire, it is evident from what has been said that the inclination of the tube must be such as to make $P S : S Q :: \text{velocity of light} : \text{velocity of the earth} :: 1 : \tan. 20''\cdot5$; and, therefore, the angle S P Q, or P S R, by

¹ This condition is indispensable. Without it we fall into all those difficulties which M. Doppler has so well pointed out in his paper on Aberration (*Abhandlungen der k. böhmischen Gesellschaft der Wissenschaften. Folge V. vol. iii.*). If light itself, or the luminiferous ether, be corporeal, the condition insisted on amounts to a formal surrender of the dogma, either of the extension or of the impenetrability of matter; at least in the sense in which those terms have been hitherto used by metaphysicians. At the point to which science is arrived, probably few will be found disposed to maintain either the one or the other.

which the axis of the telescope must deviate from the true direction of the star, must be $20''\cdot5$.

(332.) A similar reasoning will hold good when the direction of the earth's motion is not perpendicular to the visual ray. If $S B$ be the true direction of the visual ray, and $A C$ the position in which the telescope requires to be held in the apparent direction, we must still have



the proportion $B C : B A :: \text{velocity of light} : \text{velocity of the earth} :: \text{rad.} : \text{sine of } 20''\cdot5$ (for in such small angles it matters not whether we use the sines or tangents). But

we have, also, by trigonometry, $B C : B A :: \text{sine of } B A C : \text{sine of } A C B$ or $C B P$, which last is the apparent displacement caused by aberration. Thus it appears that the sine of the aberration, or (since the angle is extremely small) the aberration itself, is proportional to the sine of the angle made by the earth's motion in space with the visual ray, and is therefore a maximum when the line of sight is perpendicular to the direction of the earth's motion.

(333.) The uranographical effect of aberration, then, is to distort the aspect of the heavens, causing all the stars to crowd as it were directly toward that point in the heavens which is the vanishing point of all lines parallel to that in which the earth is for the moment moving. As the earth moves round the sun in the plane of the ecliptic, this point must lie in that plane, 90° in advance of the earth's longitude, or 90° behind the sun's, and shifts of course continually, describing the circumference of the ecliptic in a year. It is easy to demonstrate that the effect on each particular star will be to make it apparently de-

scribe a small ellipse in the heavens, having for its centre the point in which the star would be seen if the earth were at rest.

(334.) Aberration then affects the apparent right ascensions and declinations of all the stars, and that by quantities easily calculable. The formulæ most convenient for that purpose, and which, systematically embracing at the same time the corrections for precession and nutation, enable the observer, with the utmost readiness, to disencumber his observations of right ascension and declination of their influence, have been constructed by Prof. Bessel, and tabulated in the appendix to the first volume of the Transactions of the Astronomical Society, where they will be found accompanied with an extensive catalogue of the places, for 1830, of the principal fixed stars, one of the most useful and best arranged works of the kind which has ever appeared.

(335.) When the body from which the visual ray emanates is itself in motion, an effect arises which is not properly speaking aberration, though it is usually treated under that head in astronomical books, and indeed confounded with it, to the production of some confusion in the mind of the student. The effect in question (which is independent of any theoretical views respecting the nature of light^a)

^a The results of the undulatory and corpuscular theories of light, in the matter of aberration, are, in the main, the same. We say *in the main*. There is, however, a minute difference even of numerical results. In the undulatory doctrine, the propagation of light takes place with equal velocity in all directions, whether the luminary be at rest or in motion. In the corpuscular, with an excess of velocity in the direction of the motion over that in the contrary equal to twice the velocity of the body's motion. In the cases, then, of a body moving with equal velocity directly to and directly from the earth, the aberrations will be *alike* on the undulatory, but different on the corpuscular hypothesis. The utmost difference which can arise from this cause *in our system* cannot amount to above six thousandths of a second.

may be explained as follows. The ray by which we see any object is not that which it emits at the moment we look at it, but that which it *did* emit some time before, *viz.* the time occupied by light in traversing the interval which separates it from us. The aberration of such a body then arising from the earth's velocity must be applied as a correction, not to the line joining the earth's place at the moment of observation with that occupied by the body *at the same moment*, but at that antecedent instant when the ray quitted it. Hence it is easy to derive the rule given by astronomical writers for the case of a moving object. *From the known laws of its motion and the earth's, calculate its apparent or relative angular motion in the time taken by light to traverse its distance from the earth. This is the total amount of its apparent misplacement.* Its effect is to displace the body observed in a direction contrary to its apparent motion in the heavens. And it is a compound or aggregate effect consisting of two parts, one of which is the aberration, properly so called, resulting from the composition of the earth's motion with that of light, the other being what is not inaptly termed the *Equation of light*, being the allowance to be made for the *time* occupied by the light in traversing a variable space.

(336.) The complete *Reduction*, as it is called, of an astronomical observation consists in applying to the place of the observed heavenly body as read off on the instruments (supposed perfect and in perfect adjustment) five distinct and independent corrections, *viz.* those for refraction, parallax, aberration, precession, and nutation. Of these the correction for refraction enables us to declare what would have been the observed place, were there no atmosphere to displace it. That for parallax enables us to say from its

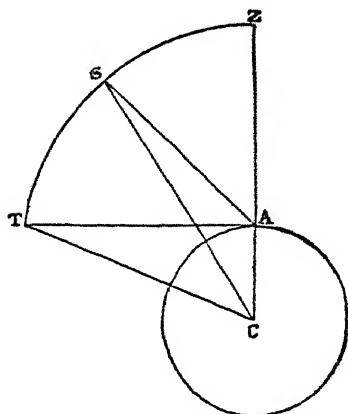
place observed at the surface of the earth, where it would have been seen if observed from the centre. That for aberration, where it would have been observed from a motionless, instead of a moving station: while the corrections for precession and nutation refer it to fixed and determinate instead of constantly varying celestial circles. The great importance of these corrections, which pervade all astronomy, and have to be applied to every observation before it can be employed for any practical or theoretical purpose, renders this recapitulation far from superfluous.

(337.) Refraction has been already sufficiently explained (art. 40), and it is only, therefore, necessary here to add that in its use as an astronomical correction its amount must be applied in a contrary sense to that in which it affects the observation; a remark equally applicable to all other corrections.

(338.) The general nature of parallax or rather of parallactic motion has also been explained in art. 80. But parallax in the uranographical sense of the word has a more technical meaning. It is understood to express that optical displacement of a body observed which is due to its being observed, not from that point which we have fixed upon as a conventional central station (from which we conceive the apparent motion would be more simple in its laws), but from some other station remote from such conventional centre: not from the centre of the earth, for instance, but from its surface: not from the centre of the sun (which, as we shall hereafter see, is for some purposes a preferable conventional station), but from that of the earth. In the former case this optical displacement is called the *diurnal* or geocentric parallax; in the latter the *annual* or heliocentric. In either case parallax is the *correction* to be ap-

plied to the apparent place of the heavenly body, as actually seen from the station of observation, to *reduce* it to its place as it would have been seen at that instant from the conventional station.

(339.) The diurnal or geocentric parallax at any place of the earth's surface is easily calculated if we know the distance



of the body, and, *vice versa*, if we know the diurnal parallax that distance may be calculated. For supposing S the object, C the centre of the earth, A the station of observation at its surface, and C A Z the direction of a perpendicular to the surface at A, then will the object be seen from A in the direction A S, and its apparent zenith distance

will be Z A S; whereas, if seen from the centre, it will appear in the direction C S, with an angular distance from the zenith of A equal to Z C S; so that Z A S—Z C S or A S C is the parallax. Now since by trigonometry $CS : CA :: \sin CAS = \sin ZAS : \sin ASC$, it follows that the sine of the parallax = $\frac{\text{Radius of earth}}{\text{Distance of body}} + \sin ZAS$.

(340.) The diurnal or geocentric parallax, therefore, at a given place, and for a given distance of the body observed, is proportional to the sine of its apparent zenith distance, and is, therefore, the greatest when the body is observed in the act of rising or setting, in which case its parallax is called its *horizontal* parallax, so that at any other zenith distance, parallax = horizontal parallax + sine of apparent

zenith distance, and since $A C S$ is always less than $Z A S$ it appears that the application of the reduction or correction for parallax always acts in diminution of the apparent zenith distance or increase of the apparent altitude or distance from the Nadir, *i.e.* in a contrary sense to that for refraction.

(341.) In precisely the same manner as the geocentric or diurnal parallax refers itself to the zenith of the observer for its direction and quantitative rule, so the heliocentric or annual parallax refers itself for its law to the point in the heavens diametrically opposite to the place of the sun as seen from the earth. Applied as a correction, its effect takes place in a plane passing through the sun, the earth, and the observed body. Its effect is always to decrease its observed distance from that point or to increase its angular distance from the sun. And its sine is given by the relation, Distance of the observed body from the sun : distance of the earth from the sun :: sine of apparent angular distance of the body from the sun (or its apparent *elongation*): sine of heliocentric parallax.*

(342.) On a summary view of the whole of the uranographical corrections, they divide themselves into two classes, those which *do*, and those which *do not*, alter the apparent configurations of the heavenly bodies *inter se*. The former are *real*, the latter *technical* corrections. The real corrections are refraction, aberration and parallax. The technical are precession and nutation, unless, indeed, we choose to consider parallax as a technical correction introduced with a view to simplification by a better choice of our point of sight.

* This account of the law of heliocentric parallax is in anticipation of what follows in a subsequent chapter, and will be better understood by the student when somewhat further advanced.

(343.) The corrections of the first of these classes have one peculiarity in respect of their law, common to them all, which the student of practical astronomy will do well to fix in his memory. *They all refer themselves to definite apexes or points of convergence in the sphere.* Thus, refraction in its apparent effect causes all celestial objects to draw together or converge toward the zenith of the observer: geocentric parallax, toward his Nadir: heliocentric, toward the place of the sun in the heavens: aberration toward that point in the celestial sphere which is the vanishing point of all lines parallel to the direction of the earth's motion at the moment, or (as will be hereafter explained) toward a point in the great circle called the ecliptic, 90° behind the sun's place in that circle. When applied as corrections to an observation, these directions are of course to be reversed.

(344.) In the quantitative law, too, which this class of corrections follow, a like agreement takes place, at least as regards the geocentric and heliocentric parallax and aberration, in all three of which the amount of the correction (or more strictly its sine) increases in the direct proportion of the sine of the apparent distance of the observed body from the *apex* appropriate to the particular correction in question. In the case of refraction the law is less simple, agreeing more nearly with the tangent than the sine of that distance, but agreeing with the others in placing the maximum at 90° from its apex.

(345.) As respects the *order* in which these corrections are to be applied to any observation, it is as follows: 1. Refraction; 2. Aberration; 3. Geocentric Parallax; 4. Heliocentric Parallax; 5. Nutation; 6. Precession. Such, at least, is the order in theoretical strictness. But as the amount of aberration and nutation is in all cases a very

minute quantity, it matters not in what order they are applied; so that for practical convenience they are always thrown together with the precession, and applied after the others.

CHAPTER VI

OF THE SUN'S MOTION AND PHYSICAL CONSTITUTION

Apparent Motion of the Sun not Uniform—Its Apparent Diameter also Variable—Variation of its Distance Concluded—Its Apparent Orbit an Ellipse about the Focus—Law of the Angular Velocity—Equable Description of Areas—Parallax of the Sun—Its Distance and Magnitude—Copernican Explanation of the Sun's Apparent Motion—Parallelism of the Earth's Axis—The Seasons—Heat Received from the Sun in Different Parts of the Orbit—Effect of Excentricity of the Orbit and Position of its Axis on Climate—Mean and True Longitudes of the Sun—Equation of the Centre—Sidereal, Tropical and Anomalistic Years—Physical Constitution of the Sun—Its Spots—Faculae—Probable Nature and Cause of the Spots—Recent Discoveries of Mr. Dawes—Of Mr. Nasmyth—Rotation of the Sun on its Axis—Its Atmosphere—Supposed Clouds—Periodical Recurrence of a More and Less Spotted State of its Surface—Temperature of its Surface—Its Expenditure of Heat—Probable Cause of Solar Radiation

(346.) IN the foregoing chapters, it has been shown that the apparent path of the sun is a great circle of the sphere, which it performs in a period of one sidereal year. From this it follows, that the line joining the earth and sun lies constantly *in one plane*; and that, therefore, whatever be the real motion from which this apparent motion arises, it must be confined to one plane, which is called the *plane of the ecliptic*.

(347.) We have already seen (art. 146) that the sun's motion in right ascension among the stars is not uniform. This is partly accounted for by the obliquity of the ecliptic, in consequence of which equal variations in longitude do

not correspond to equal changes of right ascension. But if we observe the place of the sun daily throughout the year, by the transit and circle, and from these calculate the longitude for each day, it will still be found that, even in its own proper path, its apparent angular motion is far from uniform. The change of longitude in twenty-four mean solar hours *averages* $0^{\circ} 59' 8''.33$; but about the 31st of December it amounts to $1^{\circ} 1' 9''.9$, and about the 1st of July is only $0^{\circ} 57' 11''.5$. Such are the extreme limits, and such the mean value of the sun's apparent angular velocity in its annual orbit.

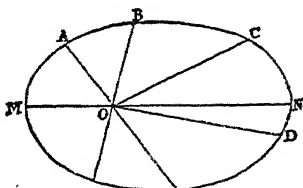
(343.) This variation of its angular velocity is accompanied with a corresponding change of its distance from us. The change of distance is recognized by a variation observed to take place in its apparent diameter, when measured at different seasons of the year, with an instrument adapted for that purpose, called the *heliometer*,¹ or, by calculating from the time which its disk takes to traverse the meridian in the transit instrument. The greatest apparent diameter corresponds to the 1st of January, or to the greatest angular velocity, and measures $32' 36''.2$, the least is $31' 32''.0$; and corresponds to the 1st of July; at which epochs, as we have seen, the angular motion is also at its extreme limit either way. Now, as we cannot suppose the sun to alter its real size periodically, the observed change of its apparent size can only arise from an actual change of distance. And the sines or tangents of such small arcs being proportional to the arcs themselves, its distances from us, at the above-named epoch, must be in the inverse proportion of the apparent diameters. It appears, therefore, that the greatest, the mean, and the least distances of the

¹ Ἡλιος the sun, and μετρεω to measure.

sun from us are in the respective proportions of the numbers 1·01679, 1·00000, and 0·98321; and that its apparent angular velocity diminishes as the distance increases, and *vice versâ*.

(349.) It follows from this, that the real orbit of the sun, as referred to the earth supposed at rest, is not a circle with the earth in the centre. The situation of the earth within it is *excentric*, the *excentricity* amounting to 0·01679 of the mean distance, which may be regarded as our unit of measure in this inquiry. But besides this, the *form* of the orbit is not circular, but elliptic. If from any point O, taken to represent the earth, we draw a line, O A, in some fixed direction, from which we then set off a series of angles, A O B, A O C, etc., equal to the observed longitudes of the sun throughout the year, and in these respective directions measure off from O the distances

O A, O B, O C, etc., representing the distances deduced from the observed diameter, and then connect all the extremities A, B, C, etc., of these lines by a continu-



ous curve, it is evident this will be a correct representation of the relative orbit of the sun about the earth. Now, when this is done, a deviation from the circular figure in the resulting curve becomes apparent; it is found to be evidently longer than it is broad—that is to say, elliptic, and the point O to occupy, not the *centre*, but one of the foci of the ellipse. The graphical process here described is sufficient to point out the general figure of the curve in question; but for the purposes of exact verification, it is necessary to recur to the properties of the ellipse,²

² See Conic Sections, by the Rev. H. P. Hamilton, or any other of the very numerous works on this subject.

and to express the distance of any one of its points in terms of the angular situation of that point with respect to the longer axis, or diameter of the ellipse. This, however, is readily done; and when numerically calculated, on the supposition of the excentricity being such as above stated, a perfect coincidence is found to subsist between the distances thus computed, and those derived from the measurement of the apparent diameter.

(350.) The mean distance of the earth and sun being taken for unity, the extremes are 1·01679 and 0·98321. But if we compare, in like manner, the mean or average angular velocity with the extremes, greatest and least, we shall find these to be in the proportions of 1·03386, 1·00000, and 0·96670. The variation of the sun's *angular velocity*, then, is much greater in proportion than that of its distance—fully twice as great; and if we examine its numerical expressions at different periods, comparing them with the mean value, and also with the corresponding distances, it will be found, that, by whatever fraction of its mean value the distance exceeds the mean, the angular velocity will fall short of *its* mean or average quantity by very nearly *twice* as great a fraction of the latter, and *vice versa*. Hence we are led to conclude that the *angular velocity* is in the inverse proportion, not of the distance simply, but of its *square*; so that, to compare the daily motion in longitude of the sun, at one point, A, of its path, with that at B, we must state the proportion thus:—

$OB^2 : OA^2 :: \text{daily motion at A} : \text{daily motion at B}.$
And this is found to be exactly verified in every part of the orbit.

(351.) Hence we deduce another remarkable conclusion—*viz.* that if the sun be supposed really to move around the

circumference of this ellipse, its actual speed cannot be uniform, but must be greatest at its least distance and less at its greatest. For, were it uniform, the apparent angular velocity would be, of course, inversely proportional to the distance; simply because the same linear change of place, being produced in the same time at different distances from the eye, must, by the laws of perspective, correspond to apparent angular displacements inversely as those distances. Since, then, observation indicates a more rapid law of variation in the angular velocities, it is evident that mere change of distance, unaccompanied with a change of actual speed, is insufficient to account for it; and that the increased proximity of the sun to the earth must be accompanied with an actual increase of its real velocity of motion along its path.

(352.) This elliptic form of the sun's path, the excentric position of the earth within it, and the unequal speed with which it is actually traversed by the sun itself, all tend to render the calculation of its longitude from theory (*i.e.* from a knowledge of the causes and nature of its motion) difficult; and indeed impossible, so long as the *law* of its actual velocity continues unknown. This *law*, however, is not immediately apparent. It does not come forward, as it were, and present itself at once, like the elliptic form of the orbit, by a direct comparison of angles and distances, but requires an attentive consideration of the whole series of observations registered during an entire period. It was not, therefore, without much painful and laborious calculation, that it was discovered by Kepler (who was also the first to ascertain the elliptic form of the orbit), and announced in the following terms:—Let a line be always supposed to connect the sun, supposed in motion, with the earth, supposed at rest; then

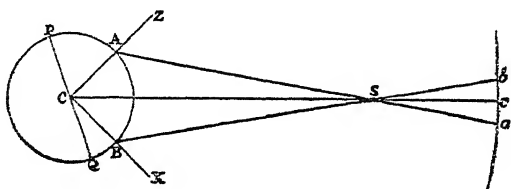
as the sun moves along its ellipse, this line (which is called in astronomy the *radius vector*) will *describe* or *sweep over* that portion of the whole *area* or *surface* of the ellipse which is included between its consecutive positions: and the motion of the sun will be such that *equal areas are thus swept over* by the revolving radius vector *in equal times*, in whatever part of the circumference of the ellipse the sun may be moving.

(353.) From this it necessarily follows, that in *unequal* times, the areas described must be proportional to the times. Thus, in the figure of art. 349 the time in which the sun moves from A to B, is to the time in which it moves from C to D, as the area of the elliptic sector A O B is to the area of the sector D O C.

(354.) The circumstances of the sun's apparent annual motion may, therefore, be summed up as follows:—It is performed in an orbit lying in one plane passing through the earth's centre, called the plane of the ecliptic, and whose projection on the heavens is the great circle so called. In this plane its motion is from west to east, or to a spectator looking down on the plane of the elliptic from the northern side, in a direction the reverse of that of the hands of a watch laid face uppermost. In this plane, however, the actual path is not circular, but elliptical; having the earth, not in its centre, but in one focus. The excentricity of this ellipse is 0.01679, in parts of a unit equal to the *mean distance*, or *half the longer diameter of the ellipse*; i.e. about one-sixtieth part of that semi-diameter; and the motion of the sun in its circumference is so regulated, that equal areas of the ellipse are passed over by the radius vector in equal times.

(355.) What we have here stated supposes no knowledge

of the sun's actual distance from the earth. nor, consequently, of the actual dimensions of its orbit, nor of the body of the sun itself. To come to any conclusions on these points, we must first consider by what means we can arrive at any knowledge of the distance of an object to which we have no access. Now, it is obvious, that its *parallax* alone can afford us any information on this subject. Suppose $P A B Q$ to represent the earth, C its centre, and S the sun, and A, B two situations of a spectator, or, which comes to the same thing, the stations of two spectators, both observing the sun S at the same instant. The spectator A will see it in the direction $A S a$, and will refer it to a point a in the infinitely distant sphere of the fixed stars, while the



spectator B will see it in the direction $B S b$, and refer it to b . The angle included between these directions, or the measure of the celestial arc $a b$, by which it is *displaced*, is equal to the angle $A S B$; and if this angle be known, and the local situations of A and B , with the part of the earth's surface $A B$ included between them, it is evident that the distance $C S$ may be calculated. Now, since $A S C$ (art. 339) is the parallax of the sun as seen from A , and $B S C$ as seen from B , the angle $A S B$, or the total apparent displacement is the sum of the two parallaxes. Suppose, then, two observers—one in the northern, the other in the southern hemisphere—at stations on the same meridian, to observe on the same day the meridian altitudes of the sun's

centre. Having thence derived the apparent zenith distances, and cleared them of the effects of refraction, if the distance of the sun were equal to that of the fixed stars, the sum of the zenith distances thus found would be precisely equal to the sum of the latitudes north and south of the places of observation. For the sum in question would then be equal to the angle $Z C X$, which is the meridional distance of the stations across the equator. But the effect of parallax being in both cases to increase the apparent zenith distances, their observed sum will be greater than the sum of the latitudes, by the sum of the two parallaxes, or by the angle $A S B$. This angle, then, is obtained by subtracting the sum of the north and south latitudes from that of the zenith distances; and this once determined, the horizontal parallax is easily found, by dividing the angle so determined by the sum of the sines of the two latitudes.

(356.) If the two stations be not exactly on the same meridian (a condition very difficult to fulfil), the same process will apply, if we take care to allow for the change of the sun's actual zenith distance in the interval of time elapsing between its arrival on the meridians of the stations. This change is readily ascertained, either from tables of the sun's motion, grounded on the experience of a long course of observations, or by actual observation of its meridional altitude on several days before and after that on which the observations for parallax are taken. Of course, the nearer the stations are to each other in longitude, the less is this interval of time, and, consequently, the smaller the amount of this correction; and, therefore, the less injurious to the accuracy of the final result is any uncertainty in the daily change of zenith distance which may arise from imperfection in the solar tables, or in the observations made to determine it.

(357.) The horizontal parallax of the sun has been concluded from observations of the nature above described, performed in stations the most remote from each other in latitude, at which observatories have been instituted. It has also been deduced from other methods of a more refined nature, and susceptible of much greater exactness, to be hereafter described. Its amount so obtained, is about $8''.6$. Minute as this quantity is, there can be no doubt that it is a tolerably correct approximation to the truth; and in conformity with it, we must admit the sun to be situated at a mean distance from us, of no less than 23984 times the length of the earth's radius, or about 95000000 miles. [See Note F.]

(358.) That at so vast a distance the sun should appear to us of the size it does, and should so powerfully influence our condition by its heat and light, requires us to form a very grand conception of its actual magnitude, and of the scale on which those important processes are carried on within it, by which it is enabled to keep up its liberal and unceasing supply of these elements. As to its actual magnitude we can be at no loss, knowing its distance, and the angles under which its diameter appears to us. An object, placed at the distance of 95000000 miles, and subtending an angle of $32' 1''$, must have a real diameter of 882000 miles. Such, then, is the diameter of this stupendous globe. If we compare it with what we have already ascertained of the dimensions of our own, we shall find that in linear magnitude it exceeds the earth in the proportion $111\frac{1}{2}$ to 1, and in bulk in that of 1384472 to 1.

(359.) It is hardly possible to avoid associating our conception of an object of definite globular figure, and of such enormous dimensions, with some corresponding attribute of

massiveness and material solidity. That the sun is not a mere phantom, but a body having its own peculiar structure and economy, our telescopes distinctly inform us. They show us dark spots on its surface, which slowly change their places and forms, and by attending to whose situation, at different times, astronomers have ascertained that the sun revolves about an axis nearly perpendicular to the plane of the ecliptic, performing one rotation in a period of about 25 days, and in the same direction with the diurnal rotation of the earth, *i.e.* from west to east. Here, then, we have an analogy with our own globe; the slower and more majestic movement only corresponding with the greater dimensions of the machinery, and impressing us with the prevalence of similar mechanical laws, and of, at least, such a community of nature as the existence of inertia and obedience to force may argue. Now, in the exact proportion in which we invest our idea of this immense bulk with the attribute of inertia, or weight, it becomes difficult to conceive its circulation round so comparatively small a body as the earth, without, on the one hand, dragging it along, and displacing it, if bound to it by some invisible tie; or, on the other hand, if not so held to it, pursuing its course alone in space, and leaving the earth behind. If we connect two solid masses by a rod, and fling them aloft, we see them circulate about a point between them, which is their common centre of gravity; but if one of them be greatly more ponderous than the other, this common centre will be proportionally nearer to that one, and even within its surface; so that the smaller one will circulate, in fact, about the larger, which will be comparatively but little disturbed from its place.

(360.) Whether the earth move round the sun, the sun round the earth, or both round their common centre of grav-

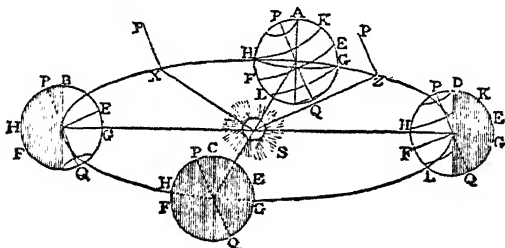
ity, will make no difference, so far as appearances are concerned, provided the stars be supposed sufficiently distant to undergo no sensible apparent *parallactic* displacement by the motion so attributed to the earth. Whether they are so or not must still be a matter of inquiry; and from the absence of any measurable amount of such displacement, we can conclude nothing but this, that the scale of the sidereal universe is so great, that the mutual orbit of the earth and sun may be regarded as an imperceptible point in comparison with the distance of its nearest members. Admitting, then, in conformity with the laws of dynamics, that two bodies connected with and revolving about each other in free space do, in fact, revolve about their common centre of gravity, which remains immovable by their mutual action, it becomes a matter of further inquiry, *whereabouts* between them this centre is situated. Mechanics teach us that its place will divide their mutual distance in the inverse ratio of their *weights* or *masses*;³ and calculations grounded on phenomena, of which an account will be given further on, inform us that this ratio, in the case of the sun and earth, is actually that of 354936 to 1—the sun being, in that proportion, more ponderous than the earth. From this it will follow that the common point about which they both circulate is only 267 miles from the sun's centre, or about $\frac{1}{354936}$ part of its own diameter.

(361.) Henceforward, then, in conformity with the above statements, and with the Copernican view of our system, we must learn to look upon the sun as the comparatively motionless centre about which the earth performs an annual elliptic orbit of the dimensions and excentricity, and with a

³ Principia, lib. i. lex. iii. cor. 14.

velocity, regulated according to the law above assigned; the sun occupying one of the foci of the ellipse, and from that station quietly disseminating on all sides its light and heat; while the earth travelling round it, and presenting itself differently to it at different times of the year and day, passes through the varieties of day and night, summer and winter, which we enjoy; its motion (art. 354) being from west to east.

(362.) In this annual motion of the earth, its axis preserves, at all times, the same direction as if the orbital movement had no existence; and is carried round parallel to itself, and pointing always to the same vanishing point in the sphere of the fixed stars. This it is which gives rise to the variety of seasons, as we shall now explain. In so doing, we shall neglect (for a reason which will be presently explained) the ellipticity of the orbit, and suppose it a circle, with the sun in the centre and the four quadrants of its orbit to be described in equal times, the motion in a circle being uniform.



(363.) Let, then, S represent the sun, and A, B, C, D, four positions of the earth in its orbit 90° apart, viz. A that which it has at the moment when the sun is opposite to the intersection of the plane of the ecliptic B G, with that of the equator F E; B that which it has a quarter of a year subsequently or 90° of longitude in advance of

A; C, 180° and D, 270° in advance of A. In each of these positions let P Q represent the axis of the earth, about which its diurnal rotation is performed without interfering with its annual motion in its orbit. Then, since the sun can only enlighten one-half of the surface at once, viz. that turned toward it, the shaded portions of the globe in its several positions will represent the dark, and the bright, the enlightened halves of the earth's surface in these positions. Now, 1st, in the position A, the sun is vertically over the intersection of the equinoctial F E and the ecliptic H G. It is, therefore, in the vernal equinox; and in this position the poles P, Q, both fall on the extreme confines of the enlightened side. In this position, therefore, it is day over half the northern and half the southern hemisphere at once; and as the earth revolves on its axis, every point of its surface describes half its diurnal course in light, and half in darkness; in other words, the duration of day and night is here equal over the whole globe: hence the term *equinox*. The same holds good at the autumnal equinox on the position C.

(364.) B is the position of the earth at the time of the *northern summer solstice*. (See art. 389.) Here the north pole P, and a considerable portion of the earth's surface in its neighborhood, as far as B, are situated *within* the enlightened half. As the earth turns on its axis in this position, therefore, the whole of that part remains constantly enlightened; therefore, at this point of its orbit, or at this season of the year, it is continual day at the north pole, and in all that region of the earth which encircles this pole as far as B—that is, to the distance of $23^\circ 27' 30''$ from the pole, or within what is called in geography the *arctic circle*. On the other hand, the opposite or south pole Q,

with all the region comprised within the *antarctic* circle, as far as $23^{\circ} 27' 30''$ from the south pole, are immersed at this season in darkness during the entire diurnal rotation, so that it is here continual night.

(365.) With regard to that portion of the surface comprehended between the arctic and antarctic circles, it is no less evident that the nearer any point is to the north pole, the larger will be the portion of its diurnal course comprised within the bright, and the smaller within the dark hemisphere; that is to say, the longer will be its day, and the shorter its night. Every station north of the equator will have a day of more and a night of less than twelve hours' duration, and *vice versâ*. All these phenomena are exactly inverted when the earth comes to the opposite point D of its orbit.

(366.) Now, the temperature of any part of the earth's surface depends mainly on its exposure to the sun's rays. Whenever the sun is above the horizon of any place, that place is receiving heat; when below, parting with it, by the process called radiation; and the whole quantities received and parted with in the year (secondary causes apart) must balance each other at every station, or the equilibrium of temperature (that is to say, the constancy which is observed to prevail in the annual averages of temperature as indicated by the thermometer) would not be supported. Whenever, then, the sun remains more than twelve hours above the horizon of any place, and less beneath, the general temperature of that place will be above the average; when the reverse, below. As the earth, then, moves from A to B, the days growing longer, and the nights shorter in the northern hemisphere, the temperature of every part of that hemisphere increases, and we pass from spring to summer;

while, at the same time, the reverse obtains in the southern hemisphere. As the earth passes from B to C, the days and nights again approach to equality—the excess of temperature in the northern hemisphere above the mean state grows less, as well as its defect in the southern; and at the autumnal equinox C, the mean state is once more attained. From thence to D, and, finally, round again to A, all the same phenomena, it is obvious, must again occur, but reversed—it being now winter in the northern and summer in the southern hemisphere.

(367.) All this is consonant to observed fact. The continual day within the polar circles in summer, and night in winter, the general increase of temperature and length of day as the sun approaches the elevated pole, and the reversal of the seasons in the northern and southern hemispheres, are all facts too well known to require further comment. The positions A, C of the earth correspond, as we have said, to the equinoxes; those at B, D to the *solstices*. This term must be explained. If, at any point, X, of the orbit, we draw X P the earth's axis, and X S to the sun, it is evident that the angle P X S will be the sun's *polar distance*. Now this angle is at its maximum in the position D, and at its minimum at B: being in the former case $= 90^\circ + 23^\circ 28' = 113^\circ 28'$, and in the latter $90^\circ - 23^\circ 28' = 66^\circ 32'$. At these points the sun ceases to approach to or to recede from the pole, and hence the name solstice.

(368 a.) Let us next consider how these phenomena are modified by the ellipticity of the earth's orbit and the position of its longer axis with respect to the line of the solstices. This ellipticity (art. 350) is about one-sixtieth of the mean distance, so that the sun, at its greatest proximity

is about one-thirtieth of its mean distance nearer us than when most remote. Since light and heat are equally dispersed from the sun in all directions, and are spread, in diverging, over the surface of a sphere enlarging as they recede from the centre, they must diminish in intensity according to the inverse proportion of the surfaces over which they are spread, *i.e.* in the inverse ratio of the squares of the distances. Hence the hemisphere opposed to the sun will receive in a given time, when nearest, two-thirtieths or one-fifteenth more heat and light than when most remote, as may be shown by an easy calculation.* Now, the sun's longitude when at its least distance from the earth (at which time it is said to be in *perigee* and the earth in its *perihelion*†) is at present $280^{\circ} 28'$ in which position it is on the 1st of January, or eleven days after the time of the *winter* solstice of the *northern* hemisphere; or, which is the same thing, the *summer* solstice of the *southern* (art. 364), while on the other hand the sun is most remote (in *apogee* or the earth in its *aphelion*‡), when in longitude $100^{\circ} 25'$ or on the 2d of July, *i.e.* eleven days after the epoch of the northern summer or southern winter solstice. We shall suppose, however, for simplicity of explanation, the perigee and apogee to be coincident with the solstice. At and about the southern summer solstice then, the whole earth is receiving *per diem* the greatest amount of heat that it can receive, and of this the southern hemisphere receives the larger share, because its pole and the whole region within the antarctic circle is in perpetual sunshine, while

* $\left(\frac{61}{30}\right)^2 = \left(\frac{63}{30}\right)^2$ very nearly, $= \left(\frac{31}{30}\right)^2 = \frac{961}{900} = \frac{96}{90}$ very nearly, $= \frac{16}{15}$.

† $\pi\epsilon\rho\epsilon$, about or in the neighborhood of; $\gamma\eta$, the earth; $\eta\lambda\iota\omicron\varsigma$, the sun.

‡ $\alpha\pi\omicron$, away from.

the corresponding northern regions lie in shadow. On the other hand, at and about the northern summer solstice, although it is true that the reverse conditions as to the regions illuminated prevail, yet the whole earth is then receiving *per diem* less heat owing to the sun's remoteness: so that on the whole *if the seasons were of equal duration*, or in other words, if the angular movement of the earth in its elliptic orbit were uniform, the southern hemisphere would receive more heat *per annum* than the northern, and would consequently have a warmer mean temperature.

(368 b.) Such, however, is not the case. The *angular* velocity of the earth in its orbit, as we have seen (art. 350), is not uniform, but varies in the inverse ratio of the square of the sun's distance, that is, in the same precise ratio as his heating power. The momentary supply of heat then received by the earth in every point of its orbit varies exactly as the momentary increase of its longitude, from which it obviously follows, that equal amounts of heat are received from the sun in passing over equal angles round it, in whatever part of the ellipse those angles may be situated. Supposing the orbit, then, to be divided into two segments by any straight line drawn through the sun, since equal angles in longitude (180°) are described on either side of this line, the amount of heat received will be equal. In passing then from either equinox to the other, the whole earth receives equal amount of heat, the inequality in the intensities of solar radiation in the two intervals being precisely compensated by the opposite inequality in the duration of the intervals themselves; which amounts to about $7\frac{1}{2}$ days, by which the northern spring and summer are together longer than the southern. For these intervals are to each other in the proportion of the two unequal seg-

ments of the whole ellipse into which the line of the equinoxes divides it. (See art. 353.)

(368 c.) In what regards the comfort of a climate and the character of its vegetation, the intensity of a summer is more naturally estimated by the temperature of its hottest day, and that of a winter by its sharpest frosts, than by the mere durations of those seasons and their total amount of heat. Supposing the excentricity of the earth's orbit were very much greater than it actually is; the position of its perihelion remaining the same; it is evident that the characters of the seasons in the two hemispheres would be strongly contrasted. In the northern, we should have a short but very mild winter with a long but very cool summer—*i.e.* an approach to perpetual spring; while the southern hemisphere would be inconvenienced and might be rendered uninhabitable by the fierce extremes caused by concentrating half the annual supply of heat into a summer of very short duration and spreading the other half over a long and dreary winter, sharpened to an intolerable intensity of frost when at its climax by the much greater remoteness of the sun.

(369.) As it is, the difference, except under peculiar circumstances, is not very striking, being masked to a certain extent by the action of another very influential cause to be explained in art. 370. This does not prevent, however, the direct impression of the solar heat in the height of summer—the glow and ardor of his rays, under a perfectly clear sky, at noon, in equal latitudes and under equal circumstances of exposure—from being materially greater in the southern hemisphere than in the northern. One-fifteenth is too considerable a fraction of the whole intensity of sunshine not to aggravate in a serious degree

the sufferings of those for instance who are exposed to it in thirsty deserts, without shelter. The accounts of these sufferings in the interior of Australia are of the most frightful kind, and would seem far to exceed what have ever been undergone by travellers in the northern deserts of Africa.*

(369 a.) It must be observed, moreover, that in estimating the effect of any additional fraction (as one-fifteenth) of solar radiation on temperature, we have to consider as our unit, not the number of degrees above a purely arbitrary zero point (such as the freezing-point of water or the zero of Fahrenheit's scale) at which a thermometer stands in a hot summer day, as compared with a cold winter one, but the thermometric interval between the temperatures it indicates in the two cases, and that which it would indicate did the sun not exist, which there is good reason to believe* would be at least as low as 239° *below zero* of Fahrenheit. And as a temperature of 100° Fahrenheit *above zero* is no uncommon one in a fair shade exposure under a sun nearly vertical, we have to take one-fifteenth of the sum of these intervals (339°), or 23° Fahrenheit, as the least variation of temperature under such circumstances which can reasonably be attributed to the actual variation of the sun's distance.

(369 b.) In what has been premised we have supposed the situation of the axis of the earth's orbit to coincide with the line of the solstices, neglecting the difference of about

* See the account of Captain Sturt's exploration in *Athenæum*, No. 1012. "The ground was almost a molten surface, and if a match accidentally fell upon it, it immediately ignited." The author has observed the temperature of the surface soil in South Africa as high as 159° Fahrenheit. An ordinary lucifer match does not ignite when simply pressed upon a smooth surface at 212° , but *in the act of withdrawing it*, it takes fire, and the slightest friction upon such a surface of course ignites it.

* See METEOROLOGY, *Encycl. Brit.* (new edition) Art. 36.

eleven days' motion at present existing between them. But this near coincidence has not always been the state of things, and will not always continue to be so. By the effect of precession (art. 312), both the line of equinoxes and those of solstices retreat on the ecliptic by an annual angular movement of $50''.1$, which cause alone would carry them round, with respect to the axis of the earth's ellipse through a complete revolution, in 25868 years. And in this period, supposing the axis to retain a fixed position, the perihelion would come to coincide successively in longitude with both the solstices and with both the equinoxes. But, besides this, owing to the operation of causes hereafter to be explained, the axis does not remain so fixed, but shifts its position, with a much slower angular movement, of $11''.8$ *per annum in the opposite direction* to that in which precession carries the line of equinoxes, and by which movement alone, if uniformly continued, the direction of the axis itself would be carried entirely round the whole circumference of the ecliptic in an immensely long period (no less than 109830 years). Thus then we see that the vernal equinox and the perihelion recede from each other by the joint annual amount of $61''.9$ or a degree in 58.16 years, which is, in effect, the same as if the perihelion made a complete revolution with reference to a fixed equinox in 20984 years. In consequence of this joint variation then, the place of the perihelion must have coincided with the vernal equinox (or have been situated in longitude 0°) about 4000 years before the Christian era, and in longitude 90° about A.D. 1250, and will be situated in longitudes 180° and 270° respectively about the years A.D. 6500 and 11700. At the latter of these epochs, the case we have considered in the foregoing articles (368 *a. et seq.*) will be reversed, and the

extreme summer and winter of the southern hemisphere will be transferred to the northern.

(369 c.) In the immense periods which geologists contemplate in the past history of the earth, this alternation of climates must have happened, not once only, but thousands of times, and it is not impossible that some of the indications which they have discovered of the prevalence at some former epoch or epochs of widely different climates from the present in the northern hemisphere, may be referable, in part at least, to this cause, though we are very far from supposing it competent (even taken in conjunction with other variations to be explained further on, which will sometimes go to exaggerate and sometimes to palliate its influence) to account for the whole of the changes which appear to have taken place.*

(370.) A conclusion of a very remarkable kind, recently drawn by Professor Dove from the comparison of thermometric observations at different seasons in very remote regions of the globe, may appear on first sight at variance with much that is above stated. That eminent meteorologist has shown, by taking at all seasons the mean of the temperatures of points diametrically opposite to each other, that the mean temperature of *the whole earth's surface* in June considerably exceeds that in December. This result,

* M. Reynaud (Extrait de Philosophie religieuse. Paris: Imprimerie Duvergne) attributes more influence to this cause, in historical times, than we should be disposed to allow it, when, for instance, he would explain by it the almost total disappearance of the date palm from Judæa since the time of Pliny, at which it appears to have flourished in perfection. At that epoch, however, the perihelion occupied a situation only 20° from the December solstice; which implies a difference between the sun's perihelial and solstitial distances not exceeding a thousandth part of its mean distance, corresponding to a difference of a five-hundredth part in the solar radiation. The effect of this, reckoned on the principles explained in the text, would not exceed two-thirds of a degree Fahr. in the midsummer temperature of Judæa at noon. See also his "Discours sur la Constitution physique de la Terre" (*Encyclopédie Nouvelle*).

which is at variance with the greater proximity of the sun in December, is, however, due to a totally different and very powerful cause—the greater amount of land in that hemisphere which has its summer solstice in June (*i.e.* the northern, see art. 362); and the fact is so explained by him. The effect of land under sunshine is to throw heat into the general atmosphere, and so distribute it by the carrying power of the latter over the whole earth. Water is much less effective in this respect, the heat penetrating its depths, and being there absorbed; so that the surface never acquires a very elevated temperature even under the equator.

(371.) The great key to simplicity of conception in astronomy, and, indeed, in all sciences where motion is concerned, consists in contemplating every movement as referred to points which are either permanently fixed, or so nearly so as that their motions shall be too small to interfere materially with and confuse our notions. In the choice of these primary points of reference, too, we must endeavor, as far as possible, to select such as have simple and symmetrical geometrical relations of situation with respect to the curves described by the moving parts of the system, and which are thereby fitted to perform the office of natural centres—advantageous stations for the eye of reason and theory. Having learned to attribute an orbital motion to the earth, it loses this advantage, which is transferred to the sun, as the fixed centre about which its orbit is performed. Precisely as, when embarrassed by the earth's diurnal motion, we have learned to transfer, in imagination, our station of observation from its surface to its centre, by the application of the diurnal parallax; so, when we come to inquire into the movements of the planets, we shall find ourselves continually embarrassed by the orbital motion of our point

of view, unless, by the consideration of the *annual or heliocentric parallax*, we consent to refer all our observations on them to the centre of the sun, or rather to the common centre of gravity of the sun, and the other bodies which are connected with it in our system.

(372.) Hence arises the distinction between the *geocentric* and *heliocentric* place of an object. The former refers its situation in space to an imaginary sphere of infinite radius, having the centre of the earth for its centre—the latter to one concentric with the sun. Thus, when we speak of the *heliocentric longitudes* and *latitudes* of objects, we suppose the spectator situated in the sun, and referring them by circles perpendicular to the plane of the ecliptic, to the great circle marked out in the heavens by the infinite prolongation of that plane.

(373.) The point in the imaginary concave of an infinite heaven, to which a spectator in the sun refers the earth, must, of course, be diametrically opposite to that to which a spectator on the earth refers the sun's centre; consequently the *heliocentric latitude* of the earth is always nothing, and *its heliocentric longitude always equal to the sun's geocentric longitude*+180°. The *heliocentric equinoxes* and *solstices* are, therefore, the same as the *geocentric* reversely named; and to conceive them, we have only to imagine a plane passing through the sun's centre, parallel to the earth's equator, and prolonged infinitely on all sides. The line of intersection of this plane and the plane of the ecliptic is the line of equinoxes, and the solstices are 90° distant from it.

(374.) Were the earth's orbit a circle, described with a uniform velocity about the sun placed in its centre, nothing could be easier than to calculate its position at any time with respect to the line of equinoxes, or its longitude, for

we should only have to reduce to numbers the proportion following; viz. One year : the time elapsed :: 360° : the arc of longitude passed over. The longitude so calculated is called in astronomy the *mean* longitude of the earth. But since the earth's orbit is neither circular, nor uniformly described, this rule will not give us the true place in the orbit at any proposed moment. Nevertheless, as the eccentricity and deviation from a circle are small, the *true place* will never deviate very far from that so determined (which, for distinction's sake, is called the *mean place*), and the former may at all times be calculated from the latter, by applying to it a *correction* or *equation* (as it is termed), whose amount is never very great, and whose computation is a question of pure geometry, depending on the equable description of areas by the earth about the sun. For since, in elliptic motion according to Kepler's law above stated, *areas* not *angles* are described uniformly, the proportion must now be stated thus;—One year : the time elapsed :: the whole *area* of the ellipse : the *area* of the sector swept over by the radius vector in that time. This area, therefore, becomes known, and it is then, as above observed, a problem of pure geometry to ascertain the *angle* about the sun (X S Z, *fig.* art. 362), which corresponds to any proposed fractional area of the whole ellipse supposed to be contained in the sector X Z S. Suppose we set out from X, the perihelion, then will the angle X S Z at first increase more rapidly than the *mean longitude*, and will, therefore, during the whole semi-revolution from A to M, exceed it in amount; or, in other words, the *true place* will be in advance of the *mean*: at M, one half the year will have elapsed, and one half the orbit have been described, whether it be circular or elliptic. Here, then, the *mean* and *true*

places coincide; but in all the other half of the orbit, from M to A, the true place will fall short of the mean, since at M the angular motion is slowest, and the true place from this point begins to lag behind the mean—to make up with it, however, as it approaches A, where it once more overtakes it.

(375.) The quantity by which the *true* longitude of the earth differs from the *mean* longitude is called the equation of the centre, and is *additive* during all the half-year in which the earth passes from A to M, beginning at $0^{\circ} 0' 0''$, increasing to a maximum, and again diminishing to zero at M; after which it becomes *subtractive*, attains a maximum of subtractive magnitude between M and A, and again diminishes to 0 at A. Its maximum, both additive and subtractive, is $1^{\circ} 55' 33''.3$.

(376.) By applying, then, to the earth's mean longitude the equation of the centre corresponding to any given time at which we would ascertain its place, the true longitude becomes known; and since the sun is always seen from the earth in 180° more longitude than the earth from the sun, in this way also the sun's true place in the ecliptic becomes known. The calculation of the equation of the centre is performed by a table constructed for that purpose, to be found in all "Solar Tables."

(377.) The maximum value of the equation of the centre depends only on the ellipticity of the orbit, and may be expressed in terms of the excentricity. *Vice versâ*, therefore, if the former quantity can be ascertained by observation, the latter may be derived from it; because, whenever the law, or numerical connection, between two quantities is known, the one can always be determined from the other. Now, by assiduous observation of the sun's transits over the merid-

ian, we can ascertain, for every day, its exact right ascension, and thence conclude its longitude (art. 309). After this, it is easy to assign the angle by which this *observed* longitude exceeds or falls short of the mean; and the greatest amount of this excess or defect which occurs in the whole year is the maximum equation of the centre. This, as a means of ascertaining the excentricity of the orbit, is a far more easy and accurate method than that of concluding the sun's distance by measuring its apparent diameter. The results of the two methods coincide, however, perfectly.

(378.) If the ecliptic coincided with the equinoctial, the effect of the equation of the centre, by disturbing the uniformity of the sun's apparent motion in longitude, would cause an inequality in its time of coming on the meridian on successive days. When the sun's centre comes to the meridian, it is *apparent noon*, and if its motion in longitude were uniform, and the ecliptic coincident with the equinoctial, this would always coincide with the *mean noon*, or the stroke of 12 on a well-regulated solar clock. But, independent of the want of uniformity in its motion, the obliquity of the ecliptic gives rise to another inequality in this respect; in consequence of which, the sun, even supposing its motion in the ecliptic uniform, would yet alternately, in its time of attaining the meridian, anticipate and fall short of the mean noon as shown by the clock. For the right ascension of a celestial object forming a side of a right-angled spherical triangle, of which its longitude is the hypotenuse, it is clear that the uniform increase of the latter must necessitate a deviation from uniformity in the increase of the former.

(379.) These two causes, then, acting conjointly, produce, in fact, a very considerable fluctuation in the time as

shown per clock, when the sun really attains the meridian. It amounts, in fact, to upward of half an hour; apparent noon sometimes taking place as much as $16\frac{1}{2}$ min. before mean noon, and at others as much as $14\frac{1}{2}$ min. after. This difference between apparent and mean noon is called the *equation of time*, and is calculated and inserted in ephemerides for every day of the year, under that title: or else, which comes to the same thing, the moment, *in mean time*, of the sun's culmination for each day, is set down as an astronomical phenomenon to be observed.

(380.) As the sun, in its apparent annual course, is carried along the ecliptic, its declination is continually varying between the extreme limits of $23^{\circ} 27' 30''$ north, and as much south, which it attains at the solstices. It is consequently always vertical over some part or other of that zone or belt of the earth's surface which lies between the north and south parallels of $23^{\circ} 27' 30''$. These parallels are called in geography the *tropics*; the northern one that of *Cancer*, and the southern, of *Capricorn*; because the sun, at the respective solstices, is situated in the divisions, or signs of the ecliptic so denominated. Of these signs there are twelve, each occupying 30° of its circumference. They commence at the vernal equinox, and are named in order—Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces.¹⁰ They are denoted also by the following symbols:—♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓. Longitude itself is also divided into signs, degrees, and minutes, etc. Thus $5^{\circ} 27' 0''$ corresponds to $177^{\circ} 0'$.

¹⁰ They may be remembered by the following memorial hexameters:—
Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo,
Libraque, Scorpius, Arcitenens, Caper, Amphora, Pisces.

(381.) These *Signs* are purely technical subdivisions of the ecliptic, commencing from the actual equinox, and are not to be confounded with the *constellations* so called (and sometimes so symbolized). The constellations of the zodiac, as they now stand arranged on the ecliptic, are all a full "sign" in advance or anticipation of their symbolic cognomens thereon marked. Thus the constellation Aries actually occupies the sign Taurus ♉, the constellation Taurus, the sign Gemini II, and so on, the *signs* having retreated¹¹ among the stars (together with the equinox their origin), by the effect of precession. The bright star Spica in the constellation Virgo (α Virginis), by the observations of Hipparchus, 128 years B.C., *preceded*, or was westward of the autumnal equinox in longitude by 6°. In 1750 it followed or stood eastward of the same equinox by 20° 21'. Its place then, as referred to the ecliptic at the former epoch, would be in longitude 5° 24' 0", or in the 24th degree of the *sign* ♍, whereas in the latter epoch it stood in the 21st degree of ♍, the equinox having retreated by 26° 21' in the interval, 1878 years, elapsed. To avoid this source of misunderstanding, the use of "signs" and their symbols in the reckoning of celestial longitudes is now almost entirely abandoned, and the ordinary reckoning (by degrees, etc., from 0 to 360) adopted in its place, and the names Aries, Virgo, etc., are becoming restricted to the constellations so called.¹²

(382.) When the sun is in either tropic, it enlightens, as we have seen, the pole on that side the equator, and shines

¹¹ *Retreated* is here used with reference to *longitude*, not to the apparent diurnal motion.

¹² When, however, the place of the sun is spoken of, the old usage prevails. Thus, if we say "the sun is in Aries," it would be interpreted to mean between 0° and 30° of longitude. So, also, "the first point of Aries" is still understood to mean the vernal, and "the first point of Libra," the autumnal equinox; and so in a few other cases.

over or beyond it to the extent of $23^{\circ} 27' 30''$. The parallels of latitude, at this distance from either pole, are called the *polar circles*, and are distinguished from each other by the names *arctic* and *antarctic*. The regions within these circles are sometimes termed *frigid zones*, while the belt between the tropics is called the *torrid zone*, and the intermediate belts *temperate zones*. These last, however, are merely names given for the sake of naming; as, in fact, owing to the different distribution of land and sea in the two hemispheres, zones of *climate* are not co-terminal with zones of *latitude*.

(383.) Our seasons are determined by the apparent passages of the sun across the equinoctial, and its alternate arrival in the northern and southern hemisphere. Were the equinox invariable, this would happen at intervals precisely equal to the duration of the sidereal year; but, in fact, owing to the slow conical motion of the earth's axis described in art. 317, the equinox retreats on the ecliptic, and *meets* the advancing sun somewhat *before* the whole sidereal circuit is completed. The annual retreat of the equinox is $50''\cdot1$, and this arc is described by the sun in the ecliptic in $20^m 19^s\cdot9$. By so much *shorter*, then, is the periodical return of our seasons than the true sidereal revolution of the earth round the sun. As the latter period, or sidereal year, is equal to $365^d 6^h 9^m 9^s\cdot6$, it follows, then, that the former must be only $365^d 5^h 48^m 49^s\cdot7$; and this is what is meant by the *tropical* year.

(384.) We have already mentioned that the longer axis of the ellipse described by the earth has a slow motion of $11''\cdot8$ per annum in advance. From this it results, that when the earth, setting out from the perihelion, has completed one sidereal period, the perihelion will have moved

forward by $11^{\circ} 8'$, which are must be described by the earth before it can again reach the perihelion. In so doing, it occupies $4^{\text{m}} 39^{\text{s}} \cdot 7$, and this must therefore be added to the sidereal period, to give the interval between two consecutive returns to the perihelion. This interval, then, is $365^{\text{d}} 6^{\text{h}} 13^{\text{m}} 49^{\text{s}} \cdot 3$,¹³ and is what is called the *anomalous year*. All these periods have their uses in astronomy; but that in which mankind in general are most interested is the *tropical year*, on which the return of the seasons depends, and which we thus perceive to be a compound phenomenon, depending chiefly and directly on the annual revolution of the earth round the sun, but subordinately also, and indirectly, on its rotation round its own axis, which is what occasions the precession of the equinoxes; thus affording an instructive example of the way in which a motion, once admitted in any part of our system, may be traced in its influence on others with which at first sight it could not possibly be supposed to have anything to do.

(§85.) As a rough consideration of the appearance of the earth points out the general roundness of its form, and more exact inquiry has led us first to the discovery of its elliptic figure, and, in the further progress of refinement, to the perception of minuter local deviations from that figure; so, in investigating the solar motions, the first notion we obtain is that of an orbit, generally speaking, round, and not far from a circle, which, on more careful and exact examination, proves to be an ellipse of small excentricity, and described in conformity with certain laws, as above stated. Still minuter inquiry, however, detects yet smaller deviations

¹³ These numbers, as well as most of the other numerical data of our system, are taken from Mr. Baily's *Astronomical Tables and Formulae*.

again from this form and from these laws, of which we have a specimen in the slow motion of the axis of the orbit spoken of in art. 372; and which are generally comprehended under the name of perturbations and secular inequalities. Of these deviations, and their causes, we shall speak hereafter at length. It is the triumph of physical astronomy to have rendered a complete account of them all, and to have left nothing unexplained, either in the motions of the sun or in those of any other of the bodies of our system. But the nature of this explanation cannot be understood till we have developed the law of gravitation, and carried it into its more direct consequences. This will be the object of our three following chapters; in which we shall take advantage of the proximity of the moon, and its immediate connection with and dependence on the earth, to render it, as it were, a stepping-stone to the general explanation of the planetary movements. We shall conclude this by describing what is known of the physical constitution of the sun.

(386.) When viewed through powerful telescopes, provided with colored glasses, to take off the heat, which would otherwise injure our eyes, the sun is observed to have frequently large and perfectly black spots upon it, surrounded with a kind of border, less completely dark, called a penumbra. Some of these are represented at *a*, *b*, *c*, *d*, in Plate I. fig. 2, at the end of this volume. They are, however, not permanent. When watched from day to day, or even from hour to hour, they appear to enlarge or contract, to change their forms, and at length to disappear altogether, or to break out anew in parts of the surface where none were before. In such cases of disappearance, the central dark spot always contracts into a point, and vanishes before the bor-

der. Occasionally they break up, or divide into two or more, and in those cases offer every evidence of that extreme mobility which belongs only to the fluid state, and of that excessively violent agitation which seems only compatible with the atmospheric or gaseous state of matter. The scale on which their movements take place is immense. A single second of angular measure, as seen from the earth, corresponds on the sun's disk to 461 miles; and a circle of this diameter (containing therefore nearly 167000 square miles) is the least space which can be distinctly discerned on the sun as a *visible area*. Spots have been observed, however, whose linear diameter has been upward of 45000 miles;¹⁴ and even, if some records are to be trusted, of very much greater extent.¹⁵ That such a spot should close up in six weeks' time (for they seldom last much longer), its borders must approach at the rate of more than 1000 miles a day.

(387.) Many other circumstances tend to corroborate this view of the subject. The part of the sun's disk not occupied by spots is far from uniformly bright. Its *ground* is finely mottled with an appearance of minute, dark dots, or *pores*, which, when attentively watched, are found to

¹⁴ Mayer, Obs. Mar. 15, 1758. "Ingens macula in sole conspiciebatur, cujus diameter = $\frac{1}{20}$ diam. solis."

¹⁵ Half the sun's disk is said in certain ancient annals to have been obscured by spots. This is monstrous—but on at least two occasions before the invention of telescopes spots have been seen with the naked eye. M. Gautier (Bibl. Univ. de Geneve, July and Aug., 1852) mentions as one of the largest spots on record, that observed by Sir W. Herschel, in 1779, which he there states to have been 470" in diameter, or 15·7 times that of the earth. I have not been able to verify the citation. The great spot of 1779 mentioned by Sir W. H. (Phil. Tr. 1795) as having been seen with the naked eye, consisted he says, of two parts, the largest of which "measured 1' 8"·06 in diameter, which is equal in length to more than 31000 miles." "Both together," he adds, "must certainly have extended above 50000." This corresponds to 113" which is not a fourth part of M. Gautier's quantity; moreover, 470" on the sun's disk corresponds, not to 15·7, but to 27·3 diameters of the earth.—(Note added in 1858.)

be in a constant state of change. There is nothing which represents so faithfully this appearance as the slow subsidence of some flocculent chemical precipitates in a transparent fluid, when viewed perpendicularly from above: so faithfully, indeed, that it is hardly possible not to be impressed with the idea of a luminous medium intermixed, but not confounded, with a transparent and non-luminous atmosphere, either floating as clouds in our air, or pervading it in vast sheets and columns like flame, or the streamers of our northern lights, directed in lines perpendicular to the surface.¹⁶ [See § 387 *a*, *b*, and *cc*, Note G.]

(388.) Lastly, in the neighborhood of great spots, or ex-

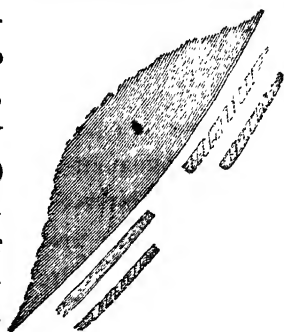
¹⁶ The light emanating immediately from the sun shows no sign of polarization whether radiating from the central or circumferential portions of its disk. This has been adduced as affording a direct experimental proof of the gaseous nature of the surface from which its light proceeds. It is argued that the light emitted by incandescent solid or fluid terrestrial bodies at great obliquities to their surfaces, is always found to be partially polarized in a plane perpendicular to that in which the angle of emanation lies, and that consequently such cannot be the nature of the solar surface. In former editions of this work, I have passed this argument *sub silentio*, and should not have thought it necessary now to enter a protest against its validity (resting as it does on authority of one of the greatest names in optical science), but that I find it prominently put forward and repeated and strongly insisted on in recent works of conspicuous merit. (Kosmos, *passim*, especially vol. iii. pp. 47, 284, and notes 99, 483, transl.; Gautier on the Sun, Bibl. Univ. 1852; Vaughan, Rep. Brit. Assoc. 1857; Athenæum, No. 1560, etc.) The fallacy consists in the assumption that the surface from which the light emanates at the borders of the sun *is* necessarily very oblique to the visual ray by which we see it; which, though true of the *general* surface regarded as a portion of a sphere 880,000 miles in diameter, is not so of each particular square foot or square inch which, not being obscured from sight by intervening protuberances, may send out rays to reach the eye of a terrestrial spectator. Supposing the sun to be an incandescent solid *not more rough than the earth or the moon*, it is obvious that whether from the centre or from the borders, the light by which we see it must consist of a mixture of rays emergent from the *local surface* at every possible angle of obliquity and in every possible plane without the smallest preference. A luminous portion of the sun's surface occupying the ten thousandth part of a square second, would correspond to a sectional area of the visual beam upward of twenty square miles in extent, admitting every variety of plain, precipice, slope, or rugged ground. The *general* surface of a forest seen on the horizon is parallel to the mathematical horizon, but who would assert that the ray by which its extreme visible leaf is seen, necessarily emanates from that leaf at any one obliquity or in any one plane of emergence rather than any other?—(Note added in 1853.)

tensive groups of them, large spaces of the surface are often observed to be covered with strongly marked curved or branching streaks, more luminous than the rest, called *faculae*, and among these, if not already existing, spots frequently break out. They may, perhaps, be regarded with most probability as the ridges of immense waves in the luminous regions of the sun's atmosphere, indicative of violent agitation in their neighborhood. They are most commonly, and best seen, toward the borders of the visible disk, and their appearance is as represented in Plate I. fig. 1.

(389.) But what *are* the spots? Many fanciful notions have been broached on this subject, but only one seems to have any degree of physical probability, viz. that they are the dark, or at least comparatively dark, solid body of the sun itself, laid bare to our view by those immense fluctuations in the luminous regions of its atmosphere, to which it appears to be subject. Respecting the manner in which this disclosure takes place, different ideas again have been advocated. Lalande (art. 3240) suggests, that eminences in the nature of mountains are actually laid bare, and project above the luminous ocean, appearing black above it, while their shoaling declivities produce the penumbra, where the luminous fluid is less deep. A fatal objection to this theory is the uniform shade of the penumbra and its sharp termination, both inward, where it joins the spot, and outward, where it borders on the bright surface. A more probable view has been taken by Sir William Herschel,¹⁷ who considers the luminous strata of the atmosphere to be sustained far above the level of the solid body by a transparent elastic medium, carrying on its upper surface (*or rather, to avoid*

¹⁷ Phil. Trans. 1801.

the former objection, *at some considerably lower level within its depth*) a cloudy stratum which, being strongly illuminated from above, reflects a considerable portion of the light to our eyes, and forms a penumbra, while the solid body shaded by the clouds reflects none. (See *fig.*) The temporary removal of both the strata, but more of the upper than the lower, he supposes effected by powerful upward currents of the atmosphere, arising, perhaps, from spiracles in the body, or from local agitations.



(389 *a.*) Such was the state of our knowledge of the appearance and constitution of the solar spots at the time when this work first issued from the press. But in 1851, a further step toward penetrating the mystery of their nature was made by that excellent and indefatigable observer Mr. Dawes, who availing himself of the ingenious contrivance described in art. 204 *e*, has been enabled to scrutinize the interior of the penumbrae of the spots, under high magnifying powers, in perfect security and with all the advantage which the absence of extraneous glare confers on the examination of feebly illuminated objects. So viewed, he has found the blacker portions occupying the middle of the penumbra, and which to former observers appeared so dark and so uniform as to lead them to believe it to be the sun's actual surface seen through an aperture in an exterior envelope—to be, itself, only an additional and inferior stratum of very feebly luminous (or illuminated) matter, which he has called “the cloudy stratum,” which again in its turn is frequently seen to be pierced with a smaller and usually

much more rounded aperture, which would seem at length to afford a view of the real solar surface of most intense blackness. Figs. 4, 5, Plate I., represent spots so seen on 23d December, 1851, and 17th January, 1852. In tracing the changes in the spots, from day to day, Mr. Dawes has also been led to conclude, that, in many instances, they have a movement of rotation about their own centres. This was particularly remarkable in the spot of 17th January, which between that date and 23d January had revolved in its own plane through an angle of more than 90° ; the "cloudy stratum," which its central aperture, presenting itself under the aspect represented at *b* fig. 5, instead of that at *a*, which it originally had, its general form remaining all the while unchanged.

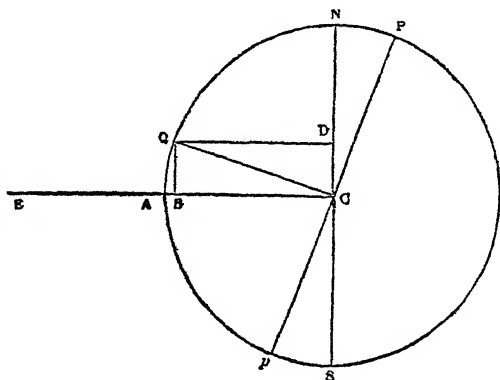
(390.) When the spots are attentively watched, their situation on the disk of the sun is observed to change. They advance regularly toward its western limb or border, where they disappear, and are replaced by others which enter at the eastern limb, and which, pursuing their respective courses, in their turn disappear at the western. The apparent rapidity of this movement is not uniform, as it would be were the spots dark bodies passing, by an independent motion of their own, between the earth and the sun; but is swiftest in the middle of their paths across the disk, and very slow at its borders. This is precisely what would be the case supposing them to appertain to and make part of the visible surface of the sun's globe, and to be carried round by a uniform rotation of that globe on its axis, so that each spot should describe a circle parallel to the sun's equator, rendered elliptic by the effect of perspective. Their apparent paths also across the disk conform to this view of their nature, being, generally speaking, ellipses,

much elongated, concentric with the sun's disk, each having one of its chords for its longer axis, and all these axes parallel to each other. At two periods of the year only do the spots appear to describe straight lines, viz. on and near to the 4th of June and 5th of December, on which days, therefore, the plane of the circle, which a spot situated on the sun's equator describes (and consequently, the plane of that equator itself), passes through the earth. Hence it is obvious, that the plane of the sun's equator is inclined to that of the ecliptic, and intersects it in a line which passes through the place of the earth on these days. The situation of this line, or *the line of the nodes of the sun's equator* as it is called, is, therefore, defined by the longitudes of the earth as seen from the sun at those epochs, which, according to Mr. Carrington, are respectively $73^{\circ} 40'$ and $253^{\circ} 40'$ ($=73^{\circ} 40' + 180^{\circ}$) for 1850, being, of course, diametrically opposite in direction.

(391.) The inclination of the sun's axis (that of the plane of its equator) to the ecliptic is determined by ascertaining the proportion of the longer and the shorter diameter of the apparent ellipse described by any remarkable, well-defined spot; in order to do which, its apparent place on the sun's disk must be very precisely ascertained by micrometric measures, repeated from day to day as long as it continues visible (usually about 12 or 13 days, according to the magnitude of the spots, which always vanish by the effect of foreshortening before they attain the actual border of the disk—but the larger spots being traceable closer to the limb than the smaller¹⁸). The *reduction* of such observations, or

¹⁸ The great spot of December, 1719, is stated to have been seen as a notch in the limb of the sun.

the conclusion from them of the element in question, is complicated with the effect of the earth's motion in the interval of the observations, and with its situation in the ecliptic, with respect to the line of nodes. For simplicity, we will suppose the earth situated as it is on the 4th of March, in a line at right angles to that of the nodes, *i.e.* in the heliocentric longitude $163^{\circ} 40'$, and to remain there stationary during the whole passage of a spot across the disk. In this case the axis of rotation of the sun will be situated in a plane passing through the earth and at right



angles to the plane of the ecliptic. Suppose C to represent the sun's centre, P C *p* its axis, E C the line of sight, P N Q A *p* S a section of the sun passing through the earth, and Q a spot situated on its equator, and in that plane, and consequently in the middle of its apparent path across the disk. If the axis of rotation were perpendicular to the ecliptic, as N S, this spot would be at A, and would be seen projected on C, the centre of the sun. It is actually at Q, projected upon D, at an apparent distance C D to the *north* of the centre, which is the apparent smaller semi-axis of the ellipse

described by the spot, which being known by micrometric measurement, the value of $\frac{CD}{CN}$ or the cosine of QCN , the inclination of the sun's equator becomes known, CN being the apparent semi-diameter of the sun at that time. At this epoch, moreover, the northern half of the circle described by the spot is visible (the southern passing behind the body of the sun), and the south pole p of the sun is within the visible hemisphere. This is the case in the whole interval from December 5th to June 4th, during which the visual ray falls upon the southern side of the sun's equator. The contrary happens in the other half year, from June 4th to December 5th, and this is what is understood when we say that the *ascending* node (denoted Ω) of the sun's equator lies in $73^\circ 40'$ longitude—a spot on the equator passing that node being then in the act of *ascending* from the southern to the northern side of the plane of the ecliptic—such being the conventional language of astronomers in speaking of these matters.

(392.) If the observations are made at other seasons (which, however, are the less favorable for this purpose the more remote they are from the epochs here assigned); when, moreover, as in strictness is necessary, the motion of the earth in the interval of the measures is allowed for (as for a change of the point of sight); the calculations requisite to deduce the situation of the axis in space, and the duration of the revolution around it, become much more intricate, and it would be beyond the scope of this work to enter into them ¹⁹ According to Mr. Carrington's deter-

¹⁹ See the theory in Lalande's *Astronomy*, art. 3258, and the formulæ of computation in a paper by Petersen, *Schumacher's Nachrichten*, No. 419.

mination, the inclination of the sun's equator to the ecliptic is about $7^{\circ} 15'$ (its nodes being as above stated), and the period of rotation 25 days 9 hours 7 minutes; the corresponding synodic period being 27 days 6 hours 36 minutes.²⁰

(393.) The region of the spots is confined, generally speaking, within about 25° on either side of the sun's equator; beyond 30° they are very rarely seen; in the polar regions, never. The actual equator of the sun is also less frequently visited by spots than the adjacent zones on either side, and a very material difference in their frequency and magnitude subsists in its northern and southern hemisphere, those on the northern preponderating in both respects. The zone comprised between the 11th and 15th degree to the northward of the equator is particularly fertile in large and durable spots. These circumstances, as well as the frequent occurrence of a more or less regular arrangement of the spots, when numerous, in the manner of belts parallel to the equator, point evidently to physical peculiarities in certain parts of the sun's body more favorable than in others to the production of the spots, on the one hand; and on the other, to a general influence of its rotation on its axis as a determining cause of their distribution and arrangement, and would appear indicative of a system of movements in the fluids which constitute its luminous surface bearing no remote analogy to our trade winds—from whatever cause arising. (See art. 239 *et seq.*)

(394.) The duration of individual spots is commonly not

²⁰ These periods are those of a spot in heliographic latitude 15° N. or S. of the sun's equator. Owing to solar atmospheric drift, the periods of rotation deduced from observations of spots in high or low heliographic latitudes differ considerably. [See Note G, § 387 *c c.*]

great; some are formed and disappear within the limit of a single transit across the disk—but such are for the most part small and insignificant. Frequently they make one or two revolutions, being recognized at their reappearance by their situation with respect to the equator, their configurations *inter se*, their size, or other peculiarities, as well as by the interval elapsing between their disappearance at one limb and reappearance on the other. In a few rare cases, however, they have been watched round many revolutions. The great spot of 1779 appeared during six months, and one and the same *groupe* was observed in 1840 by Schwabe to return eight times.²¹ It has been surmised, with considerable apparent probability, that some spots, at least, are generated again and again, at distant intervals of time, over the same identical points of the sun's body (as hurricanes, for example, are known to affect given localities on the earth's surface, and to pursue definite tracks). The uncertainty which still prevails with respect to the exact duration of its rotation renders it very difficult to obtain convincing evidence of this; nor, indeed, can it be expected, until by bringing together into one connected view the recorded state of the sun's surface during a very long period of time, and comparing together remarkable spots which have appeared *on the same parallel*, some precise periodic time shall be found which shall exactly conciliate numerous and well-characterized appearances. The inquiry is one of singular interest, as there can be no reasonable doubt that the supply of light and heat afforded to our globe stands in intimate connection with those processes which are taking

²¹ Schum. Nach. No. 418, p. 150. The recent papers of Biela, Capocci, Schwabe, Pastorff and Schmidt, in that collection, will be found highly interesting.

place on the solar surface, and to which the spots in some way or other owe their origin.

(394 a.) Meanwhile M. Schwabe, of Dessau, by comparing together the records of the general state of the sun's surface in respect of the abundance and paucity of spots exhibited by it from 1826 to 1850, has been led to a highly remarkable conclusion, viz. that their degree of copiousness is subject to a law of periodicity; alternate minima and maxima recurring at nearly equal intervals. The interval from minimum to minimum, as well as could be ascertained from the moderate interval embraced by the observations compared, was provisionally estimated by M. Schwabe at about ten years. More recently, M. Wolf, of Berne,²² from a careful assemblage and discussion of all the recorded observations of spots which could be collected from their first telescopic discovery (by Fabricius and Harriot, in 1610) to the present time, while fully confirming their periodicity, has fixed upon the somewhat longer period, from minimum to minimum, of $11^{\text{y}}.11$, being exactly at the rate of nine periods per century, the last year of each century (1700, 1800, etc.) being a year of minimum. In the minima there is for the most part an extreme paucity, and sometimes an entire absence of spots.²³ The maxima (in which they are often so copious that 50 or 100 have been counted at once on the disk) do not appear to fall exactly in the middle year between the minima, but rather earlier, about the fifth, fourth, or even the third year of the period. What is extremely remarkable, and must certainly be received as strongly corroborative, both of the

²² Rudolf Wolf. Transactions of Society of Nat. Phil. Berne. 1852.

²³ As in 1856.

general fact of periodicity and of the correctness of M. Wolf's period, is, that we find recorded in history by chroniclers and annalists on several occasions before the invention of telescopes, the appearance of spots, or groups of spots, so considerable as to have become matter of vulgar observation, as for instance in the years A.D. 807, 840, 1096 and 1607,²⁴ and several others in which, though no spots are recorded, a great deficiency in the sun's light has been remarked. Thus in the annals of the year A.D. 536, the sun is said to have suffered a great diminution of light, which continued fourteen months. From October, A.D. 626, to the following June, a defalcation of light to the extent of one-half is recorded; and in A.D. 1547, during three days, the sun is said to have been so darkened that stars were seen in the day time. Now of all these instances, supposing them all to have been owing to spots, either unusually large or numerous, there are only two, those of A.D. 807 and 1607, which deviate so much as two years from the epochs of maximum fixed as above.

(394 *b.*) Sir W. Herschel (Ph. Tr. 1801), considering the appearance of abundant spots on the sun's disk as evidence of an agitated state of its gaseous envelope, and regarding the extrication of light and heat as the results of chemical processes likely to be promoted by the more intimate mixture of heterogeneous materials having mutual affinities, has attempted to show, though from very imperfect records (such as alone could be procured by him at that date) that years of remarkably abundant or deficient spots have been also remarkable respectively for their high or low general

²⁴ Those spots were taken for planets seen on the sun; that of 840 for Venus; those of 807 and 1607 for Mercury.

temperature, and especially for abundant and deficient harvests. The point has been inquired into by M. Gautier,²⁵ who from an assemblage of meteorological averages obtained in thirty-three stations in Europe, and twenty-nine in America during eleven years of observation, finds a trifling preponderance ($0^{\circ}\cdot 11$ Fahr.) in the opposite direction. On the other hand M. Wolf, in the memoir above cited, from an examination of the Chronicles of Zurich from the year A.D. 1000 to A.D. 1800, is led to a conclusion in accordance with this speculation, and considers them as affording decisive evidence "that years rich in solar spots are in general drier and more fruitful than those of an opposite character, while the latter are wetter and more stormy than the former."

(394 c.) Although more properly belonging to the domain of general physics than of astronomy, it is impossible to omit mentioning here the singular coincidence of this period of the recurrence of the solar spots with that of those great disturbances in the magnetic system of the earth to which the epithet of "magnetic storms" has been affixed. These disturbances, during which the magnetic needle is greatly and universally agitated (not in a particular limited locality, but at one and the same instant of time over whole continents, or even over the whole earth), are found, so far as observation has hitherto extended, to maintain a parallel both in respect of their frequency of occurrence and intensity in successive years with the abundance and magnitude of the spots in the same years, too close to be regarded as fortuitous. The coincidence of the epochs of maxima and minima in the two series of phenomena amounts indeed to

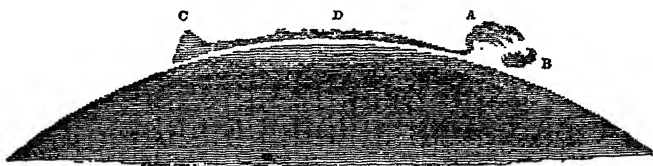
²⁵ Bibl. Univ. de Geneve. 1844.

identity, a fact evidently of most important significance, but which neither astronomical nor magnetic science is yet sufficiently advanced to interpret.

(395.) Above the luminous surface of the sun, and the region in which the spots reside, there are strong indications of the existence of a gaseous atmosphere having a somewhat imperfect transparency. When the whole disk of the sun is seen at once through a telescope magnifying moderately enough to allow it, and with a darkening glass such as to suffer it to be contemplated with perfect comfort, it is very evident that the borders of the disk are much less luminous than the centre. That this is no illusion is shown by projecting the sun's image undarkened and moderately magnified, so as to occupy a circle two or three inches in diameter, on a sheet of white paper, taking care to have it well in focus, when the same appearance will be observed.²⁶ This can only arise from the circumferential rays having undergone the absorptive action of a much greater thickness of some imperfectly transparent envelope (due to greater obliquity of their passage through it) than the central.—But a still more convincing and indeed decisive evidence is offered by the phenomena attending a total eclipse of the sun. Such eclipses (as will be shown hereafter) are produced by the interposition of the dark body of the moon between the earth and sun, the moon being large enough to cover and

²⁶ This has been denied by Arago on the evidence of certain phenomena observed with his "polariscope"; but the fact is so palpable, that it is matter of some astonishment that it could ever fail to strike the most superficial observer. The matter has been placed beyond a doubt, however, by direct experiments both photometric and thermic. The details of the latter by Sig. Secchi will be found in *Astron. Nashr.* Nos. 806, 833, and go to prove that the calorific radiation of the centre of the sun's disk is nearly double of that from its borders, and that the equatorial regions are somewhat hotter than the polar (*Comptes Rendus*, Aug. 26, 1852).—*Note added 1858.*

surpass, by a very small breadth, the whole disk of the sun. Now when this takes place, were there no vaporous atmosphere capable of reflecting any light about the sun, the sky ought to appear totally dark, since (as will hereafter abundantly appear) there is not the smallest reason for believing the *moon* to have any atmosphere capable of doing so. So far, however, is this from being the case, that a bright ring or corona of light is seen, fading gradually away, as represented in Plate I. fig. 3, which (in cases where the moon is not *centrally* superposed on the sun) is observed to be concentric with the latter, not the former body. This corona was beautifully seen in the eclipse of July 7, 1842, and with this most remarkable addition—witnessed by every spectator in



Pavia, Milan, Vienna, and elsewhere: three distinct and very conspicuous *rose-colored* protuberances (as represented in the figure cited) were seen to project beyond the dark limb of the moon, likened by some to flames, by others to mountains, but which their enormous magnitude (for to have been seen at all by the naked eye their height must have exceeded 40,000 miles), and their faint degree of illumination, clearly prove to have been cloudy masses *of the most excessive tenuity*, and which doubtless owed their support, and probably their existence, to such an atmosphere as we are now speaking of. In the total eclipse of July 28, 1851, similar *rose-colored* protuberances were observed, one in particular of a form quite decisive as to their cloudy nature, rising straight up vertically from the edge of the disk, and

then suddenly turning off at a right angle (as in the annexed figure, which represents the appearance as seen by Prof. Schmidt, at Rastenburg), just as a column of smoke rising in calm air is often seen to be drifted off horizontally when it has attained such a height as to bring it into an upper current of wind. To complete the resemblance, a detached and perfectly insulated mass B of the same rosy color was observed at some distance from the drifted train A, which was connected with another mass C, by a narrow red band or streak D.

(395 *a.*) The existence of such an atmosphere superior to the luminous envelope being admitted, affords an easy explanation of the faculæ, considered as vast waves in the photosphere (art. 388). In an atmosphere consisting of strata gradually decreasing in density, any cause of undulation acting on the inferior strata will throw them up to a vastly greater height, and therefore produce far greater waves in them than would arise from the same cause acting on the surface of a definite ocean of liquid matter, by reason of their being partially sustained against gravity, leaving their inertia free to carry them up to a higher level. The experiment is easily tried in oil floating on water, or in saline solutions increasing in density downward, and is at once amusing and instructive. [See Note O, § 395 *b b.*]

(396.) That the temperature at the visible surface of the sun cannot be otherwise than very elevated, much more so than any artificial heat produced in our furnaces, or by chemical or galvanic processes, we have indications of several distinct kinds: 1st, From the law of decrease of radiant heat and light, which, being inversely as the squares of the distances, it follows, that the heat received on a given area exposed at the distance of the earth, and on an equal area at

the visible surface of the sun, must be in the proportion of the area of the sky occupied by the sun's apparent disk to the whole hemisphere, or as 1 to about 92,000. A far less intensity of solar radiation, collected in the focus of a burning-glass, suffices to dissipate gold and platina in vapor. 2dly, From the facility with which the calorific rays of the sun traverse glass, a property which is found to belong to the heat of artificial fires in the direct proportion of their intensity.²⁷ 3dly, From the fact, that the most vivid flames disappear, and the most intensely ignited solids appear only as black spots on the disk of the sun when held between it and the eye.²⁸ From the last remark it follows, that the body of the sun, however dark it may appear when seen through its spots, *may*, nevertheless, be in a state of most intense ignition. It does not, however, follow of necessity that it *must* be so. The contrary is at least physically possible. A *perfectly reflective* canopy would effectually defend it from the radiation of the luminous regions above its atmosphere, and no heat would be conducted downward through a gaseous medium increasing rapidly in density. That the penumbral clouds *are* highly reflective, the fact of their visibility in such a situation can leave no doubt.

(397.) As the magnitude of the sun has been measured, and (as we shall hereafter see) its weight, or quantity of pon-

²⁷ By direct measurement with the *actinometer*, I find that out of 1,000 calorific solar rays, 816 penetrate a sheet of plate glass 0.12 inch thick; and that of 1,000 rays which have passed through one such plate, 859 are capable of passing through another. H. 1827.

²⁸ The ball of ignited quicklime, in Lieutenant Drummond's oxy-hydrogen lamp, gives the nearest imitation of the solar splendor which has yet been produced. The appearance of this against the sun was, however, as described in an imperfect trial at which I was present. The experiment ought to be repeated under favorable circumstances.—*Note to the ed. of 1833.* According to the more recent experiments of Messrs. Fizeau and Foucault, the intensity of the light at the surface of Drummond's lime-ball is only one-146th part of that at the surface of the sun!—*Note added 1858.*

derable matter, ascertained, so also attempts have been made, and not wholly without success, from the heat actually communicated by its rays to given surfaces of material bodies exposed to their vertical action on the earth's surface, to estimate the total expenditure of heat by that luminary in a given time. The result of such experiments has been thus announced. Supposing a cylinder of ice 45 miles in diameter to be continually darted into the sun *with the velocity of light*, and that the water produced by its fusion were continually carried off, the heat now given off constantly by radiation would then be wholly expended in its liquefaction, on the one hand, so as to leave no radiant surplus; while on the other, the actual temperature at its surface would undergo no diminution."

(397 a.) Another mode of expressing the heat generated and radiated off from the sun's surface, well calculated to impress us with an overwhelming idea of the tremendous energies there constantly in action, is that employed by Professor Thomson, who estimates the dynamical effect which would be produced in our manufactories by a consumption of fuel competent to evolve the heat given out by *each individual square yard* of that surface, at 68000 horse-power, to maintain which would require the combustion of 13500 pounds of coal per hour.³⁰

²⁹ "Results of Astronomical Observations at the Cape of Good Hope," p. 444.

³⁰ See Trans. R. S. Edin. xxi. p. 69. "On the Mechanical Energies of the Solar System," by Sir W. Thomson, Prof. Nat. Phil., Glasgow. The Professor grounds this estimate on M. Pouillet's determination of the amount of solar radiation and Mr. Joule's estimate of the mechanical equivalent of a centigrade thermal unit. The author of this work found at the Cape of Good Hope, by experiments made on six summer days, from December 23, 1836, to January 9, 1837, the sun being nearly vertical in each experiment, that in that latitude at midsummer, at noon, and at 140 feet above the sea level, the solar radiation is competent to melt an inch in thickness from a sheet of ice exposed perpendicularly to its rays (if wholly so employed) in 2h. 12m. 42s. Estimating the heat

(398.) This immense escape of heat by radiation, we may remark, will fully explain the constant state of tumultuous agitation in which the fluids composing the visible surface are maintained, and the continual generation and filling in of *the pores*, without having recourse to internal causes. The mode of action here alluded to is perfectly represented to the eye in the disturbed subsidence of a precipitate, as described in art. 387, when the fluid from which it subsides is warm, and losing heat from its surface.

(399.) The sun's rays are the ultimate source of almost every motion which takes place on the surface of the earth. By its heat are produced all winds, and those disturbances in the electric equilibrium of the atmosphere which give rise to the phenomena of lightning, and probably also to those of terrestrial magnetism and the aurora. By their vivifying action vegetables are enabled to draw support from inorganic matter, and become, in their turn, the support of animals and of man, and the sources of those great deposits of dynamical efficiency which are laid up for human use in our coal strata.³¹ By them the waters of the sea are made to circulate in vapor through the air, and irrigate the land, producing springs and rivers. By them are produced all disturbances of the chemical equilibrium of the elements of nature, which, by a series of compositions and decompositions, give rise to new products, and originate a transfer of materials. Even the slow degradation of the solid constitu-

absorbed in traversing our atmosphere at one-third of the total quantity incident on it, this gives, all reductions made, 43·39 feet in thickness of ice *per minute* melted at the sun's surface. M. Pouillet's experiments (made in June, 1837), give 11·80 metres or 38·7 feet per minute. Forty feet may therefore be taken as a probable mean, and from this the result in art. 397 is calculated.

³¹ So in the edition of 1833. The celebrated engineer Stephenson is generally, but erroneously, cited as the originator of this remark.

ents of the surface, in which its chief geological changes consist, is almost entirely due, on the one hand, to the abrasion of wind and rain, and the alternation of heat and frost; on the other, to the continual beating of the sea waves, agitated by winds, the results of solar radiation. Tidal action (itself partly due to the sun's agency) exercises here a comparatively slight influence. The effect of oceanic currents (mainly originating in that influence), though slight in abrasion, is powerful in diffusing and transporting the matter abraded; and when we consider the immense transfer of matter so produced, the increase of pressure over large spaces in the bed of the ocean, and diminution over corresponding portions of the land, we are not at a loss to perceive how the elastic power of subterranean fires, thus repressed on the one hand and relieved on the other, may break forth in points where the resistance is barely adequate to their retention, and thus bring the phenomena of even volcanic activity under the general law of solar influence.³²

(400.) The great mystery, however, is to conceive how so enormous a conflagration (if such it be) can be kept up. Every discovery in chemical science here leaves us completely at a loss, or rather, seems to remove further the prospect of probable explanation. If conjecture might be hazarded, we should look rather to the known possibility of an indefinite generation of heat by friction, or to its excitement by the electric discharge, than to any actual combustion of ponderable fuel, whether solid or gaseous, for the origin of the solar radiation.³³ Photographic repre-

³² So in the edition of 1833.

³³ Electricity traversing excessively rarefied air or vapors gives out light, and, doubtless, also heat. May not a continual current of electric matter be constantly circulating in the sun's immediate neighborhood, or traversing the planetary spaces, and exciting, in the upper regions of its atmosphere, those

sentations of the spots have been made with much success by Mr. De la Rue, with a "photohelioscope" at Kew: also by the Rev. W. Selwyn, Canon of Ely, etc.

phenomena of which, on however diminutive a scale, we have yet an unequivocal manifestation in our aurora borealis. The possible analogy of the solar light to that of the aurora has been distinctly insisted on by the late Sir W. Herschel, in his paper already cited. It would be a highly curious subject of experimental inquiry, how far a mere reduplication of sheets of flame, at a distance one behind the other (by which their *light* might be brought to any required intensity), would communicate to the *heat* of the resulting compound ray the *penetrating* character which distinguishes the solar calorific rays. We may also observe that the tranquillity of the sun's polar, as compared with its equatorial regions (if its spots be really atmospheric), cannot be accounted for by its rotation on its axis only, but *must* arise from some cause *external to the luminous surface* of the sun, as we see the belts of Jupiter and Saturn, and our trade winds, arise from a cause external to these planets, combining itself with their rotation, which *alone* can produce no motions when once the form of equilibrium is attained.

The prismatic analysis of the solar beam exhibits in the spectrum a series of "fixed lines," totally unlike those which belong to the light of any known terrestrial flame. This may hereafter lead us to a clearer insight into its origin. But, before we can draw any conclusions from such an indication, we must recollect, that previous to reaching us it has undergone the whole absorptive action of our atmosphere, as well as of the sun's. Of the latter we know nothing, and may conjecture everything; but of the blue color of the former we are sure; and if this be an inherent (*i.e.* an absorptive) color, the air must be expected to act on the spectrum after the analogy of other colored media, which often (and *especially light blue* media) leave unabsorbed portions separated by dark intervals. It deserves inquiry, therefore, whether some or all the fixed lines observed by Wollaston and Fraunhofer may not have their origin in our own atmosphere. Experiments made on lofty mountains, or the cars of balloons, on the one hand, and on the other with reflected beams which have been made to traverse several miles of additional air near the surface, would decide this point. The absorptive effect of the sun's atmosphere, and possibly also of the medium surrounding it (whatever it be) which resists the motions of comets, cannot be thus eliminated.—*Note to the edition of 1833.* The idea of referring the origin of the solar heat to friction has been worked out into an elaborate theory by Professor Thomson, in his paper already cited, of which some account will be given in a more advanced portion of this work. (1858.) The recent remarkable results of what is called "Spectrum Analysis" afford a curious commentary on this note. (1863.) [See note M. (1865.)]

NOTE ON ART. 394 a.—The year 1856 was remarkable for the absence of spots in the sun (in exact accordance with Wolf's period); during 1857 the phase of increased activity came on; and the present year (1858) is ushered in with a magnificent display of spots in the sun's southern hemisphere.—*Note added Jan. 4, 1858.*

CHAPTER VII

Of the Moon—Its Sidereal Period—Its Apparent Diameter—Its Parallax Distance, and Real Diameter—First Approximation to its Orbit—An Ellipse about the Earth in the Focus—Its Excentricity and Inclination—Motion of its Nodes and Apsides—Of Occultations and Solar Eclipses Generally—Limits within which they are Possible—They Prove the Moon to be an Opaque Solid—Its Light Derived from the Sun—Its Phases—Synodic Revolution or Lunar Month—Harvest Moon—Of Eclipses more Particularly—Their Phenomena—Their Periodical Recurrence—Physical Constitution of the Moon—Its Mountains and other Superficial Features—Indications of former Volcanic Activity—Its Atmosphere—Climate—Radiation of Heat from its Surface—Rotation on its own Axis—Libration—Appearance of the Earth from it—Probable Elongation of the Moon's Figure in the Direction of the Earth—Its Habitability not Impossible—Charts, Models and Photographs of its Surface

(401.) THE moon, like the sun, appears to advance among the stars with a movement contrary to the general diurnal motion of the heavens, but much more rapid, so as to be very readily perceived (as we have before observed) by a few hours' cursory attention on any moonlight night. By this continual advance, which, though sometimes quicker, sometimes slower, is never intermitted or reversed, it makes the tour of the heavens in a mean or average period of $27^{\text{d}} 7^{\text{h}} 43^{\text{m}} 11^{\text{s}}.5$, returning, in that time, to a position among the stars nearly coincident with that it had before, and which would be exactly so, but for reasons presently to be stated.

(402.) The moon, then, like the sun, apparently describes an orbit round the earth, and this orbit cannot be *very* different from a circle, because the apparent angular diam-

eter of the full moon is not liable to any great extent of variation.

(403.) The distance of the moon from the earth is concluded from its horizontal parallax, which may be found either directly, by observations at remote geographical stations, exactly similar to those described in art. 355, in the case of the sun, or by means of the phenomena called occultations, from which also its apparent diameter is most readily and correctly found. From such observations it results that the mean or average value of the moon's horizontal parallax is $57' 2''\cdot325$,¹ and the mean distance of the centre of the moon from that of the earth is $60\cdot255$ of the earth's equatorial radii, or about 238793 miles, taking with Mr. Adams $57' 2''\cdot325$ for the mean horizontal parallax. This distance, great as it is, is little more than one-fourth of the diameter of the sun's body, so that the globe of the sun would nearly twice include the whole orbit of the moon; a consideration wonderfully calculated to raise our ideas of that stupendous luminary!

(404.) The distance of the moon's centre from an observer at any station on the earth's surface, compared with its apparent angular diameter as measured from that station, will give its real or linear diameter. Now, the former distance is easily calculated when the distance from the earth's centre is known, and the apparent zenith distance of the moon also determined by observation; for if we turn to the figure of art. 339, and suppose S the moon, A the station, and C the earth's centre, the distance S C, and the earth's radius C A, two sides of the triangle A C S

¹ This result, recently arrived at by Mr. Adams, coincides almost precisely with that assigned by Henderson, viz. $57' 2''\cdot31$.

are given, and the angle C A S, which is the supplement of Z A S, the observed zenith distance, whence it is easy to find A S, the moon's distance from A. From such observations and calculations it results, that the real diameter of the moon is 2160 miles, or about 0·2729 of that of the earth, whence it follows that, the bulk of the latter being considered as 1, that of the former will be 0·0204, or about $\frac{1}{48}$. The difference of the apparent diameter of the moon, as seen from the earth's centre and from any point of its surface, is technically called the *augmentation of the apparent* diameter, and its maximum occurs when the moon is in the zenith of the spectator. Her mean angular diameter, as seen from the centre, is 31' 7", and is always = 0·545 \times her horizontal parallax.

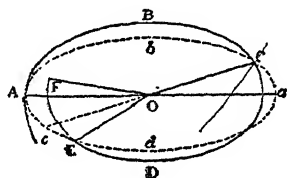
(405.) By a series of observations, such as described in art. 403, if continued during one or more revolutions of the moon, its real distance may be ascertained at every point of its orbit; and if at the same time its apparent places in the heavens be observed, and reduced by means of its parallax to the earth's centre, their angular intervals will become known, so that the path of the moon may then be laid down on a chart supposed to represent the plane in which its orbit lies, just as was explained in the case of the solar ellipse (art. 349). Now, when this is done, it is found that, neglecting certain small, though very perceptible deviations of which a satisfactory account will hereafter be rendered, the form of the apparent orbit, like that of the sun, is elliptic, but considerably more excentric, the excentricity amounting to 0·05484 of the mean distance, or the major semi-axis of the ellipse, and the earth's centre being situated in its focus.

(406.) The plane in which this orbit lies is not the

ecliptic, however, but is inclined to it at an angle of $5^{\circ} 8' 48''$, which is called the inclination of the lunar orbit, and intersects it in two opposite points, which are called its nodes—the *ascending node* being that in which the moon passes from the southern side of the ecliptic to the northern, and the *descending* the reverse. The points of the orbit at which the moon is nearest to and furthest from, the earth, are called respectively its *perigee* and *apogee*, and the line joining them and the earth the line of *apsides*.

(407.) There are, however, several remarkable circumstances which interrupt the closeness of the analogy, which cannot fail to strike the reader, between the motion of the moon around the earth, and of the earth around the sun. In the latter case, the ellipse described remains, during a great many revolutions, unaltered in its position and dimensions; or, at least, the changes which it undergoes are not perceptible but in a course of very nice observations, which have disclosed, it is true, the existence of “perturbations,” but of so minute an order, that, in ordinary parlance, and for common purposes, we may leave them unconsidered. But this cannot be done in the case of the moon. Even in a single revolution, its deviation from a perfect ellipse is very sensible. It does not return to the same exact position among the stars from which it set out, thereby indicating a continual change in the *plane* of its orbit. And, in effect, if we trace by observation, from month to month, the point where it traverses the ecliptic, we shall find that the *nodes* of its orbit are in a continual state of *retreat* upon the ecliptic. Suppose O to be the earth, and A *b a d* that portion of the plane of the ecliptic which is intersected by the moon, in its alternate passages through it, from south to north, and *vice versâ*; and let A B C D E F be a portion of the moon’s

orbit, embracing a complete sidereal revolution. Suppose it to set out from the ascending node, A; then, if the orbit lay all in one plane, passing through O, it would have α , the opposite point in the ecliptic, for its descending node; after passing which, it would again ascend at A. But, in fact, its real path carries it not to α , but along a certain curve, A B C, to C, a point in the ecliptic less than 180° distant from A; so that the angle A O C, or the arc of longitude described between the ascending and the descending node, is somewhat less than 180° . It then pursues its course below the ecliptic, along the curve C D E, and rises again above it, not at the point c, diametrically opposite to C, but at a point E, less advanced in longitude. On the whole, then, the arc described in longitude between two consecutive passages from south to north, through the plane of the ecliptic, falls short of 360° by the angle A O E; or, in other words, the ascending node appears to have retreated in one lunation on the plane of the ecliptic by that amount. To complete a sidereal revolution, then, it must still go on to describe an arc, E F, on its orbit, which will no longer, however, bring it exactly back to A, but to a point somewhat above it, or *having north latitude*.



(408.) The actual amount of this retreat of the moon's node is about $3' 10'' \cdot 64$ *per diem*, on an average, and in a period of 6793.39 mean solar days, or about 18.6 years, the ascending node is carried round in a direction contrary to the moon's motion in its orbit (or from east to west) over a whole circumference of the ecliptic. Of course, in the middle of this period the position of the orbit must have

been precisely reversed from what it was at the beginning. Its apparent path, then, will lie among totally different stars and constellations at different parts of this period; and this kind of spiral revolution being continually kept up, it will, at one time or other, cover with its disk every point of the heavens within that limit of latitude or distance from the ecliptic which its inclination permits; that is to say, a belt or zone of the heavens, of $10^{\circ} 18'$ in breadth, having the ecliptic for its middle line. Nevertheless, it still remains true that the *actual place* of the moon, in consequence of this motion, deviates in a single revolution *very little* from what it would be were the nodes at rest. Supposing the moon to set out from its node A, its latitude, when it comes to F, having completed a revolution in longitude, will not exceed $8'$; which, though small in a single revolution, accumulates in its effect in a succession of many: it is to account for, and represent geometrically, this deviation, that the *motion of the nodes* is devised.

(409.) The moon's orbit, then, is not, strictly speaking, an ellipse returning into itself, by reason of the variation of the plane in which it lies, and the motion of its nodes. But even laying aside this consideration, the axis of the ellipse is itself constantly changing its direction in space, as has been already stated of the solar ellipse, but much more rapidly; making a complete revolution, in the same direction with the moon's own motion, in 3232.5753 mean solar days, or about nine years, being about 3° of angular motion in a whole revolution of the moon. This is the phenomenon known by the name of the revolution of the moon's *apsides*. Its cause will be hereafter explained. Its immediate effect is to produce a variation in the moon's distance from the earth, which is not included in the laws of exact elliptic

motion. In a single revolution of the moon, this variation of distance is trifling; but in the course of many it becomes considerable, as is easily seen, if we consider that in four years and a half the position of the axis will be completely reversed, and the apogee of the moon will occur where the perigee occurred before.

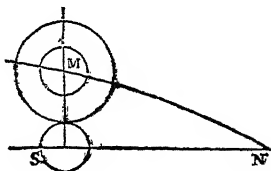
(410.) The best way to form a distinct conception of the moon's motion is to regard it as describing an ellipse about the earth in the focus, and, at the same time, to regard this ellipse itself to be in a twofold state of revolution; 1st, in its own plane, by a continual advance of its axis in that plane; and 2dly, by a continual *tilting* motion of the plane itself, exactly similar to, but much more rapid than, that of the earth's equator produced by the conical motion of its axis, described in art. 317.

(411.) As the moon is at a very moderate distance from us (astronomically speaking), and is in fact our nearest neighbor, while the sun and stars are in comparison immensely beyond it, it must of necessity happen, that at one time or other it must *pass over* and *ocult* or *eclipse* every star and planet within the zone above described (and, as seen from the *surface* of earth, even somewhat beyond it, by reason of parallax, which may throw it apparently nearly a degree either way from its place as seen from the centre, according to the observer's station). Nor is the sun itself exempt from being thus hidden, whenever any part of the moon's disk, in this her tortuous course, comes to *overlap* any part of the space occupied in the heavens by that luminary. On these occasions is exhibited the most striking and impressive of all the occasional phenomena of astronomy, an *eclipse of the sun*, in which a greater or less portion, or even in some rare conjunctures the whole, of its disk is obscured,

and, as it were, obliterated, by the superposition of that of the moon, which appears upon it as a circularly-terminated black spot, producing a temporary diminution of daylight, or even nocturnal darkness, so that the stars appear as if at midnight. In other cases, when, at the moment that the moon is centrally superposed on the sun, it so happens that her distance from the earth is such as to render her angular diameter less than the sun's, the very singular phenomenon of an *annular solar eclipse* takes place, when the edge of the sun appears for a few minutes as a narrow ring of light projecting on all sides beyond the dark circle occupied by the moon in its centre.

(412.) A solar eclipse can only happen when the sun and moon are *in conjunction*, that is to say, have the *same*, or nearly the same, position in the heavens, or the same longitude. It appears by art. 409 that this condition can only be fulfilled at the time of a *new moon*, though it by no means follows, that at *every* conjunction there *must* be an eclipse of the sun. If the lunar orbit coincided with the ecliptic, this would be the case, but as it is inclined to it at an angle of upward of 5° , it is evident that the conjunction, or equality of longitudes, may take place when the moon is in the part of her orbit too remote from the ecliptic to permit the disks to meet and overlap. It is easy, however, to assign the limits within which an eclipse is possible. To this end we must consider, that, by the effect of parallax, the moon's *apparent* edge may be thrown in *any* direction, according to a spectator's geographical station, by *any* amount not exceeding her horizontal parallax, and the same holds good of the sun, so that there is a displacement to the extent of the difference of the two parallaxes. Now, this comes to the same (so far as the possibility of an eclipse

is concerned) as if the apparent diameter of the moon, seen from the earth's centre, were dilated by twice the difference of their horizontal parallaxes; for if, when so dilated, it can touch or overlap the sun, there *must* be an eclipse at *some* part or other of the earth's surface. If, then, at the moment of the nearest conjunction, the geocentric distance of the centres of the two luminaries do not exceed the sum of their semidiameters and of the last-mentioned difference, there will be an eclipse. The sum is, at its maximum, about $1^{\circ} 34' 26''$. In the spherical triangle $S N M$, then, in which S is the sun's centre, M the moon's, $S N$ the ecliptic, $M N$ the moon's orbit, and N the node, we may suppose the angle $N S M$ a right angle, $S M = 1^{\circ} 34' 26''$, and the angle, $M N S = 5^{\circ} 8' 48''$, the inclination of the orbit. Hence we calculate $S N$, which comes out $16^{\circ} 58'$. If, then, at the moment of the new moon, the moon's node is further from the sun in longitude than this limit, there can be no eclipse; if within, there may, and probably will, at some part or other of the earth. To ascertain precisely whether there will or not, and, if there be, how great will be the part eclipsed, the solar and lunar tables must be consulted, the place of the node and the semidiameters exactly ascertained, and the local parallax, and apparent augmentation of the moon's diameter due to the difference of her distance from the observer and from the centre of the earth (which may amount to a sixtieth part of her horizontal diameter), determined; after which it is easy, from the above considerations, to calculate the amount overlapped of the two disks, and their moment of contact.



(413.) The calculation of the occultation of a star de-

pende on similar considerations. An occultation is *possible*, when the moon's course, as seen from the earth's centre, carries her within a distance from the star equal to the sum of her semidiameter and horizontal parallax; and it *will happen at any particular spot*, when her apparent path, as seen from that spot, carries her centre within a distance equal to the sum of her *augmented* semidiameter and *actual* parallax. The details of these calculations, which are somewhat troublesome, must be sought elsewhere.²

(414.) The phenomenon of a solar eclipse and of an occultation are highly interesting and instructive in a physical point of view. They teach us that the moon is an opaque body, terminated by a real and sharply defined surface intercepting light like a solid. They prove to us, also, that at those times when we cannot *see* the moon, she really exists, and pursues her course, and that when we see her only as a crescent, however narrow, the whole globular body *is there*, filling up the deficient outline, though unseen. For occultations take place indifferently at the dark and bright, the visible and invisible outline, whichever happens to be toward the direction in which the moon is moving; with this only difference, that a star *occulted* by the bright limb, if the phenomenon be watched with a telescope, gives notice, by its gradual approach to the visible edge, when to expect its disappearance, while, if occulted at the dark limb, if the moon, at least, be more than a few days old, it is, as it were, extinguished in mid-air, without notice or visible cause for its disappearance, which, as it happens *instantaneously*, and without the slightest previous diminu-

² Woodhouse's Astronomy, vol. i. See also Trans. Ast. Soc. vol. i. p. 325. See also Prof. Loomis's Introduction to Practical Astronomy, in which every detail of the calculation will be found illustrated by numerical examples.

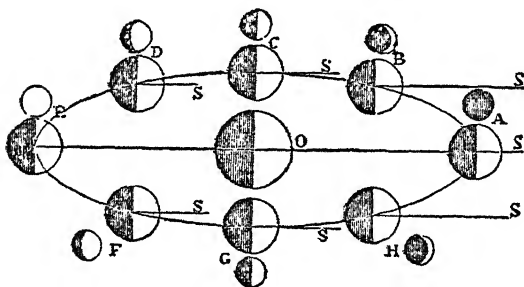
tion of its light, is always surprising; and, if the star be a large and bright one, even startling from its suddenness. The reappearance of the star, too, when the moon has passed over it, takes place in those cases when the bright side of the moon is foremost, not at the concave outline of the crescent, but at the invisible outline of the complete circle, and is scarcely less surprising, from its suddenness, than its disappearance in the other case.³

(415.) The existence of the complete circle of the disk, even when the moon is not full, does not, however, rest only on the evidence of occultations and eclipses. It may be *seen*, when the moon is crescent or waning, a few days before and after the *new moon*, with the naked eye, as a pale round body, to which the crescent seems attached, and somewhat projecting beyond its outline (which is an optical illusion arising from the greater intensity of its light). The cause of this appearance will presently be explained. Meanwhile the fact is sufficient to show that the moon is not *inherently* luminous like the sun, but that her light is of an adventitious nature. And its crescent form, increasing regularly from a narrow semicircular line to a com-

³ There is an optical illusion of a very strange and unaccountable nature which has often been remarked in occultations. The star appears to advance actually *upon* and *within* the edge of the disk before it disappears, and that sometimes to a considerable depth. I have never myself witnessed this singular effect, but it rests on most unequivocal testimony. I have called it an optical illusion; but it is *barely possible* that a star may shine on such occasions through deep fissures in the substance of the moon. The occultations of close double stars ought to be narrowly watched, to see whether *both* individuals are thus *projected*, as well as for other purposes connected with their theory. I will only hint at one, viz. that a double star, *too close* to be seen divided with any telescope, may yet be detected to be double by the mode of its disappearance. Should a considerable star, for instance, instead of undergoing instantaneous and complete extinction, go out by two distinct steps, following close upon each other; first losing a portion, then the whole remainder of its light, we may be sure it is a double star, though we cannot see the individuals separately.—*Note to the edit. of 1833.*

plete circular disk, corresponds to the appearance a globe would present, one hemisphere of which was black, the other white, when differently turned toward the eye, so as to present a greater or less portion of each. The obvious conclusion from this is, that the moon is such a globe, one-half of which is brightened by the rays of some luminary sufficiently distant to enlighten the complete hemisphere, and sufficiently intense to give it the degree of splendor we see. Now, the sun alone is competent to such an effect. Its distance and light suffice; and, moreover, it is invariably observed that, when a crescent, the bright edge is *toward the sun*, and that in proportion as the moon in her monthly course becomes more and more distant from the sun, the breadth of the crescent increases, and *vice versâ*.

(416.) The sun's distance being 23984 radii of the earth, and the moon's only 60, the former is nearly 400 times the



latter. Lines, therefore, drawn from the sun to every part of the moon's orbit may be regarded as very nearly parallel.* Suppose, now, O to be the earth, A B C D, etc., various positions of the moon in its orbit, and S the sun, at the vast distance above stated; as is shown, then, in the figure,

* The angle subtended by the moon's orbit, as seen from the sun (in the mean state of things), is only $17' 12''$.

the hemisphere of the lunar globe turned toward it (on the right) will be bright, the opposite dark, wherever it may stand in its orbit. Now, in the position A, when in conjunction with the sun, the dark part is entirely turned toward O, and the bright from it. In this case, then, the moon is not seen, it is *new moon*. When the moon has come to C, half the bright and half the dark hemisphere are presented to O, and the same in the opposite situation G: these are the first and third quarters of the moon. Lastly, when at E, the whole bright face is toward the earth, the whole dark side from it, and it is then seen wholly bright or *full moon*. In the intermediate positions B D F H, the portions of the bright face presented to O will be at first less than half the visible surface, then greater, and finally less again, till it vanishes altogether, as it comes round again to A.

(417.) These monthly changes of appearance, or *phases*, as they are called, arise, then, from the moon, an opaque body, being illuminated on one side by the sun, and reflecting from it, in all directions, a portion of the light so received. Nor let it be thought surprising that a solid substance thus illuminated should appear to *shine* and again illuminate the earth. It is no more than a white cloud does standing off upon the clear blue sky. By day, the moon can hardly be distinguished in brightness from such a cloud; and, in the dusk of the evening, clouds catching the last rays of the sun appear with a dazzling splendor, not inferior to the seeming brightness of the moon at night.*

* The actual illumination of the lunar surface is not much superior to that of weathered sandstone rock in full sunshine. I have frequently compared the moon setting behind the gray perpendicular façade of the Table Mountain illuminated by the sun just risen in the opposite quarter of the horizon, when it has scarcely been distinguishable in brightness from the rock in contact with

That the earth sends also such a light to the moon, only probably more powerful by reason of its greater apparent size,⁶ is agreeable to optical principles, and explains the appearance of the dark portion of the young or waning moon completing its crescent (art. 413). For, when the moon is nearly new to the earth, the latter (so to speak) is nearly full to the former; it then illuminates its dark half by strong *earth-light*; and it is a portion of this, reflected back again, which makes it visible to us in the twilight sky. As the moon gains age, the earth offers it a less portion of its bright side, and the phenomenon in question dies away. The light of the full moon is estimated by Bouguer on the result of photometric comparisons, as only one 300000th part of that of the sun.

(418.) The lunar month is determined by the recurrence of its phases: it reckons from new moon to new moon; that is, from leaving its conjunction with the sun to its return to conjunction. If the sun stood still, like a fixed star, the interval between two conjunctions would be the same as the period of the moon's sidereal revolution (art. 401); but, as the sun apparently advances in the heavens in the same direction with the moon, only slower, the latter has more than a complete sidereal period to perform to come up with the sun again, and will require for it a longer time, which is the lunar month, or, as it is generally termed in astronomy, a *synodical* period. The difference is easily calculated by considering that the superfluous arc (whatever it be) is described by the sun with the velocity of $0^{\circ}.98565$ *per diem*,

it. The sun and moon being nearly at equal altitudes and the atmosphere perfectly free from cloud or vapor, its effect is alike on both luminaries. (H. 1848.)

⁶ The apparent diameter of the moon is 32' from the earth; that of the earth seen from the moon is twice her horizontal parallax, or $1^{\circ} 54'$. The apparent surfaces, therefore, are as $(114)^2 : (32)^2$, or as 13 : 1 nearly.

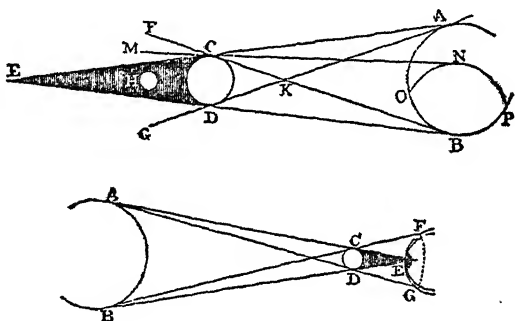
in the same time that the moon describes that arc *plus* a complete revolution, with her velocity of $13^{\circ}.17640$ *per diem*; and, the times of description being identical, the spaces are to each other in the proportion of the velocities. Let V and v be the mean angular daily motions of the sun and moon as above, x the superfluous arc; then $V : v :: 360^{\circ} + x : x$; and $V - v : v :: 360^{\circ} : x$, whence x is found; and $\frac{x}{v}$ = the time in days in which the sun describes the arc

x , that is, the synodical period $= \frac{360^{\circ}}{V - v} = \frac{P \cdot p}{p - P}$, if P, p are the periodic times of each separately, which reduced to numbers, gives, $29^d.530589 = 29^d 12^h 44^m 2^s.87$.

(419.) Supposing the position of the nodes of the moon's orbit to permit it, when the moon stands at A (or at the new moon), it will intercept a part or the whole of the sun's rays, and cause a solar eclipse. On the other hand, when at E (or at the full moon), the earth O will intercept the rays of the sun, and cast a shadow on the moon, thereby causing a lunar eclipse. And this is consonant to fact, such eclipses never happening but at the time of the full moon. But, what is still more remarkable, as confirmatory of the position of the earth's sphericity, this shadow, which we plainly see to enter upon and, as it were, eat away the disk of the moon, is always terminated by a circular outline, though, from the greater *size* of the circle, it is only partially seen at any one time. Now, a body which always casts a circular shadow must itself be spherical.

(420.) Eclipses of the sun are best understood by regarding the sun and moon as two independent luminaries, each moving according to known laws, and viewed from the earth; but it is also instructive to consider eclipses gen-

erally as arising from the shadow of one body thrown on another by a luminary *much larger than either*. Suppose, then, A B to represent the sun, and C D a spherical body, whether earth or moon, illuminated by it. If we join and prolong A C, B D; since A B is greater than C D, those lines will meet in a point E, more or less distant from the body C D, according to its size, and within the space C E D (which represents a cone, since C D and A B are spheres), there will be a total shadow. This shadow is called the *umbra*, and a spectator situated within it can see no part of the sun's disk. Beyond the umbra are two diverging spaces



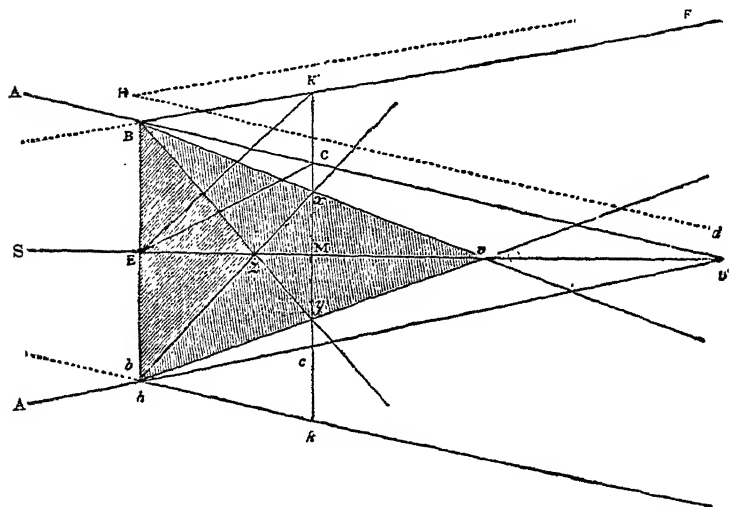
(or rather, a portion of a single conical space, having K for its vertex), where if a spectator be situated, as at M, he will see a portion only (A O N P) of the sun's surface, the rest (B O N P) being obscured by the earth. He will, therefore, receive only partial sunshine; and the more, the nearer he is to the exterior borders of that cone which is called the *penumbra*. Beyond this he will see the whole sun, and be in full illumination. All these circumstances may be perfectly well shown by holding a small globe up in the sun, and receiving its shadow at different distances on a sheet of paper.

(421.) In a lunar eclipse (represented in the upper figure), the moon is seen to enter[†] the *penumbra* first, and by degrees get involved in the *umbra*, the former bordering the latter like a smoky haze. At this period of the eclipse, and while yet a considerable part of the moon remains unobscured, the portion involved in the *umbra* is invisible to the naked eye, though still perceptible in a telescope, and of a dark gray hue. But as the eclipse advances, and the enlightened part diminishes in extent, and grows gradually more and more obscured by the advance of the *penumbra*, the eye, relieved from its glare, becomes more sensible to feeble impressions of light and color; and phenomena of a remarkable and instructive character begin to be developed. The *umbra* is seen to be very far from totally dark; and in its faint illumination it exhibits a gradation of color, being bluish, or even (by contrast) somewhat greenish, toward the borders for a space of about 4' or 5' of apparent angular breadth inward, thence passing, by delicate but rapid gradation, through rose red to a fiery or copper-colored glow, like that of dull red-hot iron. As the eclipse proceeds this glow spreads over the whole surface of the moon, which then becomes on some occasions so strongly illuminated as to cast a very sensible shadow, and allow the spots on its surface to be perfectly well distinguished through a telescope.

(422.) The cause of these singular, and sometimes very beautiful appearances, is the refraction of the sun's light in passing through our atmosphere, which at the same time becomes colored with the hues of sunset by the absorption of more or less of the violet and blue rays, as it passes

[†] The actual contact with the *penumbra* is never seen; the defalcation of light comes on so very gradually that it is not till when already deeply immersed, that it is perceived to be sensibly darkened.

through strata nearer or more remote from the earth's surface, and therefore, more or less loaded with vapor. To show this, let $A D a$ be a section of the cone of the *umbra*, and $F B h f$ of the *penumbra*, through their common axis $D E S$, passing through the centres $E S$ of the earth and sun, and let $K M k$ be a section of these cones at a distance $E M$ from E , equal to the radius of the moon's orbit,



or 60 radii of the earth.⁸ Taking this radius for unity, since $E S$, the distance of the sun, is 23984, and the semidiameter of the sun 111 such units, we readily calculate $D E = 218$, $D M = 158$, for the distances at which the apex of the *geometrical umbra* lies behind the earth and the moon respectively. We also find for the measure of the angle $E D B$, $15' 46''$, and therefore $D B E = 89^{\circ} 44' 14''$, whence also we get $M C$ (the linear semidiameter of the *umbra*) = 0.725

⁸ The figure is unavoidably drawn out of all proportion, and the angles violently exaggerated. The reader should *try* to draw the figure in its true proportions.

(or in miles 2868), and the angle $C E M$, its apparent angular semidiameter as seen from $E = 41' 32''$. And instituting similar calculations for the geometrical *penumbra* we get $M K = 1.280$ (5064 miles), and $K E M = 1^{\circ} 13' 20''$; and it may be well to remember that the doubles of these angles, or the mean angular diameters of the *umbra* and *penumbra*, are described by the moon with its mean velocity in $2^h 46^m$, and $4^h 56^m$ respectively, which are therefore the respective durations of the total and partial obscuration of any one point of the moon's disk in traversing centrally the geometrical shadow.

(423.) Were the earth devoid of atmosphere, the whole of the phenomena of a lunar eclipse would consist in these partial or total obscurations. Within the space $C c$ the whole of the light, and within $K C$ and $c k$ a greater or less portion of it, would be intercepted by the solid body $B b$ of the earth. The refracting atmosphere, however, extends from $B b$, to a certain unknown, but very small distance $B H$, $b h$, which, acting as a convex lens, of gradually (and very rapidly) decreasing density, disperses all that comparatively small portion of light which falls upon it over a space bounded externally by $H g$, parallel and very nearly coincident with $B F$, and internally by a line $B z$, the former representing the extreme exterior ray from the limb a of the sun, the latter, the extreme interior ray from the limb A . To avoid complication, however, we will trace only the courses of rays which just graze the surface at B , viz.: $B z$ from the upper border, A , and $B v$ from the lower, a , of the sun. Each of these rays is bent inward from its original course by *double* the amount of the horizontal refraction ($33'$) i. e. by $1^{\circ} 6'$, because, in passing from B out of the atmosphere, it undergoes a deviation equal to that at enter-

ing, and in the same direction. Instead, therefore, of pursuing the courses $B D$, $B F$, these rays respectively will occupy the positions $B z y$, $B v$, making, with the aforesaid lines, the angles $D B y$, $F B v$, each $1^{\circ} 6'$. Now we have found $D B E = 89^{\circ} 44' 14''$ and therefore $F B E (= D B E + \text{angular diam. of } \odot) = 90^{\circ} 16' 17''$; consequently the angles $E B y$ and $E B v$ will be respectively $88^{\circ} 38' 14''$ and $89^{\circ} 10' 17''$, from which we conclude $E z = 42.04$ and $E v = 69.14$; the former falling short of the moon's orbit by 17.96 , and the latter surpassing it by 9.14 radii of the earth.

(424.) The penumbra, therefore, of rays refracted at B , will be spread over the space $v B y$, that at H over $g H d$, and at the intermediate points, over similar intermediate spaces, and through this compound of superposed penumbrae the moon passes during the whole of its path through the geometrical shadow, never attaining the *absolute* umbra $B z b$ at all. Without going into detail as to the intensity of the refracted rays, it is evident that the totality of light so thrown into the shadow is to that which the earth intercepts, as the area of a circular section of the atmosphere to that of a diametrical section of the earth itself, and, therefore, at all events but feeble. And it is still further enfeebled by actual clouds suspended in that portion of the air which forms the visible border of the earth's disk as seen from the moon, as well as by the general want of transparency caused by invisible vapor, which is especially effective in the lowermost strata, within three or four miles of the surface, and which will impart to all the rays they transmit the ruddy hue of sunset, only of *double* the depth of tint which we admire in our glowing sunsets, by reason of the rays having to traverse twice as great a thickness of atmosphere. This redness will be most intense at the

points x , y , of the moon's path through the umbra, and will thence degrade very rapidly outwardly, over the spaces x C, y c, less so inwardly, over x y . And at C, c, its hue will be mingled with the bluish or greenish light which the atmosphere scatters by irregular dispersion, or in other words by our *twilight* (art. 44). Nor will the phenomenon be uniformly conspicuous at all times. Supposing a generally and deeply clouded state of the atmosphere around the edge of the earth's disk visible from the moon (*i.e.* around that great circle of the earth, in which, at the moment the sun is in the horizon) little or no refracted light may reach the moon.⁹ Supposing that circle partly clouded and partly clear, patches of red light corresponding to the clear portions will be thrown into the umbra, and may give rise to various and changeable distributions of light on the eclipsed disk;¹⁰ while, if entirely clear, the eclipse will be remarkable for the conspicuousness of the moon during the whole or a part of its immersion in the umbra.¹¹

(425.) Owing to the great size of the earth, the cone of its *umbra* always projects far beyond the moon; so that, if, at the time of a lunar eclipse, the moon's path be properly directed, it is sure to pass through the *umbra*. This is not, however, the case in solar eclipses. It so happens, from the adjustment of the size and distance of the moon, that the extremity of her *umbra* always falls *near* the earth, but sometimes attains and sometimes falls short of its surface. In the former case (represented in the lower figure,

⁹ As in the eclipses of June 5, 1620, April 25, 1642. Lalande, *Ast.* 1769. Also December 9, 1601, and June 10, 1816, on which occasion the moon was totally invisible even in telescopes.

¹⁰ As in the eclipse of Oct. 13, 1837, observed by the author.

¹¹ As in that of March 19, 1848, when the moon is described as giving "good light" during more than an hour after its total immersion, and some persons even doubted its being eclipsed. (*Notices of R. Ast. Soc.* viii. p. 132.)

art. 420) a black spot, surrounded by a fainter shadow, is formed, beyond which there is no eclipse on any part of the earth, but within which there may be either a total or partial one, as the spectator is within the *umbra* or *penumbra*. When the apex of the umbra falls on the surface, the moon at that point will appear, for an instant, to *just* cover the sun; but, when it falls short, there will be no total eclipse on any part of the earth; but a spectator, situated in or near the prolongation of the axis of the cone, will see the whole of the moon on the sun, although not large enough to cover it, *i.e.* he will witness an annular eclipse, a phenomenon to which much interest is attached by reason of some curious optical phenomena first observed by Mr. Baily at the moments of the forming and breaking of the annulus, like beads of light alternating with black thready elongations of the moon's limb, known by the name of "Baily's beads."

(426.) Owing to a remarkable enough adjustment of the periods in which the moon's *synodical* revolution, and that of her nodes, are performed, eclipses return after a certain period, very nearly in the same order and of the same magnitude. For 223 of the moon's mean synodical revolutions, or *lunations*, as they are called, will be found to occupy 6585·32 days, and nineteen complete synodical revolutions of the node to occupy 6585·78. The difference in the mean position of the node, then, at the beginning and end of 223 lunations, is nearly insensible; so that a recurrence of all eclipses within that interval must take place. Accordingly, this period of 223 lunations, or eighteen years and ten days, is a very important one in the calculation of eclipses. It is supposed to have been known to the Chaldeans, the earliest astronomers, the regular return of eclipses having been known as a physical fact for ages before their exact theory

was understood. In this period there occur ordinarily 70 eclipses, 29 of the moon, and 41 of the sun, visible in some part of the earth. Seven eclipses of either sun or moon at most, and two at least (both of the sun), may occur in a year.

(427.) The commencement, duration, and magnitude of a lunar eclipse are much more easily calculated than those of a solar, being independent of the position of the spectator on the earth's surface, and the same as if viewed from its centre. The common centre of the *umbra* and *penumbra* lies always in the ecliptic, at a point opposite to the sun, and the path described by the moon in passing through it is its true orbit as it stands at the moment of the full moon. In this orbit, its position, at every instant, is known from the lunar tables and ephemeris; and all we have, therefore, to ascertain, is, the *moment when* the distance between the moon's centre and the centre of the shadow is exactly equal to the sum of the semidiameters of the moon and *penumbra*, or of the moon and *umbra*, to know when it enters upon and leaves them respectively. No lunar eclipse can take place, if, at the moment of the full moon, the sun be at a greater angular distance from the node of the moon's orbit than $11^{\circ} 21'$, meaning by an *eclipse* the immersion of any part of the moon in the *umbra*, as its contact with the *penumbra* cannot be observed (see note to art. 421).

(428.) The dimensions of the shadow, at the place where it crosses the moon's path, require us to know the distances of the sun and moon at the time. These are variable; but are calculated and set down, as well as their semidiameters, for every day, in the ephemeris, so that none of the data are wanting. The sun's distance is easily calculated from its elliptic orbit; but the moon's is a matter of more difficulty,

by reason of the progressive motion of the axis of the lunar orbit. (Art. 409.) Both, however, are readily obtained from the ephemeris for every day; the sun's distance being given explicitly and the moon's implicitly, from her tabulated apparent diameter.

(428 *a.*) It deserves to be mentioned that the moon may be seen eclipsed while the sun is yet above the horizon by a spectator properly situated, so that both luminaries being on his mathematical horizon shall be raised above it by refraction, which (art. 43) exceeds the apparent diameter of either. This singular conjuncture of circumstances is said to have been observed from Montmartre, near Paris, by the assembled academicians of that city in A.D. 1668.

(428 *b.*) The full moon which happens on or nearest to the 21st of September is called the harvest moon, because it rises from night to night, after the full, more nearly after sunset than any other full moon in the year, and is therefore favorable for evening work in carrying in the late crops. Suppose the full moon to happen on that day (the time of the autumnal equinox) the sun is then entering Libra, and the moon Aries, the former setting due west, the latter rising due east; the southern half of the ecliptic is then entirely above and the northern below the horizon, and the ecliptic itself makes then the least possible angle with the horizon. In advancing then 12° , or one day's motion, along the ecliptic (or along its own orbit, which is not much inclined to it) it will become less depressed below the horizon, and have, therefore, a less hour angle to travel over by the diurnal motion after sunset the next night to bring it into view than at any other time. The most favorable harvest moon is when the full moon, falling *on the* 21st of September, happens at the same time to be *in*

the ascending node of her orbit, which then coincides with the vernal equinox.

(429.) The physical constitution of the moon is better known to us than that of any other heavenly body. By the aid of telescopes, we discern inequalities in its surface which can be no other than mountains and valleys—for this plain reason, that we see the shadows cast by the former in the exact proportion as to length which they ought to have, when we take into account the inclination of the sun's rays to that part of the moon's surface on which they stand. The convex outline of the limb turned toward the sun is always circular, and very nearly smooth; but the opposite border of the enlightened part, which (were the moon a perfect sphere) ought to be an exact and sharply defined ellipse, is always observed to be extremely ragged, and indented with deep recesses and prominent points. The mountains near this edge cast long black shadows, as they should evidently do, when we consider that the sun is in the act of rising or setting to the parts of the moon so circumstanced. But as the enlightened edge advances beyond them, *i.e.* as the sun to them gains altitude, their shadows shorten; and at the full moon, when all the light falls in our line of sight, no shadows are seen on any part of her surface. From micrometrical measures of the lengths of the shadows of the more conspicuous mountains, taken under the most favorable circumstances, the heights of many of them have been calculated. Messrs. Beer and Maedler, in their elaborate work, entitled "*Der Mond*," have given a list of heights resulting from such measurements, for no less than 1095 lunar mountains, among which occur all degrees of elevation up to 3569 toises (22,823 British feet), or about 1400 feet higher than Chimborazo in the Andes.

The existence of such mountains is further corroborated by their appearance, as small points or islands of light beyond the extreme edge of the enlightened part, which are their tops catching the sunbeams before the intermediate plain, and which, as the light advances, at length connect themselves with it, and appear as prominences from the general edge.

(430.) The generality of the lunar mountains present a striking uniformity and singularity of aspect. They are wonderfully numerous, especially toward the southern portion of the disk, occupying by far the larger portion of the surface, and almost universally of an exactly circular or cup-shaped form, foreshortened, however, into ellipses toward the limb; but the larger have for the most part flat bottoms within, from which rises centrally a small, steep, conical hill. They offer, in short, in its highest perfection, the true *volcanic* character, as it may be seen in the crater of Vesuvius, and in a map of the volcanic districts of the Campi Phlegræi¹² or the Puy de Dôme, but with this remarkable peculiarity; viz. that the bottoms of many of the craters are very deeply depressed below the general surface of the moon, the internal depth being often twice or three times the external height. In some of the principal ones, decisive marks of volcanic stratification, arising from successive deposits of ejected matter, and evident indications of lava currents streaming outward in all directions, may be clearly traced with powerful telescopes. (See Plate V. *fig. 2.*)¹³ In Lord Rosse's magnificent reflector, the flat bottom of the crater called Albategnius is seen to be strewed with blocks not visible in inferior telescopes,

¹² See Breislak's map of the environs of Naples, and Desmarest's of Auvergne.

¹³ From a drawing taken with a reflector of twenty feet focal length (*h*).

while the exterior of another (Aristillus) is all hatched over with deep gullies radiating toward its centre. What is, moreover, extremely singular in the geology of the moon is, that, although nothing having the character of seas can be traced (for the dusky spots, which are commonly called seas, when closely examined, present appearances incompatible with the supposition of deep water), yet there are large regions perfectly level, and apparently of a decided alluvial character; as there are also here and there chains of mountains whose appearance suggests no suspicion of volcanic origin. [See 430 *a*, in Note H.]

(431.) We perceive on the moon no clouds, nor any other decisive indications of an atmosphere. Were there any, it could not fail to be perceived in the occultations of stars and the phenomena of solar eclipses, as well as in a great variety of other phenomena. The moon's diameter, for example, as measured micrometrically, and as estimated by the interval between the disappearance and reappearance of a star in an occultation, ought to differ by twice the horizontal refraction at the moon's surface. No appreciable difference being perceived, we are entitled to conclude the non-existence of any atmosphere at its edge dense enough to cause a refraction of $1''$, *i. e.* having one 1980th part of the density of the earth's atmosphere. In a solar eclipse, the existence of any sensible refracting atmosphere in the moon would enable us to trace the limb of the latter beyond the cusps, externally to the sun's disk, by a *narrow, but brilliant* line of light, extending to some distance along its edge. No such phenomenon is seen. *Very* faint stars ought to be extinguished before occultation, were any appreciable amount of vapor suspended near the surface of the moon. But such is not the case; when

occulted at the bright edge, indeed, the light of the moon extinguishes small stars, and even at the dark limb, the glare in the sky caused by the near presence of the moon renders the occultation of *very* minute stars unobservable. But during the continuance of a total lunar eclipse, stars of the tenth and eleventh magnitude are seen to come up to the limb, and undergo *sudden* extinction as well as those of greater brightness.¹⁴ Hence, the climate of the moon must be very extraordinary; the alternation being that of unmitigated and burning sunshine fiercer than an equatorial noon, continued for a whole fortnight, and the keenest severity of frost, far exceeding that of our polar winters, for an equal time. Such a disposition of things must produce a constant transfer of whatever moisture may exist on its surface, from the point beneath the sun to that opposite, by distillation *in vacuo* after the manner of the little instrument called a *cryophorus*. The consequence must be absolute aridity below the vertical sun, constant accretion of hoarfrost in the opposite region, and, perhaps, a narrow zone of running water at the borders of the enlightened hemisphere.¹⁵ It is possible, then, that evaporation on the one hand, and condensation on the other, may to a certain extent preserve an equilibrium of temperature, and mitigate the extreme severity of both climates; but this process, which would imply the continual generation and destruction of an atmosphere of aqueous vapor, must, in conformity with what has been said above of a lunar atmosphere, be confined within very narrow limits.

(432.) Though the surface of the full moon exposed to us must necessarily be very much heated—*possibly* to a degree much exceeding that of boiling water—yet we *feel*

¹⁴ As observed by myself in eclipse of Oct. 13, 1837.

¹⁵ So in ed. of 1833.

no heat from it, and even in the focus of large reflectors it fails to affect the thermometer. No doubt, therefore, its heat (conformably to what is observed of that of bodies heated below the point of luminosity) is much more readily absorbed in traversing transparent media than direct solar heat, and is extinguished in the upper regions of our atmosphere, never reaching the surface of the earth at all. Some probability is given to this by the *tendency to disappearance of clouds under the full moon*, a meteorological fact (for as such we think it fully entitled to rank¹⁶) for which it is necessary to seek a cause, and for which no other rational explanation seems to offer. As for any other influence of the moon on the weather, we have no decisive evidence in its favor.

(433.) A circle of one second in diameter, as seen from the earth, on the surface of the moon, contains about a square mile. Telescopes, therefore, must yet be greatly improved, before we could expect to see signs of inhabitants, as manifested by edifices or by changes on the surface of the soil. It should, however, be observed, that, owing to the small density of the materials of the moon, and the comparatively feeble gravitation of bodies on her surface, muscular force would there go six times as far in overcoming the weight of materials as on the earth. Owing to the want of air, however, it seems impossible that any form of life, analogous to those on earth, can subsist there.

¹⁶ From my own observations, made quite independently of any knowledge of such a tendency having been observed by others. Humboldt, however, in his Personal Narrative, speaks of it as well known to the pilots and seamen of Spanish America.

M. Arago has shown from a comparison of rain, registered as having fallen during a long period, that a slight preponderance in respect of quantity falls near the new moon over that which falls near the full. This would be a natural and necessary consequence of a preponderance of a cloudless sky about the full, and forms, therefore, part and parcel of the same meteorological fact.

No appearance indicating vegetation, or the slightest variation of surface, which can, in our opinion, fairly be ascribed to change of season, can anywhere be discerned.

(434.) The lunar summer and winter arise, in fact, from the rotation of the moon on its own axis, the period of which rotation is *exactly* equal to its sidereal revolution about the earth, and is performed in a plane $1^{\circ} 30' 11''$ inclined to the ecliptic, whose *ascending* node is always precisely coincident with the *descending* node of the lunar orbit. So that the axis of rotation describes a conical surface about the pole of the ecliptic in one revolution of the node. The remarkable coincidence of the two rotations, that about the axis and that about the earth, which at first sight would seem perfectly distinct, has been asserted (but we think somewhat too hastily¹⁷) to be a consequence of the general laws to be explained hereafter. Be that how it may, it is the cause why we always see the same face of the moon, and have no knowledge of the other side.¹⁸

(435.) The moon's rotation on her axis is uniform; but since her motion in her orbit (like that of the sun) is not so, we are enabled to look a few degrees round the equatorial parts of her visible border, on the eastern or western side, according to circumstances; or, in other words, the line joining the centres of the earth and moon fluctuates a little in its position, from its mean or average intersection

¹⁷ See "Edinburgh Review," No. 175, p. 192.

¹⁸ Strange to say, there are persons who find it difficult to regard as a rotation on its own axis, that peculiarity of the moon's motion which consists in its keeping the same face always toward the earth. Should any of our readers be in this predicament, we recommend him to plant a staff upright in the ground, and, grasping it with both hands, walk round it, keeping as close to it as possible, with his face always turned toward it; when the unmistakable sensation of giddiness will effectually satisfy him of the fact of his rotation on his own axis, or he may walk round a tree, always facing it, and carrying a compass in his hand, and while watching the needle during a few circuits endeavor to persuade himself that he does *not* turn upon his own centre.

with her surface, to the east or westward. And, moreover, since the axis about which she revolves is neither exactly perpendicular to her orbit, nor holds an invariable direction in space, her poles come alternately into view for a small space at the edges of her disk. These phenomena are known by the name of *librations*. In consequence of these two distinct kinds of libration, the same identical point of the moon's surface is not always the centre of her disk, and we therefore get sight of a zone of a few degrees in breadth on all sides of the border, beyond an exact hemisphere.

(436.) If there be inhabitants in the side of the moon turned toward us, the earth must present to them the extraordinary appearance of a moon of nearly 2° in diameter, exhibiting phases complementary to those which we see the moon to do, but *immovably fixed in their sky* (or, at least, changing its apparent place only by the small amount of the libration), while the stars must seem to pass slowly beside and behind it. It will appear clouded with variable spots, and belted with equatorial and tropical zones corresponding to our trade-winds; and it may be doubted whether, in their perpetual change, the outlines of our continents and seas can ever be clearly discerned. During a solar eclipse, the earth's atmosphere will become visible as a narrow, but bright, luminous ring of a ruddy color, where it rests on the earth, gradually passing into faint blue, encircling the whole or part of the dark disk of the earth, the remainder being dark and ragged with clouds.

(436 a.) On the subject of the moon's habitability, the complete absence of air noticed in art. 431, *if general over her whole surface*, would of course be decisive. Some considerations of a contrary nature, however, suggest themselves in consequence of a remark lately made by Prof.

Hansen, viz. that the fact of the moon turning always the same face toward the earth is in all probability the result of an elongation of its figure in the direction of a line joining the centres of both the bodies acting conjointly *with a non-coincidence of its centre of gravity with its centre of symmetry*. To the middle of the length of a stick, loaded with a heavy weight at one end and a light one at the other, attach a string, and swing it round. The heavy weight will assume and maintain a position in the circulation of the joint mass further from the hand than the lighter. This is not improbably what takes place in the moon. Anticipating to a certain extent what he will find more fully detailed in the next chapter, the reader may consider the moon as retained in her orbit about the earth by some coercing power analogous to that which the hand exerts on the compound mass above described through the string. Suppose, then, its globe made up of materials not homogeneous, and so disposed in its interior that some considerable preponderance of weight should exist excentrically situated: then it will be easily apprehended that the portion of its surface nearer to that heavier portion of its solid content, under all the circumstances of a rotation so adjusted, will permanently occupy the situation most remote from the earth. Let us now consider what may be expected to be the distribution of air, water, or other fluid on the surface of such a globe, supposing its quantity not sufficient to cover and drown the whole mass. It will run toward the lowest place, that is to say, not the nearest to the centre of figure or to the central point of the mere *space* occupied by the moon, but to the centre of the *mass*, or what is called in mechanics the centre of gravity. There will be formed there an ocean, of more or less extent according to the

quantity of fluid, directly over the heavier nucleus, while the lighter portion of the solid material will stand out as a continent on the opposite side. And the height above the level of such ocean to which it will project will be greater, the greater the excentricity of the centre of gravity. Suppose then that in the case of the moon this excentricity should amount to some thirty or forty miles, such would be the general elevation of the lunar land (or the portion turned earthward) above its ocean, so that the whole of that portion of the moon we see would in fact come to be regarded as a mountainous elevation above the sea level.

(436 *b.*) In what regards its assumption of a definite level, air obeys precisely the same hydrostatical laws as water. The lunar atmosphere would rest upon the lunar ocean, and form in its basin a *lake of air*, whose upper portions at an altitude such as we are now contemplating would be of excessive tenuity, especially should the lunar provision of air be less abundant in proportion than our own. It by no means follows, then, from the absence of visible indications of water or air on this side of the moon, that the other is equally destitute of them, and equally unfitted for maintaining animal or vegetable life. Some slight approach to such a state of things actually obtains on the earth itself. Nearly all the land is collected in one of its hemispheres, and much the larger portion of the sea in the opposite (art. 284). There is evidently an excess of heavy material vertically beneath the middle of the Pacific; while not very remote from the point of the globe diametrically opposite rises the great tableland of India, and the Himalaya chain, on the summits of which the air has not more than a third of the density it has on the sea level, and from which animated existence is forever excluded.

(437.) The best charts of the lunar surface are those of Cassini, of Russel (engraved from drawings, made by the aid of a seven feet reflecting telescope), the seleno-topographical charts of Lohrmann, and the very elaborate projection of Beer and Maedler accompanying their work already cited. Madame Witte, a Hanoverian lady, has recently succeeded in producing from her own observations, aided by Maedler's charts, more than one *complete model* of the whole visible lunar hemisphere, of the most perfect kind, the result of incredible diligence and assiduity. Single craters have also been modelled on a large scale, both by her and Mr. Nasmyth. Still more recently (1851-1863), photography has been applied with success to the exact delineation of the lunar surface, by Mr. Whipple, using for this purpose the great Fraunhofer equatorial of the Observatory at Cambridge, U. S.; by Mr. Hartnup, with ~~the~~ the equatorial of the Liverpool Observatory; but more especially by Mr. De la Rue, with an equatorially mounted Newtonian reflector of 13 inches aperture and 10 feet focal length. [See § 437 *a*, in Note I.]

CHAPTER VIII

Of Terrestrial Gravity—Of the Law of Universal Gravitation—Paths of Projectiles; Apparent, Real—The Moon Retained in her Orbit by Gravity—Its Law of Diminution—Laws of Elliptic Motion—Orbit of the Earth Round the Sun in Accordance with these Laws—Masses of the Earth and Sun Compared—Density of the Sun—Force of Gravity at its Surface—Disturbing Effect of the Sun on the Moon's Motion

(438.) THE reader has now been made acquainted with the chief phenomena of the motions of the earth in its orbit round the sun, and of the moon about the earth.—We come next to speak of the physical cause which maintains and

perpetuates these motions, and causes the massive bodies so revolving to deviate continually from the directions they would naturally seek to follow, in pursuance of the first law of motion,¹ and bend their courses into curves concave to their centres.

(439.) Whatever attempts may have been made by metaphysical writers to reason away the connection of cause and effect, and fritter it down into the unsatisfactory relation of habitual sequence,² it is certain that the conception of some more real and intimate connection is quite as strongly impressed upon the human mind as that of the existence of an external world—the vindication of whose reality has (strange to say) been regarded as an achievement of no common merit in the annals of this branch of philosophy. It is our own immediate consciousness of *effort*, when we exert force to put matter in motion, or to oppose and neutralize force, which gives us this internal conviction of *power* and *causation* so far as it refers to the material world, and compels us to believe that whenever we see material objects put in motion from a state of rest, or deflected from their rectilinear paths and changed in their velocities if already in motion, it is in consequence of such an *EFFORT somehow* exerted, though not accompanied with our consciousness. That such an effort should be exerted with success through an interposed space, is no doubt diffi-

¹ Princip. Lex. I.

² See Brown "On Cause and Effect"—a work of great acuteness and subtlety of reasoning on some points, but in which the whole train of argument is vitiated by one enormous oversight; the omission, namely, of a *distinct and immediate personal consciousness of causation* in his enumeration of that *sequence of events*, by which the volition of the mind is made to terminate in the motion of material objects. I mean the consciousness of *effort*, accompanied with *intention thereby* to accomplish an end, as a thing entirely distinct from mere *desire* or *volition* on the one hand, and from mere spasmodic contraction of muscles on the other. Brown, 3d edit. Edin. 1818, p. 47. (Note to edition of 1833.)

cult to conceive. But the difficulty is no way alleviated by the interposition of any kind of material communication. The action of mind on matter admits of no explanation in words or elucidation by parallels. We know it is a fact, but are utterly incapable of analyzing it as a process.

(440.) All bodies with which we are acquainted, when raised into the air and quietly abandoned, descend to the earth's surface in lines perpendicular to it. They are therefore urged thereto by a force or effort, which it is but reasonable to regard as the direct or indirect result of a *consciousness* and a *will* existing *somewhere*, though beyond our power to trace, which force we term *gravity*, and whose tendency or direction, as universal experience teaches, is toward the earth's centre; or rather, to speak strictly, with reference to its spheroidal figure, perpendicular to the surface of still water. But if we cast a body obliquely into the air, this tendency, though not extinguished or diminished, is materially modified in its ultimate effect. The upward impetus we give the stone is, it is true, after a time destroyed, and a downward one communicated to it, which ultimately brings it to the surface, where it is opposed in its further progress, and brought to rest. But all the while it has been continually deflected or bent aside from its rectilinear progress, and made to describe a curved line concave to the earth's centre; and having a *highest point*, *vertex*, or *apogee*, just as the moon has in its orbit, where the direction of its motion is perpendicular to the radius.

(441.) When the stone which we fling obliquely upward meets and is stopped in its descent by the earth's surface, its motion is not *toward the centre*, but inclined to the earth's radius at the same angle as when it quitted our hand. As we are sure that, if not stopped by the resist-

ance of the earth, it would continue to descend, and that *obliquely*, what presumption, we may ask, is there that it would ever reach the centre toward which its motion, in no part of its visible course, was ever directed? What reason have we to believe that it might not rather circulate round it, as the moon does round the earth, returning again to the point it set out from, after completing an elliptic orbit of which the earth's centre occupies the lower focus? And if so, is it not reasonable to imagine that the same force of gravity *may* (since we know that it is exerted at all accessible heights above the surface, and even in the highest regions of the atmosphere) extend as far as 60 radii of the earth, or to the moon? and may not this be the power—for *some* power there *must* be—which deflects *her* at every instant from the tangent of her orbit, and keeps her in the elliptic path which experience teaches us she actually pursues?

(442.) If a stone be whirled round at the end of a string it will stretch the string by a *centrifugal* force, which, if the speed of rotation be sufficiently increased, will at length break the string, and let the stone escape. However strong the string, it may, by a sufficient rotary velocity of the stone, be brought to the utmost tension it will bear without breaking; and if we know what weight it is capable of carrying, the velocity necessary for this purpose is easily calculated. Suppose, now, a string to connect the earth's centre with a weight at its surface, whose strength should be just sufficient to sustain that weight suspended from it. Let us, however, for a moment imagine gravity to have no existence, and that the weight is made to revolve with the *limiting velocity* which that string can barely counteract, then will its tension be just equal to the weight of the

revolving body; and any power which should continually urge the body toward the centre with a force equal to its weight would perform the office, and might supply the place of the string, if divided. Divide it then, and in its place let gravity act, and the body will circulate as before; its tendency to the centre, or *its weight*, being just balanced by its centrifugal force. Knowing the radius of the earth, we can calculate by the principles of mechanics the periodical time in which a body so balanced must circulate to keep it up; and this appears to be $1^h 23^m 22^s$.

(443.) If we make the same calculation for a body at the distance of the moon, *supposing its weight or gravity the same as at the earth's surface*, we shall find the period required to be $10^h 45^m 30^s$. The actual period of the moon's revolution, however, is $27^d 7^h 43^m$; and hence it is clear that the moon's velocity is not nearly sufficient to sustain it against *such* a power, supposing it to revolve in a circle, or neglecting (for the present) the slight ellipticity of its orbit. In order that a body at the distance of the moon (or the moon itself) should be capable of *keeping its distance* from the earth by the outward effort of its centrifugal force, while yet its time of revolution should be what the moon's actually is, it will appear^a that *gravity*, instead of being as intense as at the surface, would require to be very nearly 3600 times less energetic; or, in other words, that its intensity is so enfeebled by the remoteness of the body on which it acts, as to be capable of producing in it, in the same time, only $\frac{1}{3600}$ th part of the motion which it would impart to the same mass of matter at the earth's surface.

(444.) The distance of the moon from the earth's centre is very nearly sixty times the distance from the centre to

^a Newton, Princip. b. i., Prop. 4, Cor. 2.

the surface, and $3600 : 1 :: 60^2 : 1^2$; so that the proportion in which we must admit the earth's gravity to be enfeebled at the moon's distance, if it be really the force which retains the moon in her orbit, must be (at least in this particular instance) that of the squares of the distances at which it is compared. Now, in such a diminution of energy with increase of distance, there is nothing *prima facie* inadmissible. Emanations from a centre, such as light and heat, do really diminish in intensity by increase of distance, and in this identical proportion; and though we cannot certainly argue much from this analogy, yet we do see that the power of magnetic and electric attractions and repulsions is actually enfeebled by distance, and much more rapidly than in the simple proportion of the increased distances. The argument, therefore, stands thus:—On the one hand, *Gravity* is a real power, of whose agency we have daily experience. We know that it extends to the greatest accessible heights, and far beyond; and we see no reason for drawing a line at any particular height, and there asserting that it must cease entirely; though we have analogies to lead us to suppose its energy may diminish as we ascend to great heights from the surface, such as that of the moon. On the other hand we are sure the moon is urged toward the earth by *some* power which retains her in her orbit and that the intensity of this power is such as would correspond to a gravity diminished in the proportion—otherwise not improbable—of the squares of the distances. If gravity be *not* that power, there must exist some other; and, besides this, gravity must cease at some inferior level, or the nature of the moon must be different from that of ponderable matter;—for if not, it would be urged by *both* powers, and therefore *too much* urged and forced inward from her path.

(445.) It is on such an argument that Newton is understood to have rested, in the first instance, and provisionally, his law of universal gravitation, which may be thus abstractly stated:—"Every particle of matter in the universe attracts every other particle, with a force directly proportioned to the mass of the attracting particle, and inversely to the square of the distance between them." In this abstract and general form, however, the proposition is not applicable to the case before us. The earth and moon are not mere *particles*, but great spherical bodies, and to such the general law does not immediately apply; and, before we can make it applicable, it becomes necessary to inquire what will be the force with which a congeries of particles, constituting a solid mass of any assigned figure, will attract another such collection of material atoms. This problem is one purely dynamical, and, in this its general form, is of extreme difficulty. Fortunately, however, for human knowledge when the attracting and attracted bodies are spheres, it admits of an easy and direct solution. Newton himself has shown (*Princip.* b. i. prop. 75) that, in that case, the attraction is precisely the same as if the whole matter of each sphere were collected into its centre, and the spheres were single particles there placed; so that, in this case, the general law applies in its strict wording. The effect of the trifling deviation of the earth from a spherical form is of too minute an order to need attention at present. It is, however, perceptible, and may be hereafter noticed.

(446.) The next step in the Newtonian argument is one which divests the law of gravitation of its provisional character, as derived from a loose and superficial consideration of the lunar orbit as a circle described with an average or mean velocity, and elevates it to the rank of a general

and primordial relation by proving its applicability to the state of existing nature in all its circumstances. This step consists in demonstrating, as he has done* (*Princip.* i. 17, i. 75), that, under the influence of such an attractive force mutually urging two spherical gravitating bodies toward each other, they will each, when moving in each other's neighborhood, be deflected into an orbit concave toward the other, and describe, one about the other regarded as fixed, or both round their common centre of gravity, curves whose forms are limited to those figures known in geometry by the general name of conic sections. It will depend, he shows, in any assigned case, upon the particular circumstances of velocity, distance, and direction, *which* of these curves shall be described—whether an ellipse, a circle, a parabola, or a hyperbola; but one or other it *must* be; and any one of any degree of excentricity it *may* be, according to the circumstances of the case; and, in all cases, the point to which the motion is referred, whether it be the centre of one of the spheres, or their common centre of gravity, will of necessity be the *focus* of the conic section described. He shows, furthermore (*Princip.* i. 1.), that, in every case, the *angular velocity* with which the line joining their centres moves, must be inversely proportional to the square of their mutual distance, and that equal areas of the curves described will be swept over by their line of junction in equal times.

* We refer for these fundamental propositions, as a point of duty, to the immortal work in which they were first propounded. It is impossible for us, in this volume, to go into these investigations: even did our limits permit, it would be utterly inconsistent with our plan; a general idea, however, of their conduct will be given in the next chapter. We trust that the careful and attentive study of the *Principia in its original form* will never be laid aside, whatever be the improvements of the modern analysis as respects facility of calculation and expression. From no other quarter can a thorough and complete comprehension of the mechanism of our system (so far as the immediate scope of that work extends), be anything like so well, and we may add, so easily obtained.

(447.) All this is in conformity with what we have stated of the solar and lunar movements. Their orbits are ellipses, but of different degrees of excentricity; and this circumstance already indicates the general applicability of the principles in question.

(448.) But here we have already, by a natural and ready implication (such is always the progress of generalization), taken a further and most important step, almost unperceived. We have extended the action of gravity to the case of the earth and sun, to a distance immensely greater than that of the moon, and to a body apparently quite of a different nature than either. Are we justified in this? or, at all events, are there no modifications introduced by the change of data, if not into the general expression, at least into the particular interpretation, of the law of gravitation? Now, the moment we come to numbers, an obvious incongruity strikes us. When we calculate, as above, from the known distance of the sun (art. 357), and from the period in which the earth circulates about it (art. 305), what must be the centrifugal force of the latter by which the sun's attraction is balanced (and which, therefore, becomes an exact measure of the sun's attractive energy as exerted on the earth), we find it to be immensely greater than would suffice to counteract the *earth's* attraction on an equal body at that distance—greater in the high proportion of 354936 to 1. It is clear, then, that if the earth be retained in its orbit about the sun by *solar attraction*, conformable in its rate of diminution with the general law, this force must be no less than 354936 times more intense than what the earth would be capable of exerting, *cæteris paribus*, at an equal distance.

(449.) What, then, are we to understand from this result? Simply this—that the sun attracts as a collection of 354936

earths occupying its place would do, or, in other words, that the sun contains 354936 times the mass or quantity of ponderable matter that the earth consists of. Nor let this conclusion startle us. We have only to recall what has been already shown in art. 358 of the gigantic dimensions of this magnificent body, to perceive that, in assigning to it so vast a mass, we are not outstepping a reasonable proportion. In fact, when we come to compare its *mass* with its *bulk* we find its density⁵ to be less than that of the earth, being no more than 0.2543. So that it must consist, in reality, of far *lighter* materials, especially when we consider the force under which its central parts must be condensed. This consideration renders it highly probable that an intense heat prevails in its interior by which its elasticity is reinforced, and rendered capable of resisting this almost inconceivable pressure without collapsing into smaller dimensions.

(450.) This will be more distinctly appreciated, if we estimate, as we are now prepared to do, the intensity of gravity at the sun's surface.

The attraction of a sphere being the same (art. 445) as if its whole mass were collected in its centre, will, of course, be proportional to the mass directly, and the square of the distance inversely; and, in this case, the distance is the radius of the sphere. Hence we conclude,⁶ that the intensities of solar and terrestrial gravity at the surfaces of the two globes are in the proportions of 27.9 to 1. A pound of terrestrial matter at the sun's surface, then, would exert a pressure equal to what 27.9 such pounds would do at the

⁵ The density of a material body is as the *mass* directly, and the volume inversely: hence density of ☉: density of ⊕:: $\frac{354936}{1384472}$:1::0.2543:1.

⁶ Solar gravity: terrestrial:: $\frac{354936}{(440000)^2}$: $\frac{1}{(4000)^2}$::27.9:1; the respective radii of the sun and earth being 440000, and 4000 miles.

earth's. The efficacy of muscular power to overcome weight is therefore proportionally nearly 28 times less on the sun than on the earth. An ordinary man, for example, would not only be unable to sustain his own weight on the sun, but would literally be crushed to atoms under the load.*

(451.) Henceforward, then, we must consent to dismiss all idea of the earth's immobility, and transfer that attribute to the sun, whose ponderous mass is calculated to exhaust the feeble attractions of such comparative atoms as the earth and moon, without being perceptibly dragged from its place. Their centre of gravity lies, as we have already hinted, almost close to the centre of the solar globe, at an interval quite imperceptible from our distance; and whether we regard the earth's orbit as being performed about the one or the other centre makes no appreciable difference in any one phenomenon of astronomy.

(452.) It is in consequence of the *mutual* gravitation of all the several parts of matter, which the Newtonian law supposes, that the earth and moon, while in the act of revolving, monthly, in their mutual orbits about their common centre of gravity, yet continue to circulate, without parting company, in a greater annual orbit round the sun. We may conceive this motion by connecting two unequal balls by a short stick, which, *at their common centre of gravity* is suspended by a long string and made to gyrate conically round a point vertically below that of suspension. Their joint system will circulate as one pendulous mass about this imaginary centre, while yet they may go on circulating round each other in subordinate gyrations, as if the stick were quite free from any such tie, and merely

* A mass weighing 12 stone or 168 lbs. on the earth would produce a pressure of 4687 lbs. on the sun.

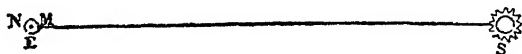
hurled through the air. If the earth alone, and not the moon, gravitated to the sun, it would be dragged away, and leave the moon behind—and *vice versa*; but, acting on both, they continue together under its attraction, just as the loose parts of the earth's surface continue to rest upon it. It is, then, in strictness, not the earth or the moon which describes an ellipse around the sun, but their common centre of gravity. The effect is to produce a small, but very perceptible, monthly *equation* in the sun's apparent motion as seen from the earth, which is always taken into account in calculating the sun's place. The moon's actual path in its compound orbit round the earth and sun is an epicycloidal curve intersecting the orbit of the earth twice in every lunar month, and alternately within and without it. But as there are not more than twelve such months in the year, and as the total departure of the moon from it either way does not exceed one 400th part of the radius, this amounts only to a slight undulation upon the earth's ellipse, so slight, indeed, that if drawn in true proportion on a large sheet of paper, no eye unaided by measurement with compasses would detect it. The real orbit of the moon is everywhere concave toward the sun.

(453.) Here moreover, *i.e.* in the attraction of the sun, we have the key to all those differences from an exact elliptic movement of the moon in her monthly orbit, which we have already noticed (arts. 407, 409), viz. to the retrograde revolution of her nodes; to the direct circulation of the axis of her ellipse; and to all the other deviations from the laws of elliptic motion at which we have further hinted. If the moon simply revolved about the earth under the influence of its gravity, none of these phenomena would take place. Its orbit would be a perfect ellipse, returning into itself,

and always lying in one and the same plane. That it is *not so*, is a proof that some cause *disturbs* it, and interferes with the earth's attraction; and this cause is no other than the sun's attraction—or rather, that part of it which is not *equally* exerted on the earth.

(454.) Suppose two stones, side by side, or otherwise situated with respect to each other, to be let fall together; then, as gravity accelerates them equally, they will retain their relative positions, and fall together as if they formed one mass. But suppose gravity to be rather more intensely exerted on one than the other; then would that one be rather more accelerated in its fall, and would gradually leave the other; and thus a relative motion between them would arise from the difference of action, however slight.

(455.) The sun is about 400 times more remote than the moon; and, in consequence, while the moon describes her monthly orbit round the earth, her distance from the sun is alternately $\frac{1}{400}$ th part greater and as much less than the earth's. Small as this is, it is yet sufficient to produce a perceptible excess of attractive tendency of the moon toward the sun, above that of the earth when in the nearer



point of her orbit, M, and a corresponding defect on the opposite part, N; and, in the intermediate positions, not only will a difference of *forces* subsist, but a difference of *directions* also; since however small the lunar orbit M N, it is not a point, and, therefore, the lines drawn from the sun S to its several parts cannot be regarded as strictly parallel. If, as we have already seen, the force of the sun were equally exerted, and in parallel directions on both, no dis-

turbance of their relative situations would take place; but from the non-verification of these conditions arises a *disturbing force*, oblique to the line joining the moon and earth, which in some situations acts to *accelerate*, in others to *retard*, her elliptic orbital motion; in some to draw the earth from the moon, in others the moon from the earth. Again, the lunar orbit, though very nearly, is yet not quite coincident with the plane of the ecliptic; and hence the action of the sun, which is very nearly parallel to the last-mentioned plane, tends to draw her somewhat *out of the plane* of her orbit, and does actually do so—producing the revolution of her nodes, and other phenomena less striking. We are not yet prepared to go into the subject of these *perturbations*, as they are called; but they are introduced to the reader's notice as early as possible, for the purpose of reassuring his mind, should doubts have arisen as to the logical correctness of our argument, in consequence of our temporary neglect of them while working our way upward to the law of gravity from a general consideration of the moon's orbit.

CHAPTER IX

OF THE SOLAR SYSTEM

Apparent Motions of the Planets—Their Stations and Retrogradations—The Sun their Natural Centre of Motion—Inferior Planets—Their Phases, Periods, etc.—Dimensions and Form of their Orbits—Transits across the Sun—Superior Planets—Their Distances, Periods, etc.—Kepler's Laws and their Interpretation—Elliptic Elements of a Planet's Orbit—Its Heliocentric and Geocentric Place—Empirical Law of Planetary Distances; Violated in the Case of Neptune—The Asteroids—Physical Peculiarities Observable in each of the Planets

(456.) THE sun and moon are not the only celestial objects which appear to have a motion independent of that

by which the great constellation of the heavens is daily carried round the earth. Among the stars there are several—and those among the brightest and most conspicuous—which, when attentively watched from night to night, are found to change their relative situations among the rest; some rapidly, others much more slowly. These are called *planets*. Four of them—Venus, Mars, Jupiter and Saturn—are remarkably large and brilliant; another, Mercury, is also visible to the naked eye as a large star, but, for a reason which will presently appear, is seldom conspicuous: a sixth, Uranus, is barely so discernible, while the rest of which about fifty are already known, and probably many more remain to be discovered, are visible only through telescopes,¹ and with one exception (that of Neptune) can only be known among the multitude of minute stars those instruments reveal to us by watching them from night to night and attending to their changes of place. All these have been discovered since the commencement of the current century, and forty-five of them since 1844. A list of their names, discoverers, and the dates of their respective discovery will be found in the Appendix. All of them but Neptune belong to a peculiar and very remarkable class or family of planets to which the name of Asteroids has been assigned.

(457.) The apparent motions of the planets are much more irregular than those of the sun or moon. Generally speaking, and comparing their places at distant times, they all advance, though with very different *average* or *mean* velocities, in the same direction as those luminaries, *i.e.* in opposition to the apparent diurnal motion, or from west

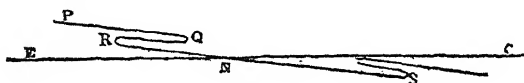
¹ One only, Vesta, is said to have been once seen by Schröter with the naked eye.

to east: all of them make the entire tour of the heavens, though under very different circumstances: and all of them, with the exception of certain among the telescopic planets, are confined in their visible paths within very narrow limits on either side the ecliptic, and perform their movements within that zone of the heavens we have called, above, the Zodiac (art. 303).

(458.) The obvious conclusion from this is, that whatever be, otherwise, the nature and law of their motions, they are performed *nearly in the plane of the ecliptic*—that plane, namely, in which our own motion about the sun is performed. Hence it follows, that we see their evolutions, not in *plan*, but in *section*; their real angular movements and linear distances being all *foreshortened* and confounded indistinguishably, while only their deviations from the ecliptic appear of their natural magnitude, undiminished by the effect of perspective.

(459.) The apparent motions of the sun and moon, though not uniform, do not deviate very greatly from uniformity; a moderate acceleration and retardation, accountable for by the ellipticity of their orbits, being all that is remarked. But the case is widely different with the planets: sometimes they advance rapidly; then relax in their apparent speed—come to a momentary stop; and then actually reverse their motion, and run back upon their former course, with a rapidity at first increasing, then diminishing, till the reversed or retrograde motion ceases altogether. Another *station*, or moment of apparent rest or indecision, now takes place; after which the movement is again reversed, and resumes its original direct character. On the whole, however, the amount of direct motion more than compensates the retrograde; and by the

excess of the former over the latter, the gradual advance of the planet from west to east is maintained. Thus, supposing the Zodiac to be unfolded into a plane surface (or represented as in Mercator's projection, art. 283, taking the



ecliptic E C for its ground line), the track of a planet when mapped down by observation from day to day, will offer the appearance P Q R S, etc.; the motion from P to Q being direct, at Q stationary, from Q to R retrograde, at R again stationary, from R to S direct, and so on.

(460.) In the midst of the irregularity and fluctuation of this motion, one remarkable feature of uniformity is observed. Whenever the planet crosses the ecliptic, as at N in the figure, it is said (like the moon) to be in its node; and as the earth necessarily lies in the plane of the ecliptic, the planet cannot be *apparently* or *uranographically* situated in the celestial circle so called, without being *really* and *locally* situated *in that plane*. The visible passage of a planet through its node, then, is a phenomenon indicative of a circumstance in its real motion quite independent of the station from which we view it. Now, it is easy to ascertain, by observation, when a planet passes from the north to the south side of the ecliptic: we have only to convert its right ascensions and declinations into longitudes and latitudes, and the change from north to south latitude on two successive days will advertise us on what *day* the transition took place; while a simple proportion, grounded on the observed state of its motion *in latitude* in the interval, will suffice to fix the precise hour and minute of its arrival on the ecliptic. Now, this being done for several

transitions from side to side of the ecliptic, and their dates thereby fixed, we find, universally, that the interval of time elapsing between the successive passages of each planet through *the same node* (whether it be the ascending or the descending) is always alike, whether the planet at the moment of such passage be direct or retrograde, swift or slow, in its apparent movement.

(461.) Here, then, we have a circumstance which, while it shows that the motions of the planets are in fact subject to certain laws and fixed periods, may lead us very naturally to suspect that the apparent irregularities and complexities of their movements may be owing to our not seeing them from their natural centre (arts. 338, 371,) and from our mixing up with their own proper motions movements of a parallactic kind, due to our own change of place, in virtue of the orbital motion of the earth about the sun.

(462.) If we abandon the earth as a centre of the planetary motions, it cannot admit of a moment's hesitation where we should place that centre with the greatest probability of truth. It must surely be the sun which is entitled to the first trial, as a station to which to refer to them. If it be not connected with them by any physical relation, it at least possesses the advantage, which the earth does not, of comparative immobility. But after what has been shown in art. 449, of the immense mass of that luminary, and of the office it performs to us as a quiescent centre of our orbital motion, nothing can be more natural than to suppose it may perform the same to other globes which, like the earth, may be revolving round it; and these globes may be visible to us by its light reflected from them, as the moon is. Now there are many facts which give a strong support to the idea that the planets are in this predicament.

(463.) In the first place, the planets really are great globes, of a size commensurate with the earth, and several of them much greater. When examined through powerful telescopes, they are seen to be round bodies, of sensible and even of considerable apparent diameter, and offering distinct and characteristic peculiarities, which show them to be material masses, each possessing its individual structure and mechanism; and *that*, in one instance at least, an exceedingly artificial and complex one. (See the representations of Mars, Jupiter and Saturn, in Plate III.) That their distances from us are great, much greater than that of the moon, and some of them even greater than that of the sun, we infer, 1st, from their being occulted by the moon, and 2dly, from the smallness of their diurnal parallax, which, even for the nearest of them, when most favorably situated, does not exceed a few seconds, and for the remote ones is almost imperceptible. From the comparison of the diurnal parallax of a celestial body, with its apparent semidiameter, we can at once estimate its real size. For the parallax is, in fact, nothing else than the apparent semidiameter of the earth as seen from the body in question (art. 339 *et seq.*); and, the intervening distance being the same, the real diameters must be to each other in the proportion of the apparent ones. Without going into particulars, it will suffice to state it as a general result of that comparison, that the planets are all of them incomparably smaller than the sun, but some of them as large as the earth, and others much greater.

(464.) The next fact respecting them is, that their distances from us, as estimated from the measurement of their angular diameters, are in a continual state of change, periodically increasing and decreasing within certain limits, but

by no means corresponding with the supposition of regular circular or elliptic orbits described by them about the earth as a centre or focus, but maintaining a constant and obvious relation to their apparent angular distances or *elongations* from the sun. For example; the apparent diameter of Mars is greatest when in opposition (as it is called) to the sun, *i.e.* when in the opposite part of the ecliptic, or when it comes on the meridian at midnight—being then about 18"—but diminishes rapidly from that amount to about 4", which is its apparent diameter when in *conjunction*, or when seen in nearly the same direction as that luminary. This, and facts of a similar character, observed with respect to the apparent diameters of the other planets, clearly point out the sun as having more than an accidental relation to their movements.

(465.) Lastly, certain of the planets (Mercury, Venus, and Mars), when viewed through telescopes, exhibit the appearance of phases like those of the moon. This proves that they are opaque bodies, shining only by reflected light, which can be no other than that of the sun's; not only because there is no other source of light external to them sufficiently powerful, but because the appearance and succession of the phases themselves are (like their visible diameters) intimately connected with their elongations from the sun, as will presently be shown.

(466.) Accordingly it is found, that, when we refer the planetary movements to the sun as a centre, all that apparent irregularity which they offer when viewed from the earth disappears at once, and resolves itself into one simple and general law, of which the earth's motion, as explained in a former chapter, is only a particular case. In order to show how this happens, let us take the case of a single

planet, which we will suppose to revolve round the sun, in a plane nearly, but not quite, coincident with the ecliptic, but passing through the sun, and of course intersecting the ecliptic in a fixed line, which is the line of the planet's nodes. This line must of course divide its orbit into two segments; and it is evident that, so long as the circumstances of the planet's motion remain otherwise unchanged, the times of describing these segments must remain the same. The interval, then, between the planet's quitting either node, and returning to *the same* node again, must be that in which it describes one complete revolution round the sun, or its periodic time; and thus we are furnished with a direct method of ascertaining the periodic time of each planet.

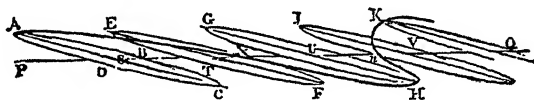
(467.) We have said (art. 457) that the planets make the entire tour of the heavens under very different circumstances. This must be explained. Two of them—Mercury and Venus—perform this circuit evidently as attendants upon the sun, from whose vicinity they never depart beyond a certain limit. They are seen sometimes to the east, sometimes to the west of it. In the former case they appear conspicuous over the western horizon, just after sunset, and are called evening stars: Venus, especially, appears occasionally in this situation with a dazzling lustre; and in favorable circumstances may be observed to cast a pretty strong shadow.² When they happen to be to the west of the sun, they rise before that luminary in the morning, and appear over the eastern horizon as morning stars: they do not, however, attain the same *elongation* from the sun.

² It must be thrown upon a white ground. An open window in a white-washed room is the best exposure. In this situation I have observed not only the shadow, but the diffracted fringes edging its outline.—*H.* Note to the edition of 1833. Venus may often be seen with the naked eye in the daytime.

Mercury never attains a greater angular distance from it than about 29° , while Venus extends her excursions on either side to about 47° . When they have receded from the sun, *eastward*, to their respective distances, they remain for a time, as it were, immovable *with respect to it*, and are carried along with it in the ecliptic with a motion equal to its own; but presently they begin to approach it, or, which comes to the same, their motion in longitude diminishes, and the sun gains upon them. As this approach goes on, their continuance above the horizon after sunset becomes daily shorter, till at length they set before the darkness has become sufficient to allow of their being seen. For a time, then, they are not seen at all, unless on very rare occasions, when they are to be observed *passing across the sun's disk as small, round, well-defined black spots*, totally different in appearance from the solar spots (art. 386). These phenomena are emphatically called *transits* of the respective planets across the sun, and take place when the earth happens to be passing the line of their nodes while they are in that part of their orbits, just as in the account we have given (art. 412) of a solar eclipse. After having thus continued invisible for a time, however, they begin to appear on the other side of the sun, at first showing themselves only for a few minutes before sunrise, and gradually longer and longer as they recede from him. At this time their motion in longitude is rapidly retrograde. Before they attain their greatest elongation, however, they become stationary in the heavens; but their recess from the sun is still maintained by the advance of that luminary along the ecliptic, which continues to leave them behind, until, having reversed their motion, and become again *direct*, they acquire sufficient speed to commence overtaking him—at which moment

they have their greatest *western* elongation; and thus is a kind of oscillatory movement kept up, while the general advance along the ecliptic goes on.

(468.) Suppose P Q to be the ecliptic, and A B C D the orbit of one of these planets (for instance, Mercury), seen almost edgewise by an eye situated very nearly in its plane; S, the sun, its centre; and A, B, C, D successive positions of the planet, of which B and D are in the nodes. If, then, the sun S stood apparently still in the ecliptic, the planets would simply appear to oscillate backward and forward from A to C, alternately passing before and behind the sun; and, if the eye happened to lie exactly in the plane of the orbit, *transiting* his disk in the former case, and being covered by it in the latter. But as the sun is not so

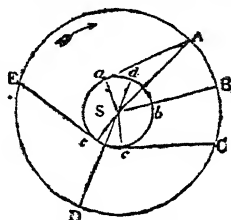


stationary, but apparently carried along the ecliptic P Q, let it be supposed to move over the spaces S T, T U, U V, while the planet in each case executes one quarter of its period. Then will its orbit be apparently carried along with the sun, into the successive positions represented in the figure; and while its real motion round the sun brings it into the respective points, B, C, D, A, its apparent movement in the heavens will seem to have been along the wavy or zigzag line A N H K. In this, its motion in longitude will have been direct in the parts A N, N H, and retrograde in the parts H n K; while at the turns of the zigzag, as at H, it will have been stationary.

(469.) The only two planets—Mercury and Venus—whose evolutions are such as above described, are called

inferior planets; their points of furthest recess from the sun are called (as above) their *greatest* eastern and western *elongations*; and their points of nearest approach to it, their *inferior* and *superior* conjunctions—the former when the planet passes between the earth and the sun, the latter when behind the sun.

(470.) In art. 467 we have traced the apparent path of an inferior planet, by considering its orbit in section, or as viewed from a point in the plane of the ecliptic. Let us now contemplate it *in plan*, or as viewed from a station above that plane, and projected on it. Suppose then, S to represent the sun, $a\ b\ c\ d$ the orbit of Mercury, and A B C D a part of that of the earth—the direction of the circulation being the same in both, viz. that of the arrow. When the planet stands at a , let the earth be situated at A, in the direction of a tangent, $a\ A$, to its orbit; then it is evident that it will appear at its *greatest elongation* from the sun—the angle $a\ A\ S$, which measures their apparent interval as seen from A, being then greater than in any other situation of a upon its own circle.



(471.) Now, this angle being known by observation, we are hereby furnished with a ready means of ascertaining, at least approximately, the distance of the planet from the sun, or the radius of its orbit, supposed a circle. For the triangle S A a is right-angled at a , and consequently we have $S\ a : S\ A :: \sin. S\ A\ a : \text{radius}$, by which proportion the radii S a , S A of the two orbits are directly compared. If the orbits were both exact circles, this would of course be a perfectly rigorous mode of proceeding: but (as is proved by the inequality of the resulting values of S a ob-

tained at different times) this is not the case; and it becomes necessary to admit an excentricity of position, and a deviation from the exact circular form in *both* orbits, to account for this difference. Neglecting, however, at present this inequality, a mean or average value of $S\alpha$ may, at least, be obtained from the frequent repetition of this process in all varieties of situation of the two bodies. The calculations being performed, it is concluded that the mean distance of Mercury from the sun is about 36000000 miles; and that of Venus, similarly derived, about 68000000; the radius of the earth's orbit being 95000000.

(472.) The sidereal periods of the planets may be obtained (as before observed), with a considerable approach to accuracy, by observing their passages through the nodes of their orbits; and indeed, when a certain very minute motion of these nodes and the apsides of their orbits (similar to that of the moon's nodes and apsides, but incomparably slower) is allowed for, with a precision only limited by the imperfection of the appropriate observations. By such observation, so corrected, it appears that the sidereal period of Mercury is $87^{\text{d}} 23^{\text{h}} 15^{\text{m}} 43.9^{\text{s}}$; and that of Venus, $224^{\text{d}} 16^{\text{h}} 49^{\text{m}} 8.0^{\text{s}}$. These periods, however, are widely different from the intervals at which the successive appearances of the two planets at their eastern and western elongations from the sun are observed to happen. Mercury is seen at its greatest splendor as an evening star, at average intervals of about 116, and Venus at intervals of about 584 days. The difference between the *sidereal* and *synodical* revolutions (art. 418) accounts for this. Referring again to the figure of art. 470, if the earth stood still at A, while the planet advanced in its orbit, the lapse of a sidereal period, which should bring it round again to α , would also produce a similar

elongation from the sun. But, meanwhile, the earth has advanced in its orbit in the same direction toward E, and therefore the next greatest elongation on the same side of the sun will happen—not in the position a A of the two bodies, but in some more advanced position, e E. The determination of this position depends on a calculation exactly similar to what has been explained in the article referred to; and we need, therefore, only here state the resulting synodical revolutions of the two planets, which come out respectively 115.877^d , and 583.920^d .

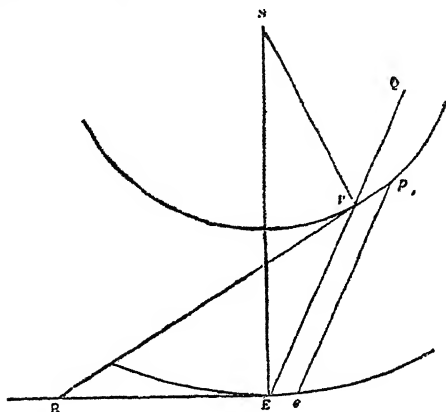
(473.) In this interval, the planet will have described a whole revolution *plus* the arc $a c e$, and the earth only the arc A C E of its orbit. During its lapse, the *inferior conjunction* will happen when the earth has a certain intermediate situation, B, and the planet has reached b , a point between the sun and earth. The greatest elongation on the opposite side of the sun will happen when the earth has come to C, and the planet to c , where the line of junction C c is a tangent to the interior circle on the opposite side from M. Lastly, the *superior conjunction* will happen when the earth arrives at D, and the planet at d in the same line prolonged on the other side of the sun. The intervals at which these phenomena happen may easily be computed from a knowledge of the synodical periods and the radii of the orbits.

(474.) The circumferences of circles are in the proportion of their radii. If, then, we calculate the circumferences of the orbits of Mercury and Venus, and the earth, and compare them with the times in which their revolutions are performed, we shall find that the actual velocities with which they move in their orbits differ greatly; that of Mercury being about 109360 miles per hour, of Venus

80000, and of the earth 68040. From this it follows, that at the inferior conjunction, or at *b*, either planet is moving in the *same* direction as the earth, but with a greater velocity; it will, therefore, leave the earth *behind* it; and the apparent motion of the planet viewed from the earth, will be *as if* the planet stood still, and the earth moved in a contrary direction from what it really does. In this situation, then, the apparent motion of the planet must be contrary to the apparent motion of the sun; and, therefore, retrograde. On the other hand, at the superior conjunction, the real motion of the planet being in the opposite direction to that of the earth, the relative motion will be the same as if the planet stood still, and the earth advanced with their united velocities in its own proper direction. In this situation, then, the apparent motion will be direct. Both these results are in accordance with observed fact.

(475.) The stationary points may be determined by the following consideration. At *a* or *c*, the points of greatest elongation, the motion of the planet is directly to or from the earth, or *along* their line of junction, while that of the earth is nearly perpendicular to it. Here, then, the apparent motion must be direct. At *b*, the inferior conjunction, we have seen that it must be retrograde, owing to the planet's motion (which is there, as well as the earth's, *perpendicular* to the line of junction) surpassing the earth's. Hence, the stationary points ought to lie, as it is found by observation they do, between *a* and *b*, or *c* and *b*, viz. in such a position that the obliquity of the planet's motion with respect to the line of junction shall just compensate for the excess of its velocity, and cause an equal advance of each extremity of that line, by the motion of the planet at one end, and of the earth at the other: so that, for an

instant of time, the whole line shall move parallel to itself. The question thus proposed is purely geometrical, and its solution on the supposition of circular orbits is easy. Let $E e$ and $P p$ represent small arcs of the orbits of the earth and planet described contemporaneously, at the moment when the latter appears stationary, about S , the sun. Pro-



duce p P and e E, tangents at P and E, to meet at R, and prolong E P backward to Q, join e p . Then since P E, p e are parallel we have by similar triangles P p : E e :: P R : R E, and since, putting v and V for the respective velocities of the planet and the earth, P p : E e :: v : V; therefore

$$\begin{aligned} v : V &:: P R : R E :: \sin. P E R : \sin. E P R \\ &:: \cos. S E P : \cos. S P Q \\ &:: \cos. S E P : \cos. (S E P + E S P) \end{aligned}$$

because the angles SER and SPR are right angles. Moreover, if r and R be the radii of the respective orbits, we have also

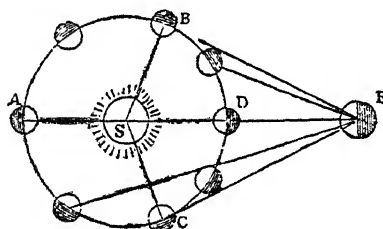
$$r : \mathbf{R} :: \sin. \text{ S E P} : \sin. (\text{S E P} + \text{E S P})$$

from which two relations it is easy to deduce the values

of the two angles SEP and ESP ; the former of which is the apparent elongation of the planet from the sun,^{*} the latter the difference of heliocentric longitudes of the earth and planet.

(476.) When we regard the orbits as other than circles (which they really are), the problem becomes somewhat complex—too much so to be here entered upon. It will suffice to state the results which experience verifies, and which assign the stationary points of Mercury at from 15° to 20° of elongation from the sun, according to circumstances; and of Venus, at an elongation never varying much from 29° . The former continues to retrograde during about 22 days; the latter, about 42.

(477.) We have said that some of the planets exhibit phases like the moon. This is the case with both Mercury and Venus; and is readily explained by a consideration of their orbits, such as we have above supposed them. In



fact, it requires little more than mere inspection of the figure annexed, to show, that to a spectator situated on the earth E , an inferior planet, illuminated by the sun, and therefore bright

on the side next to him, and dark on that turned from him, will appear *full* at the superior conjunction A ; *gibbous* (i.e. more than half full, like the moon between the first and second quarter) between that point and the

^{*} If $\frac{R}{r} = m$ and $\frac{V}{v} = n$, $SEP = \phi$, $ESP = \psi$, the equations to be resolved are

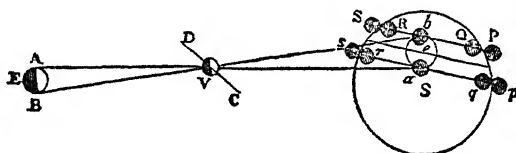
$\sin. (\phi + \psi) = m \sin. \phi$, and $\cos. (\phi + \psi) = n \cos. \psi$, which gives $\cos. \psi = \frac{1 + mn}{m + n}$

points B C of its greatest elongation; half-mooned at these points; and crescent-shaped, or horned, between these and the inferior conjunction D. As it approaches this point, the crescent ought to thin off till it vanishes altogether, rendering the planet invisible, unless in those cases where it *transits* the sun's disk, and appears on it as a black spot. All these phenomena are exactly conformable to observation.

(478.) The variation in brightness of Venus in different parts of its apparent orbit is very remarkable. This arises from two causes: 1st, the varying proportion of its visible illuminated area to its whole disk; and, 2dly, the varying angular diameter, or whole apparent magnitude of the disk itself. As it approaches its inferior conjunction from its greater elongation, the half-moon becomes a crescent, which *thins off*; but this is more than compensated, for some time, by the increasing apparent magnitude, in consequence of its diminishing distance. Thus the total light received from it goes on increasing, till at length it attains a maximum, which takes place when the planet's elongation is about 40° .

(479.) The transits of Venus are of very rare occurrence, taking place alternately at the very unequal but regularly recurring intervals of 8, 122, 8, 105, 8, 122, etc., years in succession, and always in June or December. As astronomical phenomena, they are extremely important; since they afford the best and most exact means we possess of ascertaining the sun's distance, or its parallax. Without going into the niceties of calculation of this problem, which, owing to the great multitude of circumstances to be attended to, are extremely intricate, we shall here explain its principle, which, in the abstract, is very simple and obvious. Let E be the earth, V Venus, and S the

sun, and $C D$ the portion of Venus's relative orbit which she describes while in the act of transiting the sun's disk. Suppose $A B$ two spectators at opposite extremities of that diameter of the earth which is perpendicular to the ecliptic, and, to avoid complicating the case, let us lay out of consideration the earth's rotation, and suppose A, B , to retain that situation during the whole time of the transit. Then, at any moment when the spectator at A sees the centre of Venus projected at a on the sun's disk, he at B will see it projected at b . If then one or other spectator could suddenly transport himself from A to B , he would see Venus suddenly displaced on the disk from a to b ; and if he had any means of noting accurately the place of the points on



the disk, either by micrometrical measures from its edge, or by other means, he might ascertain the angular measure of $a b$ as seen from the earth. Now, since $A V a$, $B V b$, are straight lines, and therefore make equal angles on each side V , $a b$ will be to $A B$ as the distance of Venus from the sun is to its distance from the earth, or as 68 to 27, or nearly as $2\frac{1}{2}$ to 1; $a b$ therefore occupies on the sun's disk a space $2\frac{1}{2}$ times as great as the earth's diameter; and its angular measure is therefore equal to about $2\frac{1}{2}$ times the earth's apparent diameter at the distance of the sun, or (which is the same thing) to five times the sun's horizontal parallax (art. 298). Any error, therefore, which may be committed in measuring $a b$, will entail only *one-fifth* of that error on the horizontal parallax concluded from it.

(480.) The thing to be ascertained, therefore, is, in fact, neither more nor less than the breadth of the zone $PQR S$, $pqr s$, included between the extreme apparent paths of the centre of Venus across the sun's disk, from its entry on one side to its quitting it on the other. The whole business of the observers at A, B , therefore, resolves itself into this;—to ascertain, with all possible care and precision, each at his own station, this path—where it enters, where it quits, and what segment of the sun's disk it cuts off. Now, one of the most exact ways in which (conjoined with careful micrometric measures) this can be done, is by noting the *time* occupied in the whole transit: for the relative angular motion of Venus being, in fact, very precisely known from the tables of her motion, and the apparent path being very nearly a straight line, these times give us a measure (*on a very enlarged scale*) of the lengths of the chords of the segments cut off; and the sun's diameter being known also with great precision, their versed sines, and therefore their difference, or the breadth of the zone required, becomes known. To obtain these times correctly, each observer must ascertain the instants of ingress and egress of the *centre*. To do this, he must note, 1st, the instant when the first visible impression or notch on the edge of the disk at P is produced, or the *first external contact*; 2dly, when the planet is just wholly immersed, and the broken edge of the disk just closes again at Q , or the *first internal contact*; and, lastly, he must make the same observations at the egress at R, S . The mean of the internal and external contacts, corrected for the curvature of the sun's limb in the intervals of the respective points of contact, internal and external, gives the entry and egress of the planet's centre.

(481.) The modifications introduced into this process by the earth's rotation on its axis, and by other geographical stations of the observers thereon than here supposed, are similar in their principles to those which enter into the calculation of a solar eclipse, or the occultation of a star by the moon, only more refined. Any consideration of them, however, here, would lead us too far; but in the view we have taken of the subject, it affords an admirable example of the way in which minute elements in astronomy may become magnified in their effects, and, by being made subject to measurement on a greatly enlarged scale, or by substituting the measure of time for space, may be ascertained with a degree of precision adequate to every purpose, by only watching favorable opportunities, and taking advantage of nicely adjusted combinations of circumstance. So important has this observation appeared to astronomers, that at the last transit of Venus, in 1769, expeditions were fitted out, on the most efficient scale, by the British, French, Russian, and other governments, to the remotest corners of the globe, for the express purpose of performing it. The celebrated expedition of Captain Cook to Otaheite was one of them. The general result of all the observations made on this most memorable occasion gives $8''5776$ for the sun's horizontal parallax. The next two occurrences of this phenomenon will happen on December 8, 1874, and December 6, 1882. [See Note F, § 357 *b b*.]

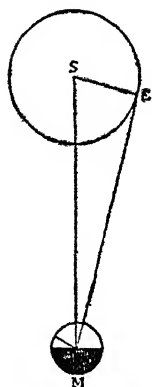
(482.) The orbit of Mercury is very elliptical, the eccentricity being nearly one-fourth of the mean distance. This appears from the inequality of the greatest elongations from the sun, as observed at different times, and which vary between the limits $16^{\circ} 12'$ and $28^{\circ} 48'$, and, from exact measures of such elongations, it is not difficult to show that the

orbit of Venus also is slightly excentric, and that both these planets, in fact, describe ellipses, having the sun in their common focus.

(483.) Transits of Mercury over the sun's disk occasionally occur, as in the case of Venus, but more frequently; those at the ascending node in November, at the descending in May. The intervals (considering each node separately) are *usually* either 13 or 7 years, and in the order 13, 13, 13, 7, etc.; but owing to the considerable inclination of the orbit of Mercury to the ecliptic, this cannot be taken as an exact expression of the said recurrence, and it requires a period of at least 217 years to bring round the transits in regular order. One will occur in the present year (1848), the next in 1861. They are of much less astronomical importance than that of Venus, on account of the proximity of Mercury to the sun, which affords a much less favorable combination for the determination of the sun's parallax.

(484.) Let us now consider the superior planets, or those whose orbits inclose on all sides that of the earth. That they do so is proved by several circumstances:—1st, They are not, like the inferior planets, confined to certain limits of elongation from the sun, but appear at all distances from it, even in the opposite quarter of the heavens, or, as it is called, in *opposition*; which could not happen, did not the earth at such times place itself between them and the sun: 2dly, They never appear horned, like Venus or Mercury, nor even semilunar. Those, on the contrary, which, from the minuteness of their parallax, we conclude to be the most distant from us, viz. Jupiter, Saturn, Uranus, and Neptune, never appear otherwise than round; a sufficient proof, of itself, that we see them always in a direction not very remote from that in which the sun's rays illuminate

them; and that, therefore, we occupy a station which is never very widely removed from the centre of their orbits, or, in other words, that the earth's orbit is entirely inclosed within theirs, and of comparatively small diameter. One only of them, Mars, exhibits any perceptible *phase*, and in its deficiency from a circular outline, never surpasses a moderately *gibbous* appearance—the enlightened portion of the disk being never less than seven-eighths of the whole.

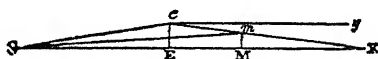


To understand this, we need only cast our eyes on the annexed figure, in which E is the earth, at its apparent greatest elongation from the sun S, as seen from Mars, M. In this position, the angle S M E, included between the lines S M and E M, is at its maximum; and therefore, in this state of things, a spectator on the earth is enabled to see a greater portion of the dark hemisphere of Mars than in any other situation.

The extent of the phase, then, or greatest observable degree of gibbosity, affords a measure—a sure, although a coarse and rude one—of the angle S M E, and therefore of the proportion of the distance S M, of Mars, to S E, that of the earth from the sun, by which it appears that the diameter of the orbit of Mars cannot be less than $1\frac{1}{2}$ times that of the earth's. The phases of Jupiter, Saturn, Uranus, and Neptune, being imperceptible, it follows that their orbits must include not only that of the earth, but of Mars also.

(485.) All the superior planets are retrograde in their apparent motions when in *opposition*, and for some time before and after; but they differ greatly from each other, both in the extent of their arc of retrogradation, in the

duration of their retrograde movement, and in its rapidity when swiftest. It is more extensive and rapid in the case of Mars than of Jupiter, of Jupiter than of Saturn, of that planet than of Uranus, and of Uranus again than Neptune. The angular velocity with which a planet appears to retrograde is easily ascertained by observing its apparent place in the heavens from day to day; and from such observations, made about the time of opposition, it is easy to conclude the relative magnitudes of their orbits, as compared with the earth's, supposing their periodical times known. For, from these, their mean angular velocities are known also, being inversely as the times. Suppose, then, $E e$ to be a very small portion of the earth's orbit, and $M m$ a corresponding portion of that of a superior planet, described



on the day of opposition, about the sun S , on which day the three bodies lie in one straight line $S E M X$. Then the angles $E S e$ and $M S m$ are given. Now, if $e m$ be joined and prolonged to meet $S M$ continued in X , the angle $e X E$, which is equal to the alternate angle $X e Y$, is evidently the retrogradation of Mars on that day, and is, therefore, also given. $E e$, therefore, and the angle $E X e$, being given in the right-angled triangle $E e X$, the side $E X$ is easily calculated, and thus $S X$ becomes known. Consequently, in the triangle $S m X$, we have given the side $S X$ and the two angles $m S X$, and $m X S$, whence the other sides, $S m$, $m X$, are easily determined. Now, $S m$ is no other than the radius of the orbit of the superior planet required, which in this calculation is supposed circular, as well as that of the earth; a supposition not exact,

but sufficiently so to afford a satisfactory approximation to the dimensions of its orbit, and which, if the process be often repeated, in every variety of situation at which the opposition can occur, will ultimately afford an average or mean value of its diameter fully to be depended upon.

(486.) To apply this principle, however, to practice, it is necessary to know the periodic times of the several planets. These may be obtained directly, as has been already stated, by observing the intervals of their passages through the ecliptic; but, owing to the very small inclination of the orbits of some of them to its plane, they cross it so obliquely that the precise moment of their arrival on it is not ascertainable, unless by very nice observations. A better method consists in determining, from the observations of several successive days, the exact moments of their arriving *in opposition* with the sun, the criterion of which is a difference of longitudes between the sun and planet of exactly 180° . The interval between successive oppositions thus obtained is nearly one *synodical* period; and would be exactly so, were the planet's orbit and that of the earth both circles, and uniformly described; but as that is found not to be the case (and the criterion is, the *inequality* of successive synodical revolutions so observed), the average of a great number, taken in all varieties of situation in which the oppositions occur, will be freed from the elliptic inequality, and may be taken as a *mean synodical* period. From this, by the considerations and by the process of calculation, indicated (art. 418) the sidereal periods are readily obtained. The accuracy of this determination will, of course, be greatly increased by embracing a long interval between the extreme observations employed. In point of fact, that interval extends to nearly 2000 years in the cases of the

planets known to the ancients, who have recorded their observations of them in a manner sufficiently careful to be made use of. Their periods may, therefore, be regarded as ascertained with the utmost exactness. Their numerical values will be found stated, as well as the mean distances, and all the other elements of the planetary orbits, in the synoptic table at the end of the volume, to which (to avoid repetition) the reader is once for all referred.

(487.) In casting our eyes down the list of the planetary distances, and comparing them with the periodic times, we cannot but be struck with a certain correspondence. The greater the distance, or the larger the orbit, evidently the longer the period. The order of the planets, beginning from the sun, is the same, whether we arrange them according to their distances, or to the time they occupy in completing their revolutions; and is as follows:—Mercury, Venus, Earth, Mars, the recently discovered family of Asteroids, Jupiter, Saturn, Uranus, and Neptune. Nevertheless, when we come to examine the numbers expressing them, we find that the relation between the two series is not that of simple *proportional* increase. The periods increase more than in proportion to the distances. Thus, the period of Mercury is about 88 days, and that of the Earth 365—being in proportion as 1 to 4.15, while their distances are in the less proportion of 1 to 2.56; and a similar remark holds good in every instance. Still, the ratio of increase of the times is not so rapid as that of the *squares* of the distances. The square of 2.56 is 6.5536, which is considerably greater than 4.15. An intermediate rate of increase, between the simple proportion of the distances and that of their squares, is therefore clearly pointed out by the sequence of the numbers; but it required no ordinary penetration in the illustrious

Kepler, backed by uncommon perseverance and industry, at a period when the data themselves were involved in obscurity, and when the processes of trigonometry and of numerical calculation were encumbered with difficulties, of which the more recent invention of logarithmic tables has happily left us no conception, to perceive and demonstrate the real law of their connection. This connection is expressed in the following proposition:—"The squares of the periodic times of any two planets are to each other, in the same proportion as the cubes of their mean distances from the sun." Take, for example, the Earth and Mars,⁴ whose periods are in the proportion of 3652564 to 6869796, and whose distance from the sun is that of 100000 to 152369; and it will be found, by any one who will take the trouble to go through the calculation, that—

$$(3652564)^2 : (6869796)^2 :: (100000)^3 : (152369)^3.$$

(488.) Of all the laws to which induction from pure observation has ever conducted man, this *third law* (as it is called) of *Kepler* may justly be regarded as the most remarkable, and the most pregnant with important consequences. When we contemplate the constituents of the planetary system from the point of view which this relation affords us, it is no longer mere analogy which strikes us—no longer a general resemblance among them, as individuals independent of each other, and circulating about the sun, each according to its own peculiar nature, and connected with it by its own peculiar tie. The resemblance is now perceived to be a true *family* likeness; they are bound up in one chain—interwoven in one web of

⁴ The expression of this law of Kepler requires a slight modification when we come to the extreme nicety of numerical calculation, for the greater planets, due to the influence of their masses. This correction is imperceptible for the Earth and Mars.

mutual relation and harmonious agreement—subjected to one pervading influence, which extends from the centre to the furthest limits of that great system, of which all of them, the earth included, must henceforth be regarded as members.

(489.) The laws of elliptic motion about the sun as a focus, and of the equable description of areas by lines joining the sun and planets, were originally established by Kepler, from a consideration of the observed motions of Mars; and were by him extended, analogically, to all the other planets. However precarious such an extension might then have appeared, modern astronomy has completely verified it as a matter of fact, by the general coincidence of its results with entire series of observations of the apparent places of the planets. These are found to accord satisfactorily with the assumption of a particular ellipse for each planet, whose magnitude, degree of eccentricity, and situation in space, are numerically assigned in the synoptic table before referred to. It is true, that when observations are carried to a high degree of precision, and when each planet is traced through many successive revolutions, and its history carried back, by the aid of calculations founded on these data, for many centuries, we learn to regard the laws of Kepler as only *first approximations* to the much more complicated ones which actually prevail; and that to bring remote observations into rigorous and mathematical accordance with each other, and at the same time to retain the extremely convenient nomenclature and relations of the ELLIPTIC SYSTEM, it becomes necessary to modify, to a certain extent, our verbal expression of the laws, and to regard the numerical data, or *elliptic elements* of the planetary orbits as not absolutely permanent, but

subject to a series of extremely slow and almost imperceptible changes. These changes may be neglected when we consider only a few revolutions; but going on from century to century, and continually accumulating, they at length produce material departures in the orbits from their original state. Their explanation will form the subject of a subsequent chapter; but for the present we must lay them out of consideration, as of an order too minute to affect the general conclusions with which we are now concerned. By what means astronomers are enabled to compare the results of the elliptic theory with observation, and thus satisfy themselves of its accordance with nature, will be explained presently.

(490.) It will first, however, be proper to point out what particular theoretical conclusion is involved in each of the three laws of Kepler, considered as satisfactorily established—what indication each of them, separately, affords of the mechanical forces prevalent in our system, and the mode in which its parts are connected—and how, when thus considered, they constitute the basis on which the Newtonian explanation of the mechanism of the heavens is mainly supported. To begin with the first law, that of the equable description of areas.—Since the planets move in curvilinear paths, they *must* (if they be bodies obeying the laws of dynamics) be deflected from their otherwise natural rectilinear progress *by force*. And from this law, taken as a matter of observed fact, it follows, that the *direction* of such force, at every point of the orbit of each planet, always *passes through the sun*. No matter from what ultimate cause the power which is called gravitation originates—be it a virtue lodged in the sun as its receptacle, or be it pressure from without, or the resultant of many pressures

or solicitations of unknown fluids, magnetic or electric ethers, or impulses—still, when finally brought under our contemplation, and summed up into a single resultant energy—its *direction* is, *from* every point on all sides, *toward the sun's centre*. As an abstract dynamical proposition, the reader will find it demonstrated by Newton, in the first proposition of the *Principia*, with an elementary simplicity to which we really could add nothing but obscurity by amplification, that any body, urged toward a certain central point by a force continually directed thereto, and thereby deflected into a curvilinear path, will describe about that centre equal areas in equal times; and *vice versâ*, that such equable description of areas is itself the essential criterion of a continual direction of the acting force toward the centre to which this character belongs. The first law of Kepler, then, gives us no information as to the nature or intensity of the force urging the planets to the sun; the only conclusion it involves is, that it does so urge them. It is a property of orbital rotation under the influence of central forces *generally*, and, as such, we daily see it exemplified in a thousand familiar instances. A similar experimental illustration of it is to tie a bullet to a thin string, and, having whirled it round with a moderate velocity in a vertical plane, to draw the end of the string through a small ring, or allow it to coil itself round the finger, or round a cylindrical rod held very firmly in a horizontal position. The bullet will then approach the centre of motion in a spiral line; and the increase of its angular velocity, and the rapid diminution of its periodic time when near the centre, will express, more clearly than any words, the compensation by which its uniform description of areas is maintained under a constantly diminishing distance. If

the motion be reversed, and the thread allowed to uncoil, beginning with a rapid impulse, the angular velocity will diminish by the same degrees as it before increased. The increasing rapidity of a dancer's *pirouette*, as he draws in his limbs and straightens his whole person, so as to bring every part of his frame as near as possible to the axis of his motion, is another instance where the connection of the observed effect with the central force exerted, though equally real, is much less obvious.

(491.) The second law of Kepler, or that which asserts that the planets describe ellipses about the sun as their focus, involves, as a consequence, the *law* of solar gravitation (so be it allowed to call the force, whatever it be, which urges them toward the sun) as exerted on each individual planet, apart from all connection with the rest. A straight line, dynamically speaking, is the only path which can be pursued by a body *absolutely free*, and under the action of *no* external force. All *deflection* into a curve is evidence of the exertion of a force; and the greater the deflection in equal times, the more intense the force. Deflection from a straight line is only another word for *curvature* of path; and as a circle is characterized by the uniformity of its curvatures in all its parts—so is every other curve (as an ellipse) characterized by the particular *law* which regulates the increase and diminution of its curvature as we advance along its circumference. The deflecting force, then, which continually bends a moving body into a curve, may be ascertained, provided its direction, in the first place, and, secondly, the law of curvature of the curve itself, be known. Both these enter as elements into the expression of the force. A body may describe, for instance, an ellipse, under a great variety of dispositions of the acting forces: it

may glide along it, for example, as a bead upon a polished wire, bent into an elliptic form; in which case the acting force is always perpendicular to the wire, and the velocity is uniform. In this case the *force* is directed to *no fixed* centre, and there is no equable description of areas at all. Or it may describe it as we may see done, if we suspend a ball by a *very* long string, and, drawing it a little aside from the perpendicular, throw it *round* with a gentle impulse. In this case the acting force is directed to the centre of the ellipse, about which areas are described equably, and to which a force *proportional* to the distance (the decomposed result of terrestrial gravity) perpetually urges it.* This is at once a very easy experiment, and a very instructive one, and we shall again refer to it. In the case before us, of an ellipse described by the action of a force directed to the *focus*, the steps of the investigation of the law of force are these: 1st, The law of the areas determines the actual *velocity* of the revolving body at every point, or the space really run over by it in a given minute portion of time; 2dly, The law of curvature of the ellipse determines the linear amount of deflection from the tangent *in the direction of the focus*, which corresponds to that space so run over; 3dly, and lastly, The laws of accelerated motion declare that the intensity of the acting force causing such deflection *in its own direction*, is measured by or proportional to the amount of that deflection, and may therefore be calculated in any particular position, or generally expressed by geometrical or algebraic symbols, *as a law* independent of particular positions, when that deflection is so

* If the suspended body be a vessel full of fine sand, having a small hole at its bottom, the elliptic trace of its orbit will be left in a sand streak on a table placed below it. This neat illustration is due, to the best of my knowledge, to Mr. Babbage.

calculated or expressed. We have here the spirit of the process by which Newton has resolved this interesting problem. For its geometrical detail, we must refer to the 3d section of his *Principia*. We know of no artificial mode of imitating this species of elliptic motion; though a rude approximation to it—enough, however, to give a conception of the alternate approach and recess of the revolving body to and from the focus, and the variation of its velocity—may be had by suspending a small steel bead to a fine and very long silk fibre, and setting it to revolve in a small orbit round the pole of a powerful cylindrical magnet, held upright, and vertically under the point of suspension.

(492.) The third law of Kepler, which connects the distances and periods of the planets by a general rule, bears with it, as its theoretical interpretation, this important consequence, viz. that it is one and the same force, modified only by distance from the sun, which retains *all* the planets in their orbits about it. That the attraction of the sun (if such it be) is exerted upon all the bodies of our system indifferently, without regard to the peculiar materials of which they may consist, in the exact proportion of their inertiae, or quantities of matter; that it is not, therefore, of the nature of the elective attractions of chemistry or of magnetic action, which is powerless on other substances than iron and some one or two more, but is of a more universal character, and extends equally to all the material constituents of our system, and (as we shall hereafter see abundant reason to admit) to those of other systems than our own. This law, important and general as it is, results, as the simplest of corollaries, from the relations established by Newton in the section of the *Principia* referred to (Prop. xv.), from which proposition it results, that if the earth were taken from its actual

orbit, and launched anew in space at the place, in the direction, and with the velocity of any of the other planets, it would describe the very same orbit, and in the same period, which that planet actually does, a minute correction of the period only excepted, arising from the difference between the mass of the earth and that of the planet. Small as the planets are compared to the sun, some of them are not, as the earth is, mere atoms in the comparison. The strict wording of Kepler's law, as Newton has proved in his fifty-ninth proposition, is applicable only to the case of planets whose proportion to the central body is absolutely inappreciable. When this is not the case, the periodic time is shortened in the proportion of the square root of the number expressing the sun's mass or inertia, to that of the sum of the numbers expressing the masses of the sun and planet; and in general, whatever be the masses of two bodies revolving round each other under the influence of the Newtonian law of gravity, the square of their periodic time will be expressed by a fraction whose numerator is the cube of their mean distance, *i.e.* the greater semi-axis of their elliptic orbit, and whose denominator is the sum of their masses. When one of the masses is incomparably greater than the other, this resolves into Kepler's law; but when this is not the case, the proposition thus generalized stands in lieu of that law. In the system of the sun and planets, however, the numerical correction thus introduced into the results of Kepler's law is too small to be of any importance, the mass of the largest of the planets (Jupiter) being much less than a thousandth part of that of the sun. We shall presently, however, perceive all the importance of this generalization, when we come to speak of the satellites.

(493.) It will first, however, be proper to explain by what

process of calculation the expression of a planet's elliptic orbit by its *elements* can be compared with observation, and how we can satisfy ourselves that the numerical data contained in a table of such elements for the whole system does really exhibit a true picture of it, and afford the means of determining its state at every instant of time, by the mere application of Kepler's laws. Now, for each planet, it is necessary for this purpose to know, 1st, the magnitude and form of its ellipse; 2dly, the situation of this ellipse in space, with respect to the ecliptic, and to a fixed line drawn therein; 3dly, the local situation of the planet in its ellipse at some known epoch, and its periodic time or mean angular velocity, or, as it is called, its mean motion.

(494.) The magnitude and form of an ellipse are determined by its greatest length and least breadth, or its two principal axes; but for astronomical uses it is preferable to use the semi-axis major (or half the greatest length), and the excentricity or distance of the focus from the centre, which last is usually estimated in parts of the former. Thus, an ellipse, whose length is 10 and breadth 8 parts of any scale, has for its major semi-axis 5, and for its excentricity 3 such parts; but when estimated in parts of the semi-axis, regarded as a unit, the excentricity is expressed by the fraction $\frac{3}{5}$.

(495.) The ecliptic is the plane to which an inhabitant of the earth most naturally refers the rest of the solar system, as a sort of ground-plane; and the axis of its orbit might be taken for a line of departure in that plane or origin of angular reckoning. Were the axis *fixed*, this would be the best possible origin of longitudes; but as it has a motion (though an excessively slow one), there is, in fact, no advantage in reckoning from the axis more than from the line of the equinoxes, and astronomers therefore prefer the latter, taking

account of its variation by the effect of precession, and restoring it, by calculation at every instant, to a fixed position. Now, to determine the situation of the ellipse described by a planet with respect to this plane, three *elements* require to be known:—1st, the *inclination* of the plane of the planet's orbit to the plane of the ecliptic; 2dly, the line in which these two planes intersect each other, which of necessity passes through the sun, and whose position with respect to the line of the equinoxes is therefore given by stating its longitude. This line is called the line of the nodes. When the planet is in this line, in the act of passing from the south to the north side of the ecliptic, it is *in its ascending node*, and its longitude at that moment is the element called the *longitude of the node*. These two data determine the situation of *the plane* of the orbit; and there only remains for the complete determination of the situation of the planet's ellipse, to know how it is placed *in* that plane, which (since its focus is necessarily in the sun) is ascertained by stating the *longitude of its perihelion*, or the place which the extremity of the axis nearest the sun occupies, when orthographically projected on the ecliptic.*

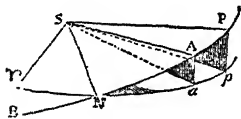
(496.) The dimensions and situation of the planet's orbit thus determined, it only remains, for a complete acquaintance with its history, to determine the circumstances of its motion in the orbit so precisely fixed. Now, for this purpose, all that is needed is to know the moment of time when it is either at the perihelion, or at any other precisely determined point of its orbit, and its whole period; for these being known, the law of the areas determines the place at

* What is most improperly called in some books the "longitude of the perihelion on the orbit" is a broken arc or an angle made up of two in different planes, viz. from the equinox to the node on the ecliptic and thence to the perihelion on the orbit.

every other instant. This moment is called (when the perihelion is the point chosen) the *perihelion passage*, or, when some point of the orbit is fixed upon, without special reference to the perihelion, the *epoch*.

(497.) Thus, then, we have *seven* particulars or elements, which must be numerically stated, before we can reduce to calculation the state of the system at any given moment. But these known, it is easy to ascertain the apparent positions of each planet, as it would be seen from the sun, or is seen from the earth at any time. The former is called the *heliocentric*, the latter the *geocentric*, place of the planet.

(498.) To commence with the heliocentric places. Let S represent the sun; P A N the orbit of the planet, being an ellipse, having the S in its focus, and A for its perihelion; and let $p a N r$ represent the projection of the orbit on the plane of the ecliptic, intersecting the line of equinoxes S r in r , which, therefore, is the origin of longitudes. Then will S N be the line of nodes; and if we suppose B to lie on the south, and A on the north side of the ecliptic, and the direction of the planet's motion to be from B to A, N will be the ascending node, and the angle $r S N$ the *longitude of the node*. In like manner, if P



be the place of the planet at any time, and if it and the perihelion A be projected on the ecliptic, upon the points p, a , the angles $r S p, r S a$, will be the respective heliocentric longitudes of the planet and of the perihelion, the former of which is to be determined, and the latter is one of the given elements. Lastly, the angle $p S P$ is the heliocentric latitude of the planet, which is also required to be known.

(499.) Now, the time being given, and also the moment

of the planet's passing the perihelion, the interval, or the time of describing the portion $A P$ of the orbit, is given, and the periodical time, and the whole area of the ellipse being known, the law of proportionality of areas to the times of their description gives the magnitude of the area $A S P$. From this it is a problem of pure geometry to determine the corresponding *angle* $A S P$, which is called the planet's *true anomaly*. This problem is of the kind called transcendental, and has been resolved by a great variety of processes, some more, some less intricate. It offers, however, no peculiar difficulty, and is practically resolved with great facility by the help of tables constructed for the purpose, adapted to the case of each particular planet.*

(500.) The true anomaly thus obtained, the planet's angular distance from the node, or the angle $N S P$, is to be found. Now, the longitudes of the perihelion and node being respectively γa and γN , which are given, their difference $a N$ is also given, and the angle N of the spherical right-angled triangle $A N a$, being the inclination of the plane of the orbit to the ecliptic, is known. Hence we calculate the arc $N A$, or the angle $N S A$, which, added to $A S P$, gives the angle $N S P$ required. And from this, regarded as the measure of the arc $N P$, forming the hypotenuse of the

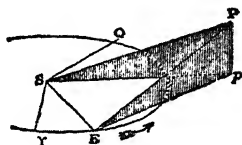
* It will readily be understood, that, except in the case of uniform circular motion, an equable description of *areas* about any centre is incompatible with an equable description of *angles*. The object of the problem in the text is to pass from the *area*, supposed known, to the *angle*, supposed unknown; in other words, to derive the true amount of angular motion from the perihelion, or the *true anomaly* from what is technically called the mean anomaly, that is, the mean angular motion which would have been performed had the motion *in angle* been uniform instead of the motion *in area*. It happens, fortunately, that this is the simplest of all problems of the transcendental kind, and can be resolved, in the most difficult case, by the rule of "false position," or trial and error, in a very few minutes. Nay, it may even be resolved approximately on inspection by a simple and easily constructed piece of mechanism, of which the reader may see a description in the Cambridge Philosophical Transactions, vol. iv. p. 425, by the author of this work.

right-angled spherical triangle $P N p$, whose angle N , as before, is known, it is easy to obtain the other two sides, $N p$ and $P p$. The latter, being the measure of the angle $p S P$, expresses the planet's heliocentric latitude; the former measures the angle $N S p$, or the planet's distance in longitude from its node, which, added to the known angle $r S N$, the longitude of the node, gives the heliocentric longitude. This process, however circuitous it may appear, when once well understood may be gone through numerically by the aid of the usual logarithmic and trigonometrical tables, in little more time than it will have taken the reader to peruse its description.

(501.) The geocentric differs from the heliocentric place of a planet by reason of that parallactic change of apparent situation which arises from the earth's motion in its orbit. Were the planets' distances as vast as those of the stars, the earth's orbital motion would be insensible when viewed from them, and they would always appear to us to hold the same relative situations among the fixed stars as if viewed from the sun, *i.e.* they would then be seen in their *heliocentric* places. The difference, then, between the heliocentric and geocentric places of a planet is, in fact, the same thing with its *parallax*, arising from the earth's removal from the centre of the system and its annual motion. It follows from this, that the first step toward a knowledge of its amount, and the consequent determination of the apparent place of each planet, as referred from the earth to the sphere of the fixed stars, must be to ascertain the proportion of its linear distances from the earth and from the sun, as compared with the earth's distance from the sun, and the angular positions of all three with respect to each other.

(502.) Suppose, therefore, S to represent the sun, E the

earth, and P the planet; S r the line of equinoxes, r E the earth's orbit, and P p a perpendicular let fall from the planet on the ecliptic. Then will the angle S P E (according to the general notion of parallax conveyed in art. 69) represent the parallax of the planet arising from the change of station from S to E ; E P will be the apparent direction of the planet seen from E ; and if S Q be drawn parallel to E p , the angle r S Q will be the geocentric longitude of the planet, while r S E represents the heliocentric longitude of the earth, r S p that of the planet. The former of these, r S E , is given by the solar tables; the latter, r S p , is found by the process above described (art. 500). Moreover, S P is the radius vector of the planet's orbit, and S E that of the earth's, both of which are determined from the known dimensions of their respective ellipses, and the places of the bodies in them at the assigned time. Lastly, the angle P S p is the planet's heliocentric latitude.



(503.) Our object, then, is, from all these data, to determine the angle r S Q , and P E p , which is the geocentric latitude. The process, then, will stand as follows:—1st, In the triangle S P p , right-angled at p , given S P , and the angle P S p (the planet's radius vector and heliocentric latitude), find S p and P p ; 2dly, In the triangle S E p , given S p (just found), S E (the earth's radius vector), and the angle E S p (the difference of heliocentric longitudes of the earth and planet), find the angle S p E , and the side E p . The former being equal to the alternate angle p S Q , is the parallactic removal of the planet in longitude, which, added to r S p , gives its geocentric longitude. The latter, E p (which is called the *curtate dis-*

tance of the planet from the earth), gives at once the geocentric latitude, by means of the right-angled triangle $P E p$, of which $E p$ and $P p$ are known sides, and the angle $P E p$ is the geocentric latitude sought.

(504.) The calculations required for these purposes are nothing but the most ordinary processes of plane trigonometry; and, though somewhat tedious, are neither intricate nor difficult. When executed, however, they afford us the means of comparing the places of the planets actually observed with the elliptic theory, with the utmost exactness, and thus putting it to the severest trial; and it is upon the testimony of such computations, so brought into comparison with observed facts, that we declare that theory to be a true representation of nature.

(505.) The planets Mercury, Venus, Mars, Jupiter and Saturn have been known from the earliest ages in which astronomy has been cultivated. Uranus was discovered by Sir W. Herschel in 1781, March 13th, in the course of a review of the heavens, in which every star visible in a telescope of a certain power was brought under close examination, when the new planet was immediately detected by its disk, under a high magnifying power. It has since been ascertained to have been observed on many previous occasions, with telescopes of insufficient power to show its disk, and even entered in catalogues as a star; and some of the observations which have been so recorded have been used to improve and extend our knowledge of its orbit. The discovery of the asteroids dates from the first day of 1801, when Ceres was discovered by Piazzi, at Palermo, a discovery speedily followed by those of Juno by Professor Harding, of Göttingen, in 1804; and of Pallas and Vesta, by Dr. Olbers, of Bremen, in 1802 and 1807 re-

spectively. It is extremely remarkable that this important addition to our system had been in some sort surmised as a thing not unlikely, on the ground that the intervals between the orbit of Mercury and the other planetary orbits, go on doubling as we recede from the sun, or nearly so. Thus, the interval between the orbits of the Earth and Mercury is nearly twice that between those of Venus and Mercury; that between the orbits of Mars and Mercury nearly twice that between the Earth and Mercury; and so on. The interval between the orbits of Jupiter and Mercury, however, is much too great, and would form an exception to this law, which is, however, again resumed in the case of the three planets next in order of remoteness, Jupiter, Saturn, and Uranus. It was therefore thrown out, by the late Professor Bode, of Berlin,^s as a possible surmise, that a planet not then yet discovered might exist between Mars and Jupiter; and it may easily be imagined what was the astonishment of astronomers on finding not one only, but four planets, differing greatly in all the other elements of their orbits, but agreeing very nearly, both *inter se*, and with the above stated empirical law, in respect of their mean distances from the sun. No account, *à priori* or from theory,

^s The progression is (rather rudely) that of the numbers 4, 4 + 3, 4 + 6, 4 + 12, etc. The empirical law itself, as we have above stated it, is ascribed by Voiron, not to Bode (who would appear, however, at all events, to have first drawn attention to this interpretation of its interruption), but to Professor Titius of Wittemberg. (Voiron, Supplement to Bailly.)

Another law has been proposed (in a letter to the writer, dated March 1, 1869), by Mr. J. Jones, of Brynhyfryd, Wrexham. If the planets' mean distances from the sun be arranged in the following orders.—Mercury, Venus, Jupiter, Saturn;—the Earth, Mars, Uranus, Neptune;—the product of the means in each group is nearly equal to the product of the extremes.

$$\frac{\text{Venus} \times \text{Jupiter}}{\text{Mercury} \times \text{Saturn}} = \frac{\text{Earth} \times \text{Neptune}}{\text{Mars} \times \text{Uranus}} = 1.$$
 In point of fact the first fraction = 1.02; and the last = $\frac{1}{1.03}$, so that the approach to verification of the law is really very near.

was to be given of this singular progression, which is not, like Kepler's laws, strictly exact in numerical verification: but the circumstances we have just mentioned tended to create a strong belief that it was something beyond a mere accidental coincidence, and bore reference to the essential structure of the planetary system. It was even conjectured that the asteroids are fragments of some greater planet which formerly circulated in that interval, but which has been blown to atoms by an explosion; an idea countenanced by the exceeding minuteness of these bodies which present disks; and it was argued that in that case innumerable more such fragments must exist and might come to be hereafter discovered. Whatever may be thought of such a speculation as a physical hypothesis, this conclusion has been verified to a considerable extent as a matter of fact by subsequent discovery, the result of a careful and minute examination and mapping down of the smaller stars in and near the zodiac, undertaken with that express object. Zodiacal charts of this kind, the product of the zeal and industry of many astronomers, have been constructed, in which every star down to the ninth, tenth, or even lower magnitudes, is inserted, and these stars being compared with the actual stars of the heavens, the intrusion of any stranger within their limits cannot fail to be noticed when the comparison is systematically conducted. The discovery of Astræa and Hebe by Professor Hencke in 1845 and 1847 revived the flagging spirit of inquiry in this direction; with what success, the list in the Appendix to this volume will best show. The labors of our indefatigable countryman, Mr. Hind, have been rewarded by the discovery of no less than eight of them.

(506.) The discovery of Neptune marks in a signal man-

ner the maturity of Astronomical science. The proof, or at least the urgent presumption of the existence of such a planet, as a means of accounting (by its attraction) for certain small irregularities observed in the motions of Uranus, was afforded almost simultaneously by the independent researches of two geometers, Messrs. Adams of Cambridge and Leverrier of Paris, who were enabled, *from theory alone*, to calculate whereabout it ought to appear in the heavens, *if visible*, the places thus independently calculated agreeing surprisingly. *Within a single degree* of the place assigned by M. Leverrier's calculations, and by him communicated to Dr. Galle of the Royal Observatory at Berlin, and within two and a half from that indicated by Mr. Adams, it was actually found by Dr. Galle on the very first night (Sept. 23, 1846) after the receipt of M. Leverrier's communication, on turning a telescope on the spot, and comparing the stars in its immediate neighborhood with those previously laid down in one of the zodiacal charts already alluded to.*

(507.) The mean distance of Neptune from the sun, however, so far from falling in with the supposed law of planetary distances above mentioned, offers a decided case of discordance. The interval between its orbit and that of Mercury, instead of being nearly double the interval

* Constructed by Dr. Bremiker, of Berlin. On reading the history of this noble discovery, we are ready to exclaim with Schiller—

“Mit dem Genius steht die Natur im ewigem Bunde,
Was der Eine verspricht leistet die Andre gewiss.”

Professor Challis, of the Cambridge Observatory, directing the Northumberland telescope of that Institution to the place assigned by Mr. Adams's calculations and its vicinity, on the 4th and 12th of August, 1846, saw the planet on both those days, and noted its place (among those of other stars) for re-observation. He, however, *postponed the comparison* of the places observed, and not possessing Dr. Bremiker's chart (which would have at once indicated the presence of an unmapped star), remained in ignorance of the planet's existence as a visible object till its announcement as such by Dr. Galle.

between those of Uranus and Mercury, does not, in fact, exceed the latter interval by much more than half its amount. This remarkable exception may serve to make us cautious in the too ready admission of empirical laws of this nature to the rank of fundamental truths, though, as in the present instance, they may prove useful auxiliaries, and serve as stepping stones, affording a temporary footing in the path to great discoveries. The force of this remark will be more apparent when we come to explain more particularly the nature of the theoretical views which led to the discovery of Neptune itself.

(508.) We shall devote the rest of this chapter to an account of the physical peculiarities and probable condition of the several planets, so far as the former are known by observation, or the latter rest on probable grounds of conjecture. In this, three features principally strike us as necessarily productive of extraordinary diversity in the provisions by which, if they be, like our earth, inhabited, animal life must be supported. These are, first, the difference in their respective supplies of light and heat from the sun; secondly, the difference in the intensities of the gravitating forces which must subsist at their surfaces, or the different ratios which, on their several globes, the *inertiæ* of bodies must bear to their *weights*; and, thirdly, the difference in the nature of the materials of which, from what we know of their mean density, we have every reason to believe they consist. The intensity of solar radiation is nearly seven times greater on Mercury than on the Earth, and on Neptune 900 times less; the proportion between the two extremes being that of upward of 5600 to 1. Let any one figure to himself the condition of our globe, were the sun to be septupled, to say nothing of the greater ratio! or

were it diminished to a seventh, or to a 900th of its actual power! It is true that owing to the remarkable difference between the properties of radiant heat as emitted from bodies of very exalted temperature, as the sun, and as from such as we commonly term *warm*, it is very possible that a dense atmosphere surrounding a planet, while allowing the access of solar heat to its surface, may oppose a powerful obstacle to its escape, and that thus the feeble sunshine on a remote planet may be retained and accumulated on its surface in the same way (and for the same reason) that a very slight amount of sunshine, or even the dispersed heat of a bright though clouded day, suffices to maintain the interior of a closed greenhouse at a high temperature. We cannot then absolutely conclude the prevalence of that excessive cold on the surface of a distant planet which its mere remoteness from the sun might lead us, *primâ facie*, to expect.

(508 b.) Again, the intensity of gravity, or its efficacy in counteracting muscular power and repressing animal activity, on Jupiter, is nearly two and a half times that on the Earth, on Mars not more than one-half, on the Moon one-sixth, and on the smaller planets probably not more than one-twentieth; giving a scale of which the extremes are in the proportion of sixty to one. Lastly, the density of Saturn hardly exceeds one-eighth of the Earth's, so that it must consist of materials not heavier on the average than dry fir wood. Now, under the various combinations of elements so important to life as these, what immense diversity must we not admit in the conditions of that great problem, the maintenance of animal and intellectual existence and happiness, which seems, so far as we can judge by what we see around us in our own planet, and by the way in which

every corner of it is crowded with living beings, to form an unceasing and worthy object for the exercise of the Benevolence and Wisdom which preside over all!

(509.) Quitting, however, the region of mere speculation, we will now show what information the telescope affords us of the actual condition of the several planets within its reach. Of Mercury we can see little more than that it is round, and exhibits phases. It is too small, and too much lost in the constant neighborhood of the Sun, to allow us to make out more of its nature. The real diameter of Mercury is about 3200 miles: its apparent diameter varies from 5" to 12". Nor does Venus offer any remarkable peculiarities: although its real diameter is 7800 miles, and although it occasionally attains the considerable apparent diameter of 61", which is larger than that of any other planet, it is yet the most difficult of them all to define with telescopes. The intense lustre of its illuminated part dazzles the sight, and exaggerates every imperfection of the telescope; yet we see clearly that its surface is not mottled over with permanent spots like the Moon; we notice in it neither mountains nor shadows, but a uniform brightness, in which sometimes we may indeed fancy, or perhaps more than fancy, brighter or obscurer portions, but can seldom or never rest fully satisfied of the fact. It is from some observations of this kind that both Venus and Mercury have been concluded to revolve on their axes in about the same time as the Earth, though in the case of Venus, Bianchini and other more recent observers have contended for a period of twenty-four times that length. The most natural conclusion, from the very rare appearance and want of permanence in the spots, is, that we do not see, as in the Moon, the real surface of these planets, but only their atmospheres, much loaded with clouds, and which may

serve to mitigate the otherwise intense glare of their sunshine.

(510.) The case is very different with Mars. In this planet we frequently discern, with perfect distinctness, the outlines of what may be continents and seas. (See Plate III. *fig. 1*, which represents Mars in its gibbous state, as seen on the 16th of August, 1830, in the 20-foot reflector at Slough.) Of these, the former are distinguished by that ruddy color which characterizes the light of this planet (which always appears red and fiery), and indicates, no doubt, an ochrey tinge in the general soil, like what the red sandstone districts on the Earth may possibly offer to the inhabitants of Mars, only more decided. Contrasted with this (by a general law in optics), the seas, as we may call them, appear greenish.¹⁰ These spots, however, are not always to be seen equally distinct, but, *when seen*, they offer the appearance of forms considerably definite and highly characteristic,¹¹ brought successively into view by the rotation of the planet, from the assiduous observation of which it has even been found practicable to construct a rude chart of the surface of the planet. The variety in the spots may arise from the planet not being destitute of atmosphere and clouds; and what adds greatly to the probability of this is the appearance of brilliant white spots at its poles—one of which appears in our figure—which have been conjectured, with some probability, to be snow; as they disappear when they have been long exposed to the sun, and are greatest when

¹⁰ I have noticed the phenomena described in the text on many occasions, but never more distinct than on the occasion when the drawing was made from which the figure in Plate I. is engraved.—*Author*.

¹¹ The reader will find many of those forms represented in Schumacher's *Astronomische Nachrichten*, No. 191, 434, and in the chart in No. 349, by Messrs. Beer and Mädler.

just emerging from the long night of their polar winter, the snow line then extending to about six degrees (reckoned on a meridian of the planet) from the pole. By watching the spots during a whole night, and on successive nights, it is found that Mars has a rotation on an axis inclined about $30^{\circ} 18'$ to the ecliptic, and in a period of $24^h 37^m 23^s.12$ in the same direction as the Earth's, or from west to east. The greatest and least apparent diameters of Mars are $4''$ and $18''$, and its real diameter about 4100 miles.

(511.) We now come to a much more magnificent planet, Jupiter, the largest of them all, being in diameter no less than 87,000 miles, and in bulk exceeding that of the Earth nearly 1300 times. It is, moreover, dignified by the attendance of four *moons*, *satellites*, or *secondary planets*, as they are called, which constantly accompany and revolve about it, as the Moon does round the Earth, and in the same direction, forming with their principal, or *primary*, a beautiful miniature system, entirely analogous to that greater one of which their central body is itself a member, obeying the same laws, and exemplifying, in the most striking and instructive manner, the prevalence of the gravitating power as the ruling principle of their motions: of these, however, we shall speak more at large in the next chapter.

(512.) The disk of Jupiter is always observed to be crossed in one certain direction by dark bends or belts presenting the appearance, in Plate III. *fig.* 2, which represents this planet as seen on the 23d of September, 1832, in the 20-foot reflector at Slough. These belts are, however, by no means alike at all times; they vary in breadth and in situation on the disk (though never in their general direc-

¹² Beer and Mädler, *Astr. Nachr.* 349. 22nd 736, Proctor, *A. S. Not.* xxix. 232.

tion). They have even been seen broken up and distributed over the whole face of the planet; but this phenomenon is extremely rare. Branches running out from them, and subdivisions, as represented in the figure, as well as evident darker spots, are by no means uncommon. But the most singular phenomenon presented by the belts of Jupiter is the occasional appearance upon them of perfectly round, well defined, bright spots, not unlike the disks of the satellites (see art. 540), as they are occasionally seen projected on the planet when passing between it and the Earth, only smaller. They vary in situation and number, as many as ten having, on one occasion (Oct. 28, 1857), been seen at once, but, so far as hitherto observed, only on the southern hemisphere of Jupiter. They were first noticed by Mr. Dawes in the spring of 1849, but first described and figured by Mr. Lassell, March 27, 1850. They have been more recently again and more distinctly and consecutively observed by the former of these observers, who has given figures of them in *Ast. Soc. Not.* xviii. pp. 8, 40.

(512 *a.*) From the appearances and configurations of the belts, attentively watched, it is concluded that this planet revolves in the surprisingly short period of $9^h 55^m 21^s.3$ (Airy), on an axis perpendicular to the direction of the belts. Now, it is very remarkable, and forms a most satisfactory comment on the reasoning by which the spheroidal figure of the Earth has been deduced from its diurnal rotation, that the outline of Jupiter's disk is evidently not circular, but elliptic, being considerably flattened in the direction of its axis of rotation. This appearance is no optical illusion, but is authenticated by micrometrical measures, which assign 106 to 100 for the proportion of the equatorial and polar diameters. And to confirm, in the strongest man-

ner, the truth of those principles on which our former conclusions have been founded, and fully to authorize their extension to this remote system, it appears, on calculation, that this is really the degree of oblateness which corresponds, on those principles, to the dimensions of Jupiter, and to the time of his rotation.

(513.) The parallelism of the belts to the equator of Jupiter, their occasional variations, and these appearances of spots seen upon them, render it extremely probable that they subsist in the atmosphere of the planet, forming tracts of comparatively clear sky, determined by currents analogous to our trade-winds, but of a much more steady and decided character, as might indeed be expected from the immense velocity of its rotation. That it is the comparatively darker body of the planet which appears in the belts is evident from this—that they do not come up in all their strength to the edge of the disk, but fade away gradually before they reach it. The round bright spots described above may therefore not impossibly be insulated masses of cloud, of local origin, analogous to the cumuli which sometimes cap ascending columns of vapor in our atmosphere. The apparent diameter of Jupiter varies from 30" to 46".¹³

(514.) A still more wonderful, and, as it may be termed, elaborately artificial mechanism, is displayed in Saturn, the next in order of remoteness to Jupiter, to which it is not much inferior in magnitude, being about 79,000 miles in diameter, nearly 1000 times exceeding the earth in bulk, and subtending an apparent angular diameter at the earth, of

¹³ Prof. P. Smyth and Mr. De la Rue have published fine representations of Jupiter, the former as seen from the Peak of Teneriffe (alt. 10,700 ft.), the latter in his observatory at Cranford.

about 18" at its mean distance. This stupendous globe, besides being attended by no less than eight satellites, or moons, is surrounded with three broad, flat, and extremely thin rings, concentric with the planet and with each other, the inner being very faint and semi-transparent; all lying in one plane, and separated by a very narrow interval from each other throughout their whole circumference, as they are from the planet by a much wider. The dimensions of this extraordinary appendage are as follows:—¹⁴

	"	Miles
Exterior diameter of exterior bright ring	40.095	= 176,418
Interior ditto	35.289	= 155,272
Exterior diameter of interior bright ring	34.475	= 151,690
Interior ditto	26.668	= 117,339
Equatorial diameter of the body	17.991	= 79,160
Interval between the planet and interior bright ring .	4.339	= 19,090
Interval of the rings	0.408	= 1,791
Thickness of the ring not exceeding		= 250

The figure (Plate III. *fig.* 3) represents Saturn surrounded by its rings, and having its body striped with dark belts, somewhat similar, but broader and less strongly marked than those of Jupiter, and owing, doubtless, to a similar cause.¹⁵ Whatever be the materials of which the ring consists (and there are strong reasons, art. 522, for believing it not to consist of solid matter), it is at least substantial enough to cast a shadow, which, when the Earth is properly situated, may be seen on the body of the planet on the side next the Sun; as also to receive one when thrown on it by the body

¹⁴ These dimensions are calculated from Prof. Struve's micrometric measures, *Mem. Ast. Soc.* iii. 301, with the exception of the thickness of the ring, which is concluded from its total disappearance in 1833, in a telescope which would certainly have shown, as a visible object, a line of light one-twentieth of a second in breadth. The interval of the rings here stated is possibly somewhat too small.

¹⁵ The equatorial bright belt is generally well seen. The subdivision of the dark one by two narrow bright bands is seldom so distinct as represented in the plate.

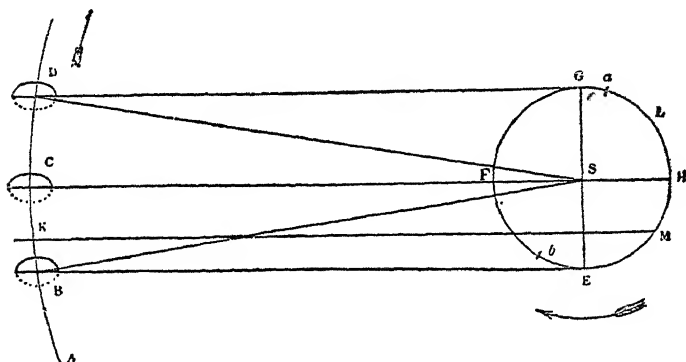
on the opposite side. The form of this latter shadow, minutely scrutinized with powerful telescopes, has led some observers to conclude that the edge of the outer ring is in some degree rounded, and that the two rings do not lie precisely in one plane.¹⁶ From the parallelism of the belts with the plane of the ring, it may be conjectured that the axis of rotation of the planet is perpendicular to that plane; and this conjecture is confirmed by the occasional appearance of extensive dusky spots on its surface, which, when watched, like the spots on Mars or Jupiter, indicate a rotation in $10^h 16^m 0^s.44$ (according to the observations of Sir Wm. Herschel) about an axis so situated.

(515.) The axis of rotation, like that of the earth, preserves its parallelism to itself during the motion of the planet in its orbit; and the same is also the case with the ring, whose plane is constantly inclined at the same, or very nearly the same, angle to that of the orbit, and, therefore, to the ecliptic, viz. $28^\circ 11'$; and intersects the latter plane in a line, which makes at present¹⁷ an angle with the line of equinoxes of $167^\circ 31'$. So that the *nodes of the ring* lie in $167^\circ 31'$ and $347^\circ 31'$ of longitude. Whenever, then, the planet happens to be situated in one or other of these longitudes, as at C, the plane of the ring passes through the sun, which then illuminates only the edge of it. And if the earth at that moment be in F, it will see the ring edgewise, the planet being in opposition, and therefore most favorably situated (*cæteris paribus*) for ob-

¹⁶ The excessive thinness of the rings leads us to demur to the former of these conclusions as a result of observation, though fully admitting it as theoretically probable.

¹⁷ According to Bessel, the longitude of the node of the ring increases by $46''.462$ per annum. In 1800 it was $166^\circ 53' 8''.9$.

servation. Under these circumstances the ring, if seen at all, can only appear as a very narrow straight line of light projecting on either side of the body as a prolongation of its diameter. In fact, it is quite invisible in any but telescopes of extraordinary power.¹⁸ This remarkable phenomenon takes place at intervals of fifteen years nearly (being a semi-period of Saturn in its orbit). One disappearance at least must take place whenever Saturn passes either node of its orbit; but three must frequently happen, and two are possible. To show this, suppose S to be the



sun, A B C D part of Saturn's orbit situated so as to include the node of the ring (at C); E F G H the Earth's orbit; S C the line of the node; E B, G D parallel to S C touching the Earth's orbit in E G; and let the direction of motion of both bodies be that indicated by the arrow. Then since the ring preserves its parallelism, its plane can nowhere intersect the Earth's orbit, and therefore no disappearance can take place, unless the planet be between B and

¹⁸ Its disappearance was *complete* when observed with a reflector eighteen inches in aperture and twenty feet in focal length on the 29th of April, 1833, by the author.

D: and, on the other hand, a disappearance is possible (if the Earth be rightly situated) during the whole time of the description of the arc B D. Now, since S B or S D, the distance of Saturn from the Sun, is to S E or S G, that of the Earth, as 9·54 to 1, the angle C S D or C S B = $6^{\circ} 1'$, and the whole angle B S D = $12^{\circ} 2'$, which is described by Saturn (on an average) in 359·46 days, wanting only 5·8 days of a complete year. The Earth then describes very nearly an entire revolution within the limits of time when a disappearance is possible; and since, in either half of its orbit E F G or G H E, it may equally encounter the plane of the ring, one such encounter at least is unavoidable within the time specified.

(516.) Let $G a$ be the arc of the Earth's orbit described from G in 5·8 days. Then if, at the moment of Saturn's arrival at B, the Earth be at a , it will encounter the plane of the ring advancing parallel to itself and to B E to meet it, somewhere in the quadrant H E, as at M, after which it will be behind that plane (with reference to the direction of Saturn's motion) through all the arc M E F G up to G, where it will again overtake it at the very moment of the planet quitting the arc B D. In this state of things there will be two disappearances. If, when Saturn is at B, the Earth be anywhere in the arc a H E, it is equally evident that it will *meet* and pass through the advancing plane of the ring somewhere in the quadrant H E, that it will again *overtake* and pass through it somewhere in the semicircle E F G, and again meet it in some point of the quadrant G H, so that three disappearances will take place. So, also, if the Earth be at E when Saturn is at B, the motion of the Earth being at that instant directly toward B, the plane of the ring will for a short time leave it behind; but the ground

so lost being rapidly regained, as the Earth's motion becomes oblique to the line of junction, it will soon overtake and pass through the plane in the early part of the quadrant E F, and passing on through G before Saturn arrives at D, will meet the plane again in the quadrant G H. The same will continue up to a certain point *b*, at which, if the Earth be initially situated, there will be but two disappearances—the plane of the ring there overtaking the Earth for an instant, and being immediately again left behind by it, to be again encountered by it in G H. Finally, if the initial place of the Earth (when Saturn is at B) be in the arc *b* F *a*, there will be but one passage through the plane of the ring, viz. in the semicircle G H E, the Earth being in advance of that plane throughout the whole of *b* G.

(517.) The appearances will moreover be varied according as the Earth passes from the enlightened to the unenlightened side of the ring, or *vice versâ*. If C be the ascending node of the ring, and if the under side of the paper be supposed south and the upper north of the ecliptic, then, when the Earth *meets* the plane of the ring in the quadrant H E, it passes from the bright to the dark side: where it *overtakes* it in the quadrant E F, the contrary. *Vice versâ*, when it overtakes it in F G, the transition is from the bright to the dark side, and the contrary where it meets it in G H. On the other hand when the Earth is overtaken by the ring-plane in the interval E *b*, the change is from the bright to the dark side. When the dark side is exposed to sight, the aspect of the planet is very singular. It appears as a bright round disk, with its belts, etc., but crossed equatorially by a narrow and perfectly black line. This can never of course happen when the planet is more than $6^{\circ} 1'$ from the node of the ring. Generally, the northern side is enlightened and

visible when the heliocentric longitude of Saturn is between $173^{\circ} 32'$ and $341^{\circ} 30'$, and the southern when between $353^{\circ} 32'$ and $161^{\circ} 30'$. The greatest opening of the ring occurs when the planet is situated at 90° distance from the node of the ring, or in longitudes $77^{\circ} 31'$ and $257^{\circ} 31'$, and at these points the longer diameter of its apparent ellipse is almost exactly double the shorter.

(518.) It will naturally be asked how so stupendous an arch, if composed of solid and ponderous materials, can be sustained without collapsing and falling in upon the planet? The answer to this is to be found in a swift rotation of the ring in its own plane, which observation has detected, owing to some portion of the ring being a little less bright than others, and assigned its period at $10^h 32^m 15^s$, which, from what we know of its dimensions, and of the force of gravity in the Saturnian system, is very nearly the periodic time of a satellite revolving at the same distance as the middle of its breadth. It is the centrifugal force, then, arising from this rotation, which sustains it; and although no observations nice enough to exhibit a difference of periods between the outer and inner rings have hitherto been made, it is more than probable that such a difference does subsist as to place each independently of the other in a similar state of equilibrium. Still, it might be urged, such is the thinness of the rings that it may very well be doubted, whether the strain brought upon either of them by the difference of its interior and exterior centrifugal forces, if solid, would not suffice to tear it in pieces. A fluid constitution would obviate this difficulty; and indeed it is very possible that the rings may be gaseous, or rather such a mixture of gas and vapor as consists with our idea of a cloud.

(519) Although the rings are, as we have said, very

nearly concentric with the body of Saturn, yet micrometrical measurements of extreme delicacy¹⁹ have demonstrated that the coincidence is not mathematically exact, but that the centre of gravity of the rings oscillates round that of the body, describing a very minute orbit, probably under laws of much complexity. Trifling as this remark may appear, it is of the utmost importance to the stability of the system of the rings, if solid and coherent. Supposing them mathematically perfect in their circular form, and exactly concentric with the planet, it is demonstrable that they would form a system in a state of *unstable equilibrium*, which the slightest external power would subvert—not by causing a rupture in the substance of the rings—but by precipitating them *unbroken* on the surface of the planet. For the attraction of such a ring or rings on a point or sphere excentrically within them, is not the same in all directions, but tends to draw the point or sphere toward the nearest part of the ring, or away from the centre. Hence, supposing the body to become, from any cause, ever so little excentric to the ring, the tendency of their mutual gravity is not to correct but to increase this excentricity, and to bring the nearest parts of them together. Now, external powers, capable of producing such excentricity, exist in the attractions of the satellites, as will be shown in Chapter XII.; and in order that the system may be *stable*, and possess within itself a power of resisting the first inroads of such a tendency, while yet nascent and feeble, and opposing them by an opposite or maintaining power, it has been shown that it is sufficient to admit the rings, if solid,

¹⁹ By Struve, confirming a suspicion suggested by the eye-observations of M. Schwabe.

to be *loaded* in some part of their circumference, either by some minute inequality of thickness, or by some portions being denser than others. Such a load would give to the whole ring to which it was attached somewhat of the character of a heavy and sluggish satellite maintaining itself in an orbit with a certain energy sufficient to overcome minute causes of disturbance, and establish an average bearing on its centre. But even without supposing the existence of any such load—of which, after all, we have no proof—and granting, in its full extent, the general instability of the equilibrium, we think we perceive, in the rapid periodicity of all the causes of disturbance, a sufficient guarantee of its preservation. However homely be the illustration, we can conceive nothing more apt, in every way, to give a general conception of this maintenance of equilibrium under a constant tendency to subversion, than the mode in which a practiced hand will sustain a long pole in a perpendicular position resting on the finger, by a continual and almost imperceptible variation of the point of support. Be that, however, as it may, the observed oscillation of the centres of the rings about that of the planet is in itself the evidence of a perpetual contest between conservative and destructive powers—both extremely feeble, but so antagonizing one another as to prevent the latter from ever acquiring an uncontrollable ascendancy, and rushing to a catastrophe.

(520.) This is also the place to observe, that as the smallest difference of velocity between the body and the rings must infallibly precipitate the latter on the former, never more to separate (for they would, once in contact, have attained a position of *stable equilibrium*, and be held together ever after by an immense force); it follows, either that their motions in their common orbit round the sun must

have been adjusted to each other by an external power, with the minutest precision, or that the rings must have been formed about the planet while subject to their common orbital motion, and under the full and free influence of all the acting forces.

(521.) [The exterior ring of Saturn is described by many observers as rather less luminous than the interior, and the inner portion of this latter than its outer. On the night of November 11, 1850, however, Mr. G. B. Bond, of the Harvard Observatory (Cambridge, U. S.), using the great Fraunhofer equatorial of that institution, became aware of a line of demarcation between these two portions so definite, and an extension inward of the dusky border to such an extent (one-fifth, *by measurement*, of the joint breadth of the two old rings), as to justify him in considering it as a newly-discovered ring. On the nights of the 25th and 29th of the same month, and without knowledge of Mr. Bond's observations, Mr. Dawes, at his observatory at Wateringbury, by the aid of an exquisite achromatic by Merz, of $6\frac{1}{2}$ inches aperture, observed the very same fact, and even more distinctly, so as to be sure of a decidedly darker interval between the old and new rings, and even to subdivide the latter into two of unequal degrees of obscurity, separated by a line more obscure than either.

(522.) Dr. Galle of Berlin, however, would appear to have been the first to notice (June 10, 1838) a faint extension of the inner ring toward the body of the planet, to about half the interval between the then recognized inner ring and the body, *as shown by micrometrical measures*. But this result remained unpublished (or at least not generally known) until after the observations of Messrs. Bond and Dawes. The most remarkable feature of this singular dis-

covery is, that subsequent observations, from many quarters, have concurred in showing the new ring to consist of *semi-transparent materials through which the limb of the planet may be seen up to the edge of the interior bright ring*. Dark lines (apparently of a transitory nature) have been observed on the bright rings parallel to the permanent dark interval dividing them. All these indications taken in conjunction with what is said in art. 518 decidedly point to a vaporous constitution of these wonderful appendages.²⁰]

(522 a.) Still it has been thought remarkable that this new ring, or appendage to the rings, should not have been discovered earlier; and it has even been conjectured that the breadth of the ring has been gradually increasing inward since the time of Huyghens, its first discoverer: and this conjecture for a while appeared to be supported by micro-metrical measures obtained by M. Otto Struve (with whom the conjecture originated), which seemed to show a still further diminution of the interval between the rings and the ball. The question, however, appears to be definitively settled in the negative by the elaborate micrometrical measures of Mr. Main, at the Royal Observatory at Greenwich,²¹ and by the discussions entered into by M. Kaiser.²²

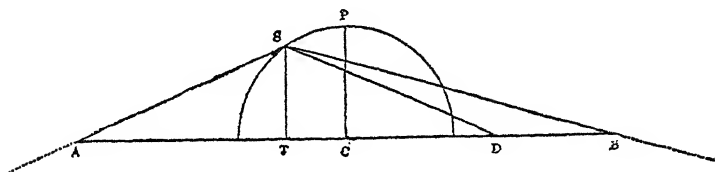
(522 b.) The rings of Saturn must present a magnificent spectacle from those regions of the planet which lie above their enlightened sides, being seen as vast luminous arches, spanning the sky from horizon to horizon, and holding an

²⁰ The passage of Saturn across any considerable star would afford an admirable opportunity of testing the existence of fissures in the rings, as it would flash in succession through them. The opportunity of watching for such occultations—when Saturn traverses the Milky Way, for instance—should not be neglected.

²¹ Mem. Ast. Soc. xxv.

²² Ast. Soc. Notice, xvi. 66.

almost invariable situation among the stars. To a spectator situated anywhere in the axis of the planet, it is evident that their interior and exterior outlines must both appear as circles corresponding to parallels of declination, and must occasion a permanent eclipse of every heavenly body lying between these parallels. It is otherwise to a spectator situated on the planet's surface. To such a one the interior and exterior outline of each ring would, by the effect of perspective, be thrown into nonconcentric ellipses, so that (supposing he could see through the whole planet and obtain a view of the whole ring) it would appear broader on the side



nearest to him than on that most remote. These ellipses, moreover, when traced along the heavens, would not coincide with parallels of declination;²³ but would deviate from such parallels toward the elevated pole, as is evident, if we consider that a perpendicular ST from any point S on the planet's surface to the plane of the ring AB is parallel to the axis of rotation; so that the right cone ASD , generated by the revolution of AS round ST , traces on the heavens a circle of declination, having the edge A of the ring for its upper culminating point: whereas the oblique cone ASB , tracing the visible course of the ring in the heavens, though

²³ The circumstances have been traced in minute detail by Dr. Lardner, who first, I believe, drew attention to the effect of situation on the surface of the planet in modifying the phenomena presented by the rings.

coincident with the former at its upper culmination A, lies elsewhere wholly exterior to it, and has its inferior culmination B nearer to the elevated pole by the angle B S D, the difference of the angles of the two cones. The apparent course of either edge of the ring, then, is a curve touching the circle of declination at which that edge culminates, but receding from it toward the elevated pole, so as to allow stars or the Sun to be visible at certain seasons *under* the ring at their rising—to be eclipsed wholly or partially by it at its under edge, and again to emerge before setting. This will not prevent, however, some considerable regions of Saturn from suffering very long total interception of the solar beams, affording, to our ideas, but an inhospitable asylum to animated beings, ill compensated by the feeble light of the satellites. But we shall do wrong to judge of the fitness or unfitness of their condition from what we see around us, when perhaps the very combinations which convey to our minds only images of horror, may be, in reality, theatres of the most striking and glorious displays of beneficent contrivance.

(523.) Of Uranus we see nothing but a small round uniformly illuminated disk, without rings, belts, or discernible spots. Its apparent diameter is about 4", from which it never varies much, owing to the smallness of our orbit in comparison of its own. Its real diameter is about 35,000 miles, and its bulk 82 times that of the earth. It is attended by four satellites, whose existence may be considered as conclusively established (and more have been suspected).

(524.) The discovery of Neptune is so recent, and its situation in the ecliptic at present so little favorable for seeing it with perfect distinctness, that nothing very positive can be stated as to its physical appearance. It was at first

suspected to have a ring, but the suspicion has not been verified. It is attended by at least one satellite, the existence of which has been demonstrated by the observations of Mr. Lassell, M. Otto Struve, and Mr. Bond.

(525.) If the immense distance of Neptune precludes all hope of coming at much knowledge of its physical state, the minuteness of the Asteroids is no less a bar to any inquiry into theirs. One of them, Pallas, has been said to have somewhat of a nebulous or hazy appearance, indicative of an extensive and vaporous atmosphere, little repressed and condensed by the inadequate gravity of so small a mass. It is probable, however, that the appearance in question has originated in some imperfection in the telescope employed, or other temporary causes of illusion. In Vesta and Pallas only have sensible disks been hitherto observed, and those only with very high magnifying powers. Vesta was once seen by Schröter with the naked eye. No doubt the most remarkable of their peculiarities must lie in this condition of their state. A man placed on one of them would spring with ease 60 feet high, and sustain no greater shock in his descent than he does on the earth from leaping a yard. On such planets giants might exist; and those enormous animals, which on earth require the buoyant power of water to counteract their weight, might there be denizens of the land. From some recent researches of M. Leverrier, it appears that we shall be warranted in attributing to the *totality* of the Asteroids a quantity of matter quite insignificant.

(525 a.) There is a remarkable division of the planetary system into two families or classes of planets, the large, and the small. To the latter family belong those interior to the orbits of Jupiter, viz. Mercury, Venus, the Earth, and Mars, with the Asteroids. To the former, all exterior to the orbits

of the first class—Jupiter, Saturn, Uranus, and Neptune. The Asteroids themselves, however, may be considered as forming a family apart, their magnitudes being as much inferior to those of the interior planets as these are to the exterior, or in a still lower ratio. Not less remarkable is the circumstance that while all the interior planets revolve on their axes (so far as is known) in about the same time (24^h), the exterior (as is certain in the case of Jupiter and Saturn at least) have periods of rotation less than half that length. In point of density, too, as we shall see further on, an equally marked distinction of specific character is preserved, all the interior ones having about the same density as the Earth, while that of all the exterior is very much less, not exceeding a quarter of the Earth's, and agreeing (in the cases of Jupiter and Uranus) very closely with that of the Sun.

(526.) We shall close this chapter with an illustration calculated to convey to the minds of our readers a general impression of the relative magnitudes and distances of the parts of our system. Choose any well levelled field or bowling-green. On it place a globe, two feet in diameter; this will represent the Sun; Mercury will be represented by a grain of mustard seed, on the circumference of a circle 164 feet in diameter for its orbit; Venus a pea, on a circle of 284 feet in diameter; the Earth also a pea, on a circle of 430 feet; Mars a rather large pin's head, on a circle of 654 feet; the Asteroids, grains of sand, in orbits of from 1000 to 1200 feet; Jupiter a moderate-sized orange, in a circle nearly half a mile across; Saturn a small orange, on a circle of four-fifths of a mile; Uranus a full-sized cherry, or small plum, upon the circumference of a circle more than a mile and a half; and Neptune a good-sized plum, on a circle

about two miles and a half in diameter. As to getting correct notions on this subject by drawing circles on paper, or, still worse, from those very childish toys called orreries, it is out of the question. To imitate the motions of the planets, in the above-mentioned orbits, Mercury must describe its own diameter in 41 seconds; Venus, in $4^m 14^s$; the Earth, in 7 minutes; Mars, in $4^m 48^s$; Jupiter, $2^h 56^m$; Saturn, in $3^h 13^m$; Uranus, in $2^h 16^m$; and Neptune, in $3^h 30^m$.²⁴

²⁴ In the "Penny Encyclopædia," vol. 22, p. 197, the diameters of the orbits of the planets here set down, are quoted as their *distances* from the centre, and the size of the sun is enlarged to four feet, while the sizes of the planets are unaltered.

CHAPTER X

OF THE SATELLITES

Of the Moon, as a Satellite of the Earth—General Proximity of Satellites to their Primaries, and Consequent Subordination of their Motions—Masses of the Primaries Concluded from the Periods of their Satellites—Maintenance of Kepler's Laws in the Secondary Systems—Of Jupiter's Satellites—Their Eclipses, etc.—Velocity of Light Discovered by their Means—Satellites of Saturn—Of Uranus—Of Neptune

(527.) IN the annual circuit of the earth about the sun, it is constantly attended by its satellite, the moon, which revolves round it, or rather both round their common centre of gravity; while this centre, strictly speaking, and not either of the two bodies thus connected, moves in an elliptic orbit, undisturbed by their mutual action, just as the centre of gravity of a large and small stone tied together and flung into the air describes a parabola as if it were a real material substance under the earth's attraction, while the stones circulate round it or round each other, as we choose to conceive the matter.

(528.) If we trace, therefore, the *real* curve actually described by either the moon's or the earth's centres, in virtue of this compound motion, it will appear to be, not an exact ellipse, but an undulated curve, like that represented in the figure to article 324, only that the number of undulations in a whole revolution is but 13, and their actual deviation from the general ellipse, which serves them as a central line, is comparatively very much smaller—so much so, indeed,

that every part of the curve described by either the earth or moon is *concave* toward the sun. The excursions of the earth on either side the ellipse, indeed, are so very small as to be hardly appreciable. In fact, the centre of gravity of the earth and moon lies always *within* the surface of the earth, so that the monthly orbit described by the earth's centre about the common centre of gravity is comprehended within a space less than the size of the earth itself. The effect *is*, nevertheless, sensible, in producing an apparent monthly displacement of the sun in longitude, of a parallactic kind, which is called the *menstrual equation*; whose greatest amount is, however, less than the sun's horizontal parallax, or than $8.6''$.

(529.) The moon, as we have seen, is about 60 radii of the earth distant from the centre of the latter. Its proximity, therefore, to its centre of attraction, thus estimated, is much greater than that of the planets to the sun; of which Mercury, the nearest, is 84, and Uranus 2026 solar radii from *its* centre. It is owing to this proximity that the moon remains attached to the earth as a satellite. Were it much further, the feebleness of its gravity toward the earth would be inadequate to produce that alternate acceleration and retardation in its motion about the sun, which divests it of the character of an independent planet, and keeps its movements subordinate to those of the earth. The one would outrun, or be left behind the other, in their revolutions round the sun (by reason of Kepler's third law), according to the relative dimensions of their heliocentric orbits, after which the whole influence of the earth would be confined to producing some considerable periodical disturbance in the moon's motion, as it passed or was passed by it in each synodical revolution.

(530.) At the distance at which the moon really is from us, its gravity toward the earth is actually less than toward the sun. That this is the case appears sufficiently from what we have already stated, that the moon's *real* path, even when between the earth and sun, is *concave toward the latter*. But it will appear still more clearly if, from the known periodic times¹ in which the earth completes its annual and the moon its monthly orbit, and from the dimensions of those orbits, we calculate the amount of deflection in either, from their tangents, in equal very minute portions of time, as one second. These are the versed sines of the arcs described in that time in the two orbits, and these are the measures of the acting forces which produce those deflections. If we execute the numerical calculation in the case before us, we shall find 2.233 : 1 for the proportion in which the intensity of the force which retains the earth in its orbit round the sun actually exceeds that by which the moon is retained in *its* orbit about the earth.

(531.) Now the sun is about 400 times more remote from the earth than the moon is. And, as gravity increases as the squares of the distances decrease, it must follow that at *equal* distances, the intensity of solar would exceed that of terrestrial gravity in the above proportion, augmented in the further ratio of the square of 400 to 1; that is, in the

¹ R and r radii of two orbits (supposed circular), P and p the periodic times; then the arcs in question (A and a) are to each other as $\frac{R}{P}$ to $\frac{r}{p}$; and since the versed sines are as the squares of the arcs directly and the radii inversely, these are to each other as $\frac{R}{P^2}$ to $\frac{r}{p^2}$; and in this ratio are the forces acting on the revolving bodies in either case.

proportion of 355,000 to 1; and therefore, if we grant that the intensity of the gravitating energy is commensurate with the mass or inertia of the attracting body, we are compelled to admit the mass of the earth to be no more than $\frac{1}{355000}$ of that of the sun.²

(532.) The argument is, in fact, nothing more than a recapitulation of what has been adduced in Chap. VIII (art. 448). But it is here re-introduced, in order to show how the mass of a planet which is attended by one or more satellites can be as it were weighed against the sun, provided we have learned, from observation, the dimensions of the orbits described by the planet about the sun, and by the satellites about the planet, and also the periods in which these orbits are respectively described. It is by this method that the masses of Jupiter, Saturn, Uranus, and Neptune have been ascertained, and from which their densities are concluded. (See art. 561.)

(533.) Jupiter, as already stated, is attended by four satellites; Saturn by eight; Uranus certainly by four; and Neptune by one, or possibly more. These, with their respective *primaries* (as the central planets are called) form in each case miniature systems entirely analogous, in the general laws by which their motions are governed, to the great system in which the sun acts the part of the primary, and the planets of its satellites. In each of these systems the laws of Kepler are obeyed, in the sense, that is to say, in which they are obeyed in the planetary system—approximately, and without prejudice to the effects of mutual per-

² In the synoptic table at the end of this volume, the mass of the sun is taken somewhat higher, according to the most recent determination. It has not been thought worth while to alter all the figures of the text in conformity with that estimate.

turbation, of extraneous interference, if any, and of that small but not imperceptible correction which arises from the elliptic form of the central body. Their orbits are circles or ellipses of very moderate excentricity, the primary occupying one focus. About this they describe areas very nearly proportional to the times; and the squares of the periodical times of all the satellites belonging to each planet are in proportion to each other as the cubes of their distances. The tables at the end of the volume exhibit a synoptic view of the distances and periods in these several systems, so far as they are at present known; and to all of them it will be observed that the same remark respecting their proximity to their primaries holds good, as in the case of the moon, with a similar reason for such close connection.

(534.) Of these systems, however, the only one which has been studied with attention to all its details, is that of Jupiter; partly on account of the conspicuous brilliancy of its four attendants, which are large enough to offer visible and measurable disks in telescopes of great power; but more for the sake of their eclipses, which, as they happen very frequently, and are easily observed, afford signals of considerable use for the determination of terrestrial longitudes (art. 286). This method, indeed, until thrown into the background by the greater facility and exactness now attainable by lunar observations (art. 287), was the best, or rather the only one which could be relied on for great distances and long intervals.

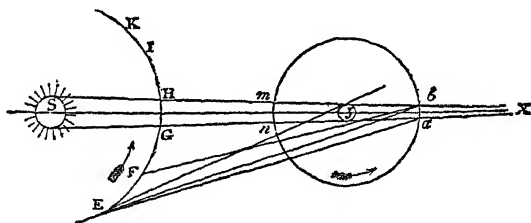
(535.) The satellites of Jupiter revolve from west to east (following the analogy of the planets and moon), in planes very nearly, although not exactly, coincident with that of the equator of the planet, or parallel to its belts. This latter

plane is inclined $3^{\circ} 5' 30''$ to the orbit of the planet, and is therefore but little different from the plane of the ecliptic. Accordingly, we see their orbits projected very nearly into straight lines, in which they appear to oscillate to and fro, sometimes passing before Jupiter, and casting shadows on his disk (which are very visible in good telescopes, like small round ink spots, the circular form of which is very evident), and sometimes disappearing behind the body, or being eclipsed in its shadow at a distance from it. It is by these eclipses that we are furnished with accurate data for the construction of tables of the satellites' motions, as well as with signals for determining differences of longitude.

(536.) The eclipses of the satellites, in their general conception, are perfectly analogous to those of the moon, but in their detail they differ in several particulars. Owing to the much greater distance of Jupiter from the sun, and its greater magnitude, the cone of its shadow or umbra (art. 420) is greatly more elongated, and of far greater dimensions, than that of the earth. The satellites are, moreover, much less in proportion to their primary, their orbits less inclined to *its* ecliptic, and (comparatively to the diameter of the planet) of smaller dimensions, than is the case with the moon. Owing to these causes, the three interior satellites of Jupiter pass through the shadow, and are totally eclipsed, every revolution; and the fourth, though, from the greater inclination of its orbit, it sometimes escapes eclipse, and *may* occasionally graze as it were the border of the shadow, and suffer partial eclipse, yet does so comparatively seldom, and, ordinarily speaking, its eclipses happen, like those of the rest, each revolution.

(537.) These eclipses, moreover, are not seen, as is the case with those of the moon, from the centre of their motion, but from a remote station, and one whose situation with respect to the line of shadow is variable. This, of course, makes no difference in the *times* of the eclipses, but a very great one in their visibility, and in their apparent situations with respect to the planet at the moments of their entering and quitting the shadow.

(538.) Suppose S to be the sun, E the earth in its orbit E F G K, J Jupiter, and $a b$ the orbit of one of its satellites. The cone of the shadow, then, will have its vertex at X, a point far beyond the orbits of all the satellites; and the penumbra, owing to the great distance of the sun, and the consequent smallness of the angle (about $6'$ only) its disk subtends at Jupiter, will hardly extend, within the limits of the satellites' orbits, to any perceptible distance beyond the



shadow—for which reason it is not represented in the figure. A satellite revolving from west to east (in the direction of the arrows) will be eclipsed when it enters the shadow at a , but not suddenly, because, like the moon, it has a considerable diameter seen from the planet; so that the time elapsing from the first perceptible loss of light to its total extinction will be that which it occupies in describing about Jupiter an angle equal to its apparent diameter as seen from the centre of the planet, or rather somewhat more, by

reason of the penumbra; and the same remark applies to its emergence at *b*. Now, owing to the difference of telescopes and of eyes, it is not possible to assign the *precise* moment of incipient obscuration, or of total extinction at *a*, nor that of the first glimpse of light falling on the satellite at *b*, or the complete recovery of its light. The observation of an eclipse, then, in which only the immersion, or only the emersion, is seen, is incomplete, and inadequate to afford any precise information, theoretical or practical. But, if both the immersion and emersion can be observed *with the same telescope and by the same person*, the interval of the times will give the duration, and their mean the exact middle of the eclipse, when the satellite is in the line *S J X*, *i.e.* the true moment of its opposition to the sun. Such observations, and such only, are of use for determining the periods and other particulars of the motions of the satellites, and for affording data of any material use for the calculation of terrestrial longitudes. The intervals of the eclipses, it will be observed, give the *synodic* periods of the satellites' revolutions; from which their sidereal periods must be concluded by the method in art. 418.

(539.) It is evident, from a mere inspection of our figure, that the eclipses take place to the west of the planet, when the earth is situated to the west of the line *S J*, *i.e.* before the opposition of Jupiter; and to the east, when in the other half of its orbit, or after the opposition. When the earth approaches the opposition, the visual line becomes more and more nearly coincident with the direction of the shadow, and the apparent place where the eclipses happen will be continually nearer and nearer to the body of the planet. When the earth comes to *F*, a point determined by drawing

b F to touch the body of the planet, the *emersions* will cease to be visible, and will thenceforth, up to the time of the opposition, happen *behind* the disk of the planet. Similarly, from the opposition till the time when the earth arrives at I, a point determined by drawing *a* I tangent to the eastern limb of Jupiter, the *immersions* will be concealed from our view. When the earth arrives at G (or H) the immersion (or emersion) will happen at the very edge of the visible disk, and when between G and H (a very small space), the satellites will *pass uneclipsed behind the limb* of the planet.

(540.) Both the satellites and their shadows are frequently observed to *transit* or pass across the disk of the planet. When a satellite comes to *m*, its shadow will be thrown on Jupiter, and will appear to move across it as a black spot till the satellite comes to *n*. But the satellite itself will not appear to enter on the disk till it comes up to the line drawn from E to the eastern edge of the disk, and will not leave it till it attains a similar line drawn to the western edge. It appears then that the shadow will *precede* the satellite in its progress over the disk *before* the opposition of Jupiter, and *vice versâ*. In these transits of the satellites, which, with very powerful telescopes, may be observed with great precision, it frequently happens that the satellite itself is discernible *on* the disk as a bright spot if projected on a dark belt; but occasionally also as a dark spot of smaller dimensions than the shadow. This curious fact (observed by Schröter and Harding) has led to a conclusion that certain of the satellites have occasionally *on their own bodies*, or in their atmospheres, obscure spots of great extent. We say of great extent; for the satellites of Jupiter, small as they appear to us, are really bodies

of considerable size, as the following comparative table will show:—³

	Mean apparent diameter as seen from the earth.	Mean apparent diameter as seen from Jupiter's centre.	Diameter in miles.	Mass.
Jupiter	39".91		91128	1·0000000
1st satellite	1·105	33' 11"	2508	0·0000173
2d ———	0·911	17 35	2068	0·0000232
3d ———	1·488	18 0	3277	0·0000885
4th ———	1·273	8 46	2890	0·0000427

From which it follows, that the first satellite appears, when on Jupiter's horizon, as large as our moon to us; the second and third nearly equal to each other, and of somewhat more than half the apparent diameter of the first, and the fourth about one quarter of that diameter. So seen, they will frequently, of course, eclipse one another, and cause eclipses of the sun (the latter visible, however, only over a very small portion of the planet), and their motions and aspects with respect to each other must offer a perpetual variety and singular and pleasing interest to the inhabitants of their primary.

(541.) Besides the eclipses and the transits of the satellites across the disk, they may also disappear to us when not eclipsed, by passing *behind* the body of the planet. Thus, when the earth is at E, the immersion of the satellite will be seen at *a*, and its emersion at *b*, both to the west of the planet, after which the satellite, still continuing its course in the direction *b*, will pass behind the body, and again emerge on the opposite side, after an interval of occultation greater or less according to the distance of the satellite. This interval (on account of the great distance of the earth

³ Struve, Mem. Art. Soc. iii. 301. Main. Do. xxv. p. 51.

⁴ Laplace, Mec. Cel. liv. viii. § 27.

compared with the radii of the orbits of the satellites) varies but little in the case of each satellite, being nearly equal to the time which the satellite requires to describe an arc of its orbit, equal to the angular diameter of Jupiter as seen from its centre, which time, for the several satellites, is as follows: viz. for the first, $2^h 20^m$; for the second, $2^h 56^m$; for the third, $3^h 43^m$; and for the fourth, $4^h 56^m$; the corresponding diameters of the planet as seen from these respective satellites being, $19^\circ 49'$; $12^\circ 25'$; $7^\circ 47'$; and $4^\circ 25'$.⁵ Before the opposition of Jupiter, these occultations of the satellites happen *after* the eclipses: after the opposition (when, for instance, the earth is in the situation K), the occultations take place before the eclipses. It is to be observed, that, owing to the proximity of the orbits of the first and second satellites to the planet, *both* the immersion and emersion of either of them can never be observed in any single eclipse, the immersion being concealed by the body, if the planet be past its opposition, the emersion if not yet arrived at it. So also of the occultation. The commencement of the occultation, or the passage of the satellite behind the disk, takes place while obscured by the shadow, before opposition, and its re-emergence after. All these particulars will be easily apparent on mere inspection of the figure (art. 536). It is only during the short time that the earth is in the arc G H, *i.e.* between the sun and Jupiter, that the cone of the shadow converging (while that of the visual rays diverges) behind the planet, permits their occultations to be completely observed both at ingress and egress, unobscured, the eclipses being then invisible.

⁵ These data are taken approximately from Mr. Woolhouse's paper in the supplement to the Nautical Almanack for 1835.

(542.) An extremely singular relation subsists between the mean angular velocities or *mean motions* (as they are termed) of the three first satellites of Jupiter. If the mean angular velocity of the first satellite be added to twice that of the third, the sum will equal three times that of the second. From this relation it follows, that if from the mean longitude of the first, added to twice that of the third, be subducted three times that of the second, the remainder will always be the same, or constant, and observation informs us that this constant is 180° , or two right angles; so that the situations of any two of them being given, that of the third may be found. It has been attempted to account for this remarkable fact, on the theory of gravity by their mutual action; and Laplace has demonstrated, that if this relation were at any one epoch approximately true, the mutual attractions of the satellites would, in process of time, render it exactly so. One curious consequence is, that these three satellites cannot be all eclipsed at once; for, in consequence of the last-mentioned relation, when the second and third lie in the *same* direction from the centre, the first must lie on the *opposite*; and therefore, when at such a conjunction the first is eclipsed, the other two must lie between the sun and planet, throwing their shadows on the disk, and *vice versa*.

(543.) Although, however, for the above-mentioned reason, the satellites cannot be all *eclipsed* at once, yet it may happen, and occasionally does so, that all are either eclipsed, occulted, or projected on the body, in which case they are, generally speaking, equally invisible, since it requires an excellent telescope to discern a satellite on the body, except in peculiar circumstances. Instances of the actual observation of Jupiter thus denuded of its usual attendance and offering the appearance of a solitary disk, though rare, have

been more than once recorded. The first occasion in which this was noticed was by Molyneux, on November 2 (old style), 1681.⁶ A similar observation is recorded by Sir W. Herschel as made by him on May 23, 1802. The phenomenon has also been observed by Mr. Wallis, on April 15, 1826 (in which case the deprivation continued two whole hours); and lastly by Mr. H. Griesbach, on September 27, 1843.

(544.) The discovery of Jupiter's satellites, one of the first fruits of the invention of the telescope, and of Galileo's early and happy idea of directing its new-found powers to the examination of the heavens, forms one of the most memorable epochs in the history of astronomy. The first astronomical solution of the great problem of "*the longitude*"—practically the most important for the interests of mankind which has ever been brought under the dominion of strict scientific principles, dates immediately from their discovery. The final and conclusive establishment of the Copernican system of astronomy may also be considered as referable to the discovery and study of this exquisite miniature system, in which the laws of the planetary motions, as ascertained by Kepler, and especially that which connects their periods and distances, were speedily traced, and found to be satisfactorily maintained. And (as if to accumulate historical interest on this point), it is to the observation of their eclipses that we owe the grand discovery of the successive propagation of light, and the determination of the enormous velocity of that wonderful element. This we must explain now at large.

(545.) The earth's orbit being concentric with that of Jupiter and interior to it (see *fig.* art. 536), their mutual

⁶ Molyneux, *Optics*, p. 271.

distance is continually varying, the variation extending from the *sum* to the *difference* of the radii of the two orbits; and the difference of the greater and least distances being equal to a diameter of the earth's orbit. Now, it was observed by Roemer (a Danish astronomer, in 1675), on comparing together observations of eclipses of the satellites during many successive years, that the eclipses at and about the opposition of Jupiter (or its nearest point to the earth) took place *too soon*—sooner, that is, than, by calculation from an average, he expected them; whereas those which happened when the earth was in the part of its orbit most remote from Jupiter were always *too late*. Connecting the observed error in their computed times with the variation of distance, he concluded, that, to make the calculation on an average period correspond with fact, an allowance in respect of time behooved to be made proportional to the excess or defect of Jupiter's distance from the earth above or below its average amount, and such that a difference of distance of one diameter of the earth's orbit should correspond to $16^m\ 26^s\cdot6$ of time allowed. Speculating on the probable physical cause, he was naturally led to think of a gradual instead of an instantaneous propagation of light. This explained every particular of the observed phenomenon, but the velocity required (192,000 miles per second) was so great as to startle many, and, at all events, to require confirmation. This has been afforded since, and of the most unequivocal kind, by Bradley's discovery of the aberration of light (art. 329). The velocity of light deduced from this last phenomenon differs by less than one-eightieth of its amount from that calculated from the eclipses, and even this difference will no doubt be destroyed by nicer and more rigorously reduced observations. The velocity

has also been determined by M. Fizeau (by direct experiments with a reflecting apparatus on a most ingenious principle, suggested by Mr. Wheatstone for measuring the velocity of the electric current) at 70,000 geographical leagues, 25 to the degree=194,600 statute miles per second.

(546.) The orbits of Jupiter's satellites are but little excentric; those of the two interior, indeed, have no perceptible excentricity. Their mutual action produces in them perturbations analogous to those of the planets about the sun, and which have been diligently investigated by Laplace and others. By assiduous observation it has been ascertained that they are subject to marked fluctuations in respect of brightness, and that these fluctuations happen periodically, according to their position with respect to the sun. From this it has been concluded, apparently with reason, that they turn on their axes, like our moon, in periods equal to their respective sidereal revolutions about their primary.

(547.) The satellites of Saturn have been much less studied than those of Jupiter, being far more difficult to observe. The most distant has its orbit materially inclined (no less than $12^{\circ} 14'$) to the plane of the ring, with which the orbits of all the rest nearly coincide. Nor is this the only circumstance which separates it by a marked difference of character from the system of the seven inferior ones, and renders it in some sort an anomalous member of the Saturnian system. Its distance from the planet's centre is no less than 64 times the radius of the globe of Saturn, a distance from the primary to which our own moon (at 60 radii)

[†] Lalande, *Astron.*, Art. 3075.

offers the only parallel. Its variation of light also in different parts of its orbit is very much greater than in the case of any other secondary planet. Dominic Cassini indeed (its first discoverer, A.D. 1671) found it to disappear for nearly half its revolution when to the east of Saturn, and though the more powerful telescopes now in use enable us to follow it round the whole of its circuit, its diminution of light is so great in the eastern half of its orbit as to render it somewhat difficult to perceive. From this circumstance (*viz.* from the defalcation of light occurring constantly on the same side of Saturn *as seen from the earth*, the visual ray from which is never very oblique to the direction in which the sun's light falls on it) it is presumed, with much certainty, that this satellite revolves on its axis in the exact time of rotation about the primary; as we know to be the case with the moon, and as there is considerable ground for believing to be so with all secondaries.

(548.) The next satellite in order, proceeding inward (as it used to be considered until the recent discovery of an intermediate one), was the first to be detected.⁸ It is by far the largest and most conspicuous of all, and is probably not much inferior to Mercury in size. It is the only one of the number whose theory and perturbations have been at all inquired into⁹ further than to verify Kepler's law of the periodic times, which holds good, *mutatis mutandis*, and under the requisite reservations, in this, as in the system of Jupiter. The next three satellites, still proceeding inward,¹⁰ are very minute, and require pretty powerful telescopes to see them; while the two interior satellites, which

⁸ By Huyghens, March 25, 1655.

⁹ By Bessel, *Astr. Nachr.* Nos. 193, 214.

¹⁰ Discovered by Dominic Cassini in 1672 and 1684.

just skirt the edge of the ring,¹¹ can only be seen with telescopes of extraordinary power and perfection, and under the most favorable atmospheric circumstances. At the epoch of their discovery they were seen to thread, like beads, the almost infinitely thin fibre of light to which the ring, then seen edgewise, was reduced, and for a short time to advance off it at either end, speedily to return, and hastening to their habitual concealment behind or on the body. An eighth very faint satellite has been recently detected (between the two outermost of the *old* satellites) simultaneously (within the same hour) by Mr. Dawes, Mr. Lassell, and Professor Bond,¹² the two former observing together in Mr. Lassell's observatory at Starfield, the latter in that of Cambridge, U. S.

(549.) Owing to the obliquity of the ring and of the orbits of the satellites to Saturn's ecliptic, there are no eclipses,

¹¹ Discovered by Sir William Herschel in 1789.

¹² On the night of the 19th of September, 1848. Considerable confusion used already to prevail, before the discovery of this satellite, in the nomenclature of the saturnian system, owing to the order of discovery not coinciding with that of distances. Astronomers were not agreed whether to call the two interior satellites the 6th and 7th (reckoning inward) and the older ones the 1st, 2d, 3d, 4th and 5th, reckoning outward; or to commence with the innermost and reckon outward, from 1 to 7. This confusion, which the introduction of an eighth would have rendered intolerable, has been obviated by a mythological nomenclature, suggested in a former edition of this work, and which has been generally accepted, in consonance with that at length completely established for the primary planets. Taking the names of the Titanian divinities, the following verses (pardoning false quantities) afford an easy artificial memory.

Iapetus cunctos supra rotat, huncce sequuntur
Hyperion, Titan, Rhêa, Dionê, Têthys,
Enceladus, Mimas—

It is worth remarking that Simon Marius, who disputed the priority of the discovery of Jupiter's satellites with Galileo, proposed for them mythological names, viz. Io, Europa, Ganymede and Callisto. The revival of these names would savor of a preference of Marius's claim, which, even if an absolute priority were conceded (which it is not), would still leave Galileo's general claim to the use of the telescope as a means of astronomical discovery intact. But in the case of Jupiter's satellites there exists no confusion to rectify. They are constantly referred to by their numerical designations in every almanac.

occultations, or transits of these bodies or their shadows across the disk of their primary (the interior ones excepted), until near the time when the ring is seen edgewise, and when they do take place their observation is attended with too much difficulty to be of any practical use, like the eclipses of Jupiter's satellites for the determination of longitudes, for which reason they have been hitherto little attended to by astronomers.

(550.) A remarkable relation subsists between the periodic times of the two interior satellites of Saturn and those of the two next in order of distance; viz. that the period of the third (Tethys) is double that of the first (Mimas), and that of the fourth (Dione) double that of the second (Enceladus). The coincidence is exact in either case to about one 800th part of the larger period.

(551.) The satellites of Uranus require very powerful and perfect telescopes for their observation. Four are certainly known to exist, to which (proceeding from without, inward in succession) the names Oberon, Titania, Umbriel, and Ariel, of the fairies, sylphs, and gnomes of Shakespeare and Pope, have been assigned respectively. Of these Oberon and Titania are tolerably conspicuous in a reflecting telescope of 18 or 20 inches in aperture. They were discovered by Sir W. Herschel in 1787, and have since been reobserved by the author of this work, and subsequently by Messrs. Lassell, Otto Struve, and Lamont. Umbriel (a much fainter object) was also very probably seen by Sir W. Herschel, and described by him as "an interior satellite," but his observations of it were not sufficiently numerous and precise to place its existence, at that time, beyond question. It was rediscovered, however, by M. Otto Struve,¹³ and observed

¹³ October 8, 1847.

subsequently, on numerous occasions, by Mr. Lassell, to whom we also owe the first discovery of Ariel,¹⁴ as well as a fine series of observations and micrometrical measures of all four, obtained at his observatory at Liverpool and during his residence in Malta in 1852-53, which forms a remarkable epoch in the history of astronomical observation. Three other satellites, one intermediate between Oberon and Titania, the others exterior to both, were suspected by Sir W. Herschel, but their existence has not been confirmed. The periods and distances of the four known satellites will be found in the synoptic table at the end of the volume.

(552.) The orbits of these satellites offer remarkable, and, indeed, quite unexpected and unexampled peculiarities. Contrary to the unbroken analogy of the whole planetary system—whether of primaries or secondaries—the planes of their orbits *are nearly perpendicular to the ecliptic*, being inclined no less than $78^{\circ} 58'$ to that plane, and in these orbits their motions *are retrograde*; that is to say, their positions, when projected on the ecliptic, instead of advancing *from west to east* round the centre of their primary, as is the case with every other planet and satellite, move in the opposite direction. Their orbits are nearly or quite circular, and they do not appear to have any sensible, or, at least, any rapid motion of nodes, or to have undergone any material change of inclination, in the course, at least, of half a revolution of their primary round the sun. When the earth is in the plane of their orbits, or nearly so, their apparent paths are straight lines or very elongated ellipses, in which case they become invisible, their feeble light being effaced by the superior light of the planet, long before they

¹⁴ September 14, 1847.

come up to its disk, so that the observations of any eclipses or occultations they may undergo is quite out of the question with our present telescopes.

(553.) If the observation of the satellites of Uranus be difficult, those of Neptune, owing to the immense distance of that planet, may be readily imagined to offer still greater difficulties. Of the existence of two, discovered by Mr. Lassell,¹⁵ there can remain no doubt, having also been observed by other astronomers, both in Europe and America. According to M. Otto Struve¹⁶ the orbit of the first is inclined to the ecliptic at the considerable angle of 35° ; but whether, as in the case of the satellites of Uranus, the direction of its motion be retrograde it is not possible to say until it shall have been longer observed.

¹⁵ On July 7, 1847 (suspected in 1846), and August 14, 1850.

¹⁶ *Astron. Nachr.* No. 629, from his own observation, September 11 to December 20, 1847.

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